Resource Allocation Algorithms Supporting Coexistence of Cognitive Vehicular and IEEE 802.22 Networks

You Han, *Student Member, IEEE*, Eylem Ekici, *Fellow, IEEE*, Haris Kremo, *Member, IEEE*, and Onur Altintas, *Member, IEEE*

***Abstract*— Many studies show that the dedicated short range communication (DSRC) band is insufficient to carry increasing wireless traffic demands in vehicular networks. The release of TV white space band by the Federal Communications Commission (FCC) for cognitive access provides additional bandwidth to solve the DSRC spectrum scarcity problem. However, FCC requires portable devices to use significantly lower transmitting power than fixed devices, which creates a challenging coexistence environment for portable (e.g., vehicular) and fixed (e.g., IEEE 802.22) networks. In this paper, we address the coexistence problem between a vehicular and an 802.22 network via resource allocation. We first formulate the coexistence problem as a mixed-integer nonlinear programming (MINLP) problem, to which three algorithms are developed. The first algorithm converts the MINLP into a convex program and obtains a near-optimal solution to the initial MINLP. In the other two algorithms, we first convert the MINLP into an integer programming (IP) problem. Then, we solve the linear program relaxation of the IP and obtain a fractional solution. Thereafter, two rounding algorithms are developed to round the fractional solution based on column-sparse packing and dependent rounding techniques, respectively. Finally, we compare the performance of the proposed algorithms with an optimal MINLP solver through numerical examples.**

***Index Terms*— Cognitive vehicular networks, channel allocation, linear programming, submodular set function.**

I. Introduction

The Dedicated Short Range Communication (DSRC) band with 75 MHz bandwidth was allocated to vehicular communications in the U.S. more than a decade ago. Then, the IEEE Wireless Access in Vehicular Environment standard stack (e.g., including IEEE 802.11p and IEEE 1609.4) was developed to support vehicular communications in the DSRC band. However, both theoretical analysis [1] and simulation results [1], [2] demonstrate that the DSRC band is insufficient to support reliable safety message transmissions. Moreover, some studies (e.g., [3]) show that non-safety use of the DSRC band has to be severely restricted during peak hours of traffic to guarantee reliable transmission of safety messages. More importantly, the spectrum scarcity problem is becoming severer due to the growth of radio-equipped vehicles as well as wireless vehicular applications, such as collision avoidance, safety warning, remote vehicle diagnostic, file downloading, web browsing, and video streaming [4].

A potential solution to address the DSRC spectrum scarcity problem is to use the TV White Space (TVWS) band that has been released by the Federal Communications Commission (FCC) for cognitive access in the U.S. [5], [6]. Although the TVWS band is a promising spectrum expansion for vehicular networks, it also raises novel challenges, one of which is the power asymmetry policy for fixed and portable devices. Specifically, since portable TVWS devices are required to use at most 100 mW transmit power while fixed devices can use up to 4 W transmit power [7], the 802.22 network can easily starve vehicular networks due to lack of coordination between the two networks. To the best of our knowledge, this is the first work dealing with the coexistence issue between an IEEE 802.22 and a cognitive vehicular network (CVN). Most existing works on cognitive radio are dedicated to the coexistence between Secondary User (SU) networks and Primary User (PU) networks (e.g., [2], [3], [8], [9]) while the coexistence of heterogeneous SU networks has not received many attentions. A critical difference between PU-SU coexistence and SU-SU coexistence is that PU-SU coexistence only considers the protection of PUs, while the SU-SU coexistence needs to consider not only the protection of PUs, but also the mutual interference among SU networks.

Although IEEE 802.22 includes an inter-network coexistence scheme, it is dedicated to the coexistence of multiple IEEE 802.22 networks. In addition, the IEEE 802.19.1 standard proposes a framework for the coexistence of heterogeneous SU networks in the TVWS band. However, it heavily relies on coexistence entities, which makes it too expensive for implementation. Although some coexistence mechanisms have been proposed to enable the coexistence of heterogeneous networks in the ISM band [10], [11], the proposed methods depends on the specific network type and include no cognitive functionality. Finally, the novelty of our work also lies in that we consider specific PHY and MAC properties of the

Manuscript received January 11, 2016; revised May 12, 2016 and September 7, 2016; accepted November 20, 2016. Date of publication December 7, 2016; date of current version February 9, 2017. The associate editor coordinating the review of this paper and approving it for publication was B. Hamdaoui.

Y. Han and E. Ekici are with the Department of Electrical and Computer Engineering, The Ohio State University, Columbus, OH 43210 USA (e-mail: han.639@osu.edu ekici@ece.osu.edu).

H. Kremo was with the Toyota InfoTechnology Center, Co., Ltd., 6-6-20 Tokyo, Japan. He is now with the CONNECT Center, Trinity College Dublin, Dublin 2, Ireland (e-mail: kremoh@tcd.ie).

O. Altintas is with Toyota InfoTechnology Center, USA, Inc., Mountain View, CA 94043 USA (e-mail: onur@us.toyota-itc.com).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TWC.2016.2636280

IEEE 802.22 standard and FCC's rules on TVWS access, which makes our work more relevant for implementation.

In this paper, we propose an 802.22-CVN coexistence framework defined as follows. In an 802.22 network, the upstream scheduling information of 802.22 Consumer Premise Equipment (CPE) devices is included in the downstream messages that are periodically broadcast by the 802.22 base station (BS). After receiving the downstream messages, the CPE devices access the TVWS channel according to the upstream scheduling information. Since the vehicular network is deployed in the same area as the 802.22 network, the vehicles can also obtain the 802.22 upstream scheduling information by listening to the downstream messages from the BS. As an add-on to the high-power 802.22 network, the low-power vehicular network coexists with the 802.22 network through reusing the same TVWS channel via adaptive resource allocation based on the 802.22 upstream scheduling information. Since the coexistence is solely performed in the vehicular network, no changes are required to the existing 802.22 standard. More importantly, the vehicular and the 802.22 networks are still separately managed and no additional signaling overhead will be generated between the two networks.

The theoretical contributions of our work are twofold. Firstly, we formulate the coexistence problem between a CVN and an IEEE 802.22 network as a resource allocation problem, which belongs to NP-complete Mixed-Integer Nonlinear Programming (MINLP) problems. Secondly, since MINLP problems are usually intractable, our second contribution is the development of three algorithms with provable performance guarantees. Note that the resource allocation problem studied in our work bears similarities with the conventional joint power control and channel allocation problem in wireless communication networks. Since such problems usually belong to NP-complete MINLP problems, most existing works only propose heuristic or greedy algorithms without overall performance guarantees while only a few works propose algorithms with certain performance guarantees (e.g., [12]–[14]). Although an approximation algorithm with performance guarantees is developed in [12], the approximation ratio is quite low. In addition, the dual methods used in [13] and [14] are proved to achieve the optimum of the resource allocation problems, but their time complexity is too high due to slow convergence of the sub-gradient algorithms used in these methods. More importantly, the algorithms proposed in [12]–[14] are not suitable to solve our 802.22-CVN coexistence problem because they do not consider the PHY/MAC layer features of the IEEE 802.22 standard and FCC's regulations on the protection of PUs in the TVWS band. Hence, development of efficient algorithms with provable performance guarantees for this class of problems is still an open research issue. Our problem formulation is more general than the standard joint power control and channel allocation problem as we consider multiple sources of interference in our constraints as shown in our formulated problem (P.1). As such, the three algorithms we develop can also solve the standard joint power control and channel allocation problems as their special cases.

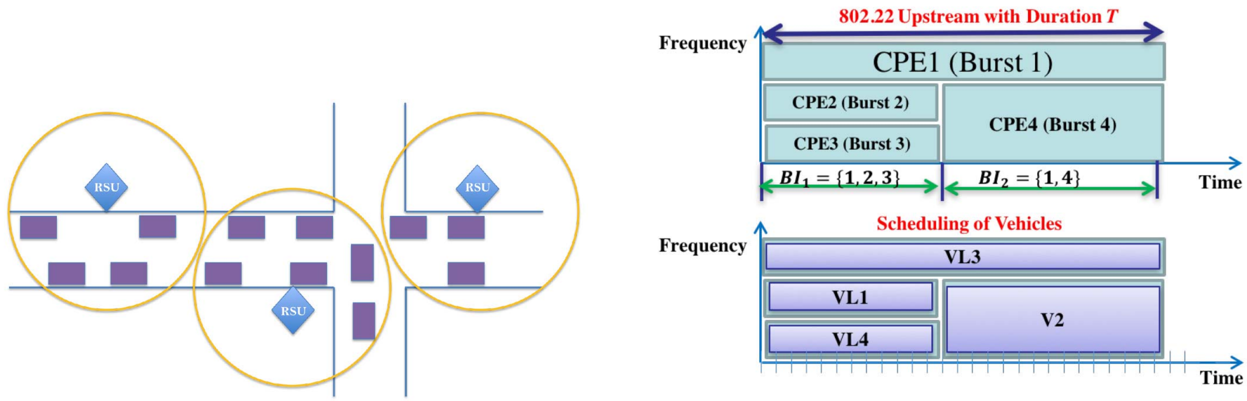
The remainder of the paper is organized as follows. The system model and problem formulation are described in Section II. In Section III, we propose a dual method and analyze its performance. In Section IV, the coexistence problem is reformulated as an integer program, and two algorithms are proposed to solve the integer program in Section IV and V, respectively. We present numerical results in Section VI, and conclude our work in Section VII.

II. System Model and Problem Formulation

The IEEE 802.22 standard was developed to provide broadband access in low population density areas using the TVWS band. Every 802.22 network consists of one Base Station (BS) and multiple CPEs. According to the standard, an MAC layer frame is 10 milliseconds long. The frame is dynamically partitioned into a downstream sub-frame and an upstream sub-frame by the BS. Upstream scheduling information (e.g., power and burst assignment) is included in downstream messages, which is broadcast by the BS every 10 milliseconds.

Similar to existing works on cognitive vehicular networks [2], [3], [15] as well as the IEEE 802.22 standard, we consider a centralized and periodic scheduling model where the resource allocation in a vehicular network is performed by a Road Side Unit (RSU). As shown in Fig. (1a), a road is divided into segments with an RSU in each segment. Whenever a vehicle moves from a segment to another segment, it must register with the RSU in the new segment to send resource allocation requests and obtain scheduling results. For example, if the average speed of a vehicle is 20 m/s and average transmission range of RSUs is 500 meters, then the vehicle needs to register with RSUs every 50 seconds. At the beginning of each scheduling period, the RSU listens to the downstream messages from the 802.22 base station, performs the resource allocation, and broadcasts the scheduling information to the vehicles in its segment. In IEEE 1609.4 (a standard for vehicular communications in the DSRC band), the MAC layer scheduling period (i.e., length of MAC layer frame) is 50 milliseconds. Similarly, we also use a periodic scheduling model, where the scheduling period is 10 milliseconds to be consistent with the IEEE 802.22 standard.

Our objective is to schedule vehicles to 802.22 upstream sub-frames to maximize the total weighted throughput of the vehicular network. Let N be the number vehicles and M be the number of CPE bursts. In 802.22, a "burst" is defined as a two-dimensional segment of OFDM sub-carriers (frequency domain) and symbols (time domain). For example, there are four bursts allocated to four CPEs in Fig. (1b). According to the standard, there are two types of upstream bursts. Type 1 burst is mapped over the full upstream sub-frame in the time domain (e.g., burst 1 in Fig. (1b)) while a normal type 2 burst is mapped over an interval with 7 upstream slots (e.g., burst 2 and 3 in Fig. (1b)). But if a type 2 burst is the last burst of an upstream, its duration can be 7 to 13 slots (e.g., burst 4 in Fig. (1b)) [7]. Based on the length of a normal type 2 burst, the upstream sub-frame is split into "burst intervals" (BIs) in the time domain, and let L



(a) System Model (b) Problem Formulation

Fig. 1. System model and problem formulation.

be the number of BIs in the upstream sub-frame. According IEEE 802.22, $L \in \{1, 2, 3, 4\}$ [7]. For example, there are two BIs in Fig. (1b). Let $C_{jl} \in \{0, 1\}$ be a BI indicator, and $C_{jl} = 1$ if burst j belongs to BI_l. Moreover, FCC requires that total transmit power of portable SUs on a single TVWS channel must be less than a threshold denoted as P_{max} (currently being 100 mW [5]). Since multiple vehicles share one channel, the total transmit power of vehicles must be less than the threshold in every BI. For example, in Fig. (1b), both total transmit power of $V1, V3, V4$, and total transmit power of $V2, V3$ must be less than P_{max}. Let T_j and B_j be the duration and bandwidth of burst j, respectively. Let p_j^c be the transmit power of CPE j on burst j, and G_{ji} be the channel gain from CPE j to vehicle i. We assume that all channel gain information can be obtained by either vehicles' measurements or from the BS. In IEEE 802.11p, packets are classified into four Access Categories with decreasing priority: $AC[0] \cdots AC[3]$, so we associate each priority class $AC[p]$ with a weight A_p such that higher priority packets have larger weights. The maximum transmit rate of vehicle i on burst j is determined by its transmit power and the transmit power of CPE j, i.e., $R_{ij} = B_j \log_2\left(1 + \frac{p_{ij}G_{ii}}{p_j^c G_{ji} + N_0}\right)$, where p_{ij}, G_{ii} denote transmit power of vehicle i on burst j and channel gain from vehicle i to its receiver, and N_0 is the noise power spectrum density.

PUs' occupancy of a TVWS channel is modeled via a random variable t_p, which is defined as residual time until the return of a PU to the channel. We assume that probability density function (PDF) $f_j(t_p)$ and cumulative density function (CDF) $F_j(t_p)$ of t_p is known by RSUs. Since exact behaviors of PUs are unknown, vehicular transmissions scheduled on a burst can be interrupted by PU transmissions with non-zero probabilities. In our model, transmissions of a vehicle before a PU returns are assumed to be successful and all remaining packets are assumed to be lost (one possible reason is that high transmit power of PUs blocks vehicular transmissions). Therefore, let t_j be the time between the starting time of burst j and the starting time of the current upstream sub-frame. If PUs return to the burst after the vehicle finishes its transmission, the valid transmission time of the vehicle is T_j (i.e., $T_j \cdot 1_{\{t_p \geq t_j + T_j\}}$ in the first equation of Equation (1), as shown at the bottom of this page). However, if the PUs return to the burst during the vehicle's transmission (e.g., at time t_p), then the valid transmission time of the vehicle is only $t_p - t_j$ (i.e., $(t_p - t_j) \cdot 1_{\{t_j \leq t_p \leq t_j + T_j)\}}$ in the first equation of Equation (1)). Since t_p is a random variable, the expected transmission time of a vehicle on burst j can be computed in Equation (1). Since t_j, T_j and $F(t_p)$ are all known at the RSUs, and thus $\bar{T_j}$ is a constant.

Given the above definitions, our objective is to maximize the weighted expected throughput (interchangeable with "utility") of vehicles via joint burst allocation and power control. Let $x_{ij} \in \{0, 1\}, \forall i, j$ be assignment variables (i.e., $x_{ij} = 1$ if vehicle i is scheduled to burst j, and $x_{ij} = 0$ otherwise), and the problem can be formulated as

$$\max_{\substack{x_{ij} \in \{0,1\} \\ p_{ij} \geq 0}} \sum_{i=1}^{N} \sum_{j=1}^{M} \frac{1}{T} A_i x_{ij} R_{ij} \bar{T_j}$$

$$\textbf{s.t.: } C1: \sum_{i=1}^{N} x_{ij} p_{ij} G_i^{BS} \leq \beta_j, \quad \forall j \in \{1, 2 \cdots M\},$$

$$\begin{aligned}
\bar{T_j} &= \int_0^\infty (T_j \cdot 1_{\{t_p \geq t_j + T_j\}} + (t_p - t_j) \cdot 1_{\{t_j \leq t_p \leq t_j + T_j)\}}) f(t_p) dt_p \\
&= \int_{t_j + T_j}^\infty T_j \cdot f(t_p) dt_p + \int_{t_j}^{t_j + T_j} (t_p - t_j) f(t_p) dt_p = T_j[1 - F(t_j + T_j)] + \int_{t_j}^{t_j + T_j} (t_p - t_j) dF(t_p) \\
&= T_j[1 - F(t_j + T_j)] + T_j F(t_j + T_j) - \int_{t_j}^{t_j + T_j} F(t_p) dt_p = T_j - \int_{t_j}^{t_j + T_j} F(t_p) dt_p. \qquad (1)
\end{aligned}$$

$$C2: \sum_{i=1}^{N}\sum_{j=1}^{M} C_{jl}x_{ij}p_{ij} \le P_{max}, \forall l \in \{1,2\cdots L\},$$
$$C3: \sum_{j=1}^{M} x_{ij} \le 1, \quad \forall i \in \{1,2\cdots N\},$$
$$C4: \sum_{i=1}^{N} x_{ij} \le 1, \quad \forall j \in \{1,2\cdots M\}, \quad \text{(P.1)}$$

where G_i^{BS} denotes the channel gain from vehicle i to the 802.22 BS, and T represents the total length of the current upstream sub-frame. $C1$ means that the interference caused by vehicular transmissions on the upstream transmission of CPE j cannot be larger than β_j. $C2$ is to meet FCC's requirement that total transmit power of SUs on a TVWS channel must be less than a threshold. $C3$ constraints mean that a vehicle can be scheduled to at most one burst for fairness among vehicles, and $C4$ constraints mean that a burst can be be assigned to at most one vehicle to avoid mutual interference.

The 802.22-CVN coexistence problem (P.1) is different from the conventional PU-SU coexistence problem as follows. Firstly, the PU-SU coexistence problem is usually formulated to maximize SU network throughput subject to PU protection [16]–[18]. In addition to PU protection, we also guarantee that the low-power CVN safely coexists with the high-power 802.22 network. More importantly, we consider specific network types such that we have to consider PHY/MAC features of the CVN and 802.22 network, while most existing works like [16]–[18] only consider general PU and SU networks. Secondly, the 802.22-CVN coexistence problem is faced with novel technical difficulties as follows. Firstly, in the PU-SU coexistence problem, throughput at a SU i is usually computed as $R_i \times \lambda_i(t) \times P_{off}(t)$, where R_i is a rate constant, $\lambda_i(t) \in \{0,1\}$ denotes its channel access decision at slot t, and $P_{off}(t) \in [0,1]$ represents the probability that PUs are not using the channel at slot t [16]–[18]. We can see that this throughput function only has binary decision variable $\lambda_i(t)$, and is linear over $\lambda_i(t)$. In contrast, the throughput function in our work has both binary decision variable x_{ij} and continuous decision variable $p_{ij} \ge 0$, and is nonlinear over these decision variables. Secondly, existing PU-SU coexistence problems are usually formulated as decentralized spectrum sensing problem [17], cooperative spectrum sensing problem [16], or power control problem [18], while our 802.22-CVN coexistence problem is a joint power control and burst allocation problem.

We can see that (P.1) belongs to MINLP problems that are generally NP-complete [19]. Hence, these problems are usually solved using numerical algorithms such as branch and bound, Bender's decomposition, outer approximation and extended cutting plane [19]. Although most of these solutions are guaranteed to attain near-optimal solutions, they have exponential time complexity. In the following three sections, we propose three efficient algorithms with provable performance guarantees and more favorable time complexity.

III. Dual Method

The difficulty of MINLP problems results from integer constraints and non-convexity. Given this observation, our dual method works as follows. Firstly, all integer constraints of (P.1) are relaxed. Next, the problem is converted to a convex program by replacing variables p_{ij} with $y_{ij} = x_{ij}p_{ij}, \forall i, j$. Then, we propose a dual algorithm to solve the convex program. Specifically, we first optimize y_{ij} variables given the assignment and dual variables, which enables us to obtain the optimal power allocation policy (denoted by $p_{ij}^*, \forall i, j$). Then, we plug $p_{ij}^*, \forall i, j$ back to the Lagrangian function and optimize assignment variables $x_{ij}, \forall i, j$, which turns out to be the linear programming relaxation of the classic assignment problem. The last step is to optimize dual variables, which is proved to be a non-smooth convex optimization problem. Using an accelerated sub-gradient method [20] enables us to obtain a near-optimal solution to the dual problem. Since the assignment variables are guaranteed to be integers, the obtained solution is also a near-optimal solution to (P.1).

A. Convert (P.1) to a Convex Program

By relaxing the integer constraints and replacing power variables p_{ij} with $y_{ij} = p_{ij}x_{ij}, \forall i, j$ in (P.1), we have

$$\max_{\substack{x_{ij}\in[0,1]\\ y_{ij}\ge 0}} \sum_{i=1}^{N}\sum_{j=1}^{M} \frac{1}{T} A_i x_{ij} R'_{ij} \bar{T}_j$$
$$\textbf{s.t.:} \sum_{i=1}^{N} y_{ij} G_i^{BS} \le \beta_j, \quad \forall j; \sum_{i=1}^{N}\sum_{j=1}^{M} C_{jl} y_{ij} \le P_{max}, \ \forall l,$$
$$\sum_{j=1}^{M} x_{ij} \le 1, \quad \forall i; \sum_{i=1}^{N} x_{ij} \le 1, \ \forall j, \quad \text{(P.2)}$$

where $R'_{ij} = 0$ if $x_{ij} = 0$ and $R'_{ij} = B_j \log_2\left(1 + \frac{y_{ij}G_{ii}}{x_{ij}(p_j^c G_{ji} + N_0)}\right)$ if $x_{ij} > 0$.

Lemma 1: (P.2) is a convex program without duality gap.

Proof: Firstly, domain of (P.2) (i.e., $x_{ij} \in [0,1]$, $y_{ij} \ge 0, \forall i, j$) forms a convex set. Secondly, all functions in the constraints are linear (thus convex). Finally, we show the objective function is concave. Since the objective function is sum of single utility functions $f_{ij}(x_{ij}, y_{ij}) = \frac{1}{T} A_i x_{ij} R'_{ij} \bar{T}_j$, we only need to show $f_{ij}(x_{ij}, y_{ij})$ is concave. Since $f_{ij}(x_{ij}, y_{ij}) = 0$ at $x_{ij} = 0$, we only need to consider $x_{ij} > 0$. In this case, $f_{ij}(x_{ij}, y_{ij})$ is the perspective function of a concave function $g_{ij}(y_{ij}) = \frac{1}{T} A_i \bar{T}_j B_j \log_2\left(1 + \frac{y_{ij}G_{ii}}{p_j^c G_{ji} + N_0}\right)$. Therefore, $f_{ij}(x_{ij}, y_{ij})$ is also concave. Next, we show that (P.2) has no duality gap by proving it satisfies Slater's condition. Let $x_{ij} = 0, y_{ij} = 0, \forall i, j$ and we obtain strict inequalities in all the constraints, and thus the Slater's condition holds. □

Next, we compute Lagrangian function of (P.2) by dualizing its $C1$ and $C2$ constraints

$$\begin{aligned} L(\vec{x}, \vec{y}, \vec{u}, \vec{v}) = &\sum_{i=1}^{N}\sum_{j=1}^{M}\frac{1}{T}A_i x_{ij} R'_{ij}\bar{T}_j \\ &- \sum_{j=1}^{M} u_j \left(\sum_{i=1}^{N} y_{ij} G_i^{BS} - \beta_j\right) \\ &- \sum_{l=1}^{L} v_l \left(\sum_{i=1}^{N}\sum_{j=1}^{M} C_{jl} y_{ij} - P_{max}\right), \quad (2) \end{aligned}$$

where $\vec{u} := \{u_j | u_j \geq 0, \forall j\}$, $\vec{v} := \{v_l | v_l \geq 0, \forall l\}$ are Lagrangian multipliers.

B. Optimize Power Control

In this step, we find the optimal $\vec{y}^*$ that maximizes the Lagrangian function given the assignment and dual variables. Firstly, when $x_{ij} = 0$, we have $y_{ij}^* = x_{ij} p_{ij}^* = 0$. For the case of $x_{ij} > 0$, the optimal y_{ij}^* can be obtained by letting $\frac{\partial L(\vec{x},\vec{y},\vec{u},\vec{v})}{\partial y_{ij}^*} = 0$, which is

$$y_{ij}^* = x_{ij}\left(\frac{A_i B_j \bar{T}_j}{(u_j G_i^{BS} + \sum_{l=1}^{L} v_l C_{jl}) T \ln 2} - \frac{p_j^c G_{ji} + N_0}{G_{ii}}\right). \quad (3)$$

Since $y_{ij}^* = x_{ij} p_{ij}^*$, we can obtain the optimal power allocation policy

$$p_{ij}^* = \left(\frac{A_i B_j \bar{T}_j}{(u_j G_i^{BS} + \sum_{l=1}^{L} v_l C_{jl}) T \ln 2} - \frac{p_j^c G_{ji} + N_0}{G_{ii}}\right)^+, \quad (4)$$

where $(x)^+ := \max\{x, 0\}$, and p_{ij}^* only depends on dual variables u_j and $\vec{v}$. Hence, we have obtained the optimal power control policy given the dual variables.

C. Optimize Burst Assignment

In this step, we plug the optimal power values back to the Lagrangian function, and continue to find the optimal burst assignment policy. Before we solve this problem, we introduce some new notations to make the problem formulation more concise. Define d_{ij} and E as

$$\begin{aligned} d_{ij} := &\frac{A_i \bar{T}_j}{T}\log_2\left(1 + \frac{p_{ij}^* G_{ii}}{p_j^c G_{ji} + N_0}\right) \\ &- p_{ij}^*(u_j G_i^{BS} + \sum_{l=1}^{L} v_l C_{jl}),\ E := \sum_{j=1}^{M} u_j \beta_j + \sum_{k=1}^{K} v_l P_{max}. \end{aligned} \quad (5)$$

Then, the problem of optimizing over burst assignment variables can be formulated as

$$\begin{aligned} &\max_{x_{ij}\in[0,1]} \sum_{i=1}^{N}\sum_{j=1}^{M} d_{ij} x_{ij} - E, \\ &\textbf{s.t.:} \sum_{j=1}^{M} x_{ij} \leq 1, \quad \forall i; \sum_{i=1}^{N} x_{ij} \leq 1,\ \forall j, \end{aligned} \quad (P.3)$$

where both $d_{ij}, \forall i, j$ and E become constants given the dual variables. This problem is the linear programming (LP) relaxation of the classic assignment problem, the objective of which is to assign a set of people to different jobs to maximize total profit. The LP relaxation of the assignment problem has the property that if it has a feasible solution at all, then it has an optimal integer solution [21]. Hence, an optimal integer solution can be acquired by solving (P.3). Many algorithms have been proposed to solve this problem like the Hungarian algorithm whose time complexity is $O(n^3)$ [21] ($n = \max\{M, N\}$). Although we can obtain the optimal burst assignment policy, there is no explicit formula for the optimal assignment variables $x_{ij}^*, \forall i, j$.

D. Solve the Dual Problem

After optimizing over all primal variables, we continue to solve the following dual problem

$$\begin{aligned} &\min_{\vec{u},\vec{v}} L(\vec{x}^*(\vec{u},\vec{v}), \vec{y}^*(\vec{u},\vec{v}), \vec{u}, \vec{v}) \\ &\textbf{s.t.:} \ \vec{u} \geq \vec{0}; \quad \vec{v} \geq \vec{0}, \end{aligned} \quad (P.4)$$

where $y_{ij}^* = x_{ij}^* p_{ij}^*, \forall i, j$. Since we cannot obtain closed form solutions to (P.3), (P.4) becomes a non-smooth convex problem. Sub-gradient based algorithms have been widely used to solve non-smooth convex problems. Although those algorithms can obtain near-optimal solutions, their convergence rate can be rather slow [20]. Hence, we solve (P.4) using an "accelerated" sub-gradient method [20].

Before presenting our proposed algorithm, we first discuss fundamental ideas of the accelerated sub-gradient methods to solve a general non-smooth optimization problem

$$\min_{\vec{x}\in Q} f(\vec{x}), \quad (P.5)$$

where $f(\vec{x})$ is a non-smooth convex function and Q is a nonempty closed convex set. (P.5) can be solved via a dual method. Different from normal sub-gradient algorithms, the accelerated sub-gradient algorithm requires two different updating sequences of parameters $\{\lambda_t\}, \{\theta_t\}$ with different functionality. Similar to the normal sub-gradient algorithms, the $\{\lambda_t\}$ sequence satisfies $\lambda_t > 0, \lambda_t \to 0, \sum_{t=0}^{\infty}\lambda_t = \infty$, which is used to guarantee the convergence of the updating process $x_{t+1} = x_t - \lambda_t g(x_t)$. Here $g(x_t)$ denotes sub-gradient of $f(x)$ at iteration t. However, further theoretical analysis demonstrates that new sub-gradients have diminishing weights in the updating process, which contradicts to general principle of iterative schemes that new information is more important than old information [22], [23]. Given this observation, another sequence $\{\theta_t\}$ is used to attach larger weights to newer sub-gradients. Specifically, the $\{\theta_t\}$ sequence satisfies $\theta_0 = 1, \theta_{t+1} = \theta_t + \frac{1}{\theta_t}$. In addition, let $d(x)$ be any strongly convex and continuously differentiable function on Q, and accelerated sub-gradient algorithm updates $\{x_t\}$ be

$$\begin{aligned} x_{t+1} = &\operatorname*{argmin}_{x\in Q}(\lambda_t[f(x_t) + g(x_t)(x - x_t)] \\ &+ \theta_t d(x) - \theta_{t-1}[d(x_t) + \dot{d}(x_k)(x - x_t)]), \end{aligned} \quad (6)$$

Algorithm 1 Dual Algorithm

Require: Initial $\vec{u}$ and $\vec{v}$ values, $\{\lambda_t\}$, $\{\theta_t\}$, $f(x)$, $d(x)$, ϵ
1: Solve for $p^*_{ij}, \forall i, j$ and $x^*_{ij}, \forall i, j$ using Equation (4) and Hungarian algorithm, respectively
2: **for** $j = 1$ **to** M **do**
3: sub-gradients: $g_j(\vec{u}, \vec{v}) = \sum_{i=1}^{N} y^*_{ij} G_i^{BS} - \beta_j$
4: **end for**
5: **for** $l = 1$ **to** L **do**
6: sub-gradients: $g_l(\vec{u}, \vec{v}) = \sum_{i,j} C_{jl} y^*_{ij} - P_{max}$
7: **end for**
8: **while** $|q(\vec{u}(t+1), \vec{v}(t+1)) - q(\vec{u}(t), \vec{v}(t))| > \epsilon$ **do**
9: Update $\vec{u}(t+1)$ and $\vec{v}(t+1)$ according to Equation (6)
10: Repeat Step 1 to Step 7
11: **end while**
12: **return** $x^*_{ij}, \forall i, j$ and $p^*_{ij}, \forall i, j$

where $\dot{d}(x)$ denotes derivative of $d(x)$ [20]. The accelerated sub-gradient method achieves an ϵ-solution (i.e., $|f(\vec{x}) - f^*| \leq \epsilon$) within $O(\frac{J^2 R^2}{\epsilon^2})$ iterations, where f^* is the optimal solution of (P.5), $J := \sup\{||g(\vec{x})||\}$, and $R := \sup\{||f(\vec{x})||\}$ [20]. This method is proved to achieve the least complexity among all sub-gradient algorithms attaining ϵ-solutions [20].

E. The Whole Dual Algorithm

Given the above analysis, our proposed dual algorithm is shown in Algorithm 1. Since the complexity of the Hungarian algorithm is $O(n^3)$, total complexity of the dual algorithm is $O(\frac{J^2 R^2 n^3}{\epsilon^2})$, where $n = \max\{M, N\}$. Since the complexity of Algorithm 1 can be fairly high for large scale coexistence problems, we propose another two algorithms with more favorable complexity based on integer programming.

IV. Sparse Packing Based Method

Similar to [8], [24], we assume that only a fixed number of transmit power levels are supported in the CVN, which are $\{p_k | \forall k \in 1, 2 \cdots K\}$ with $p_1 = 0$ and $p_K = P_{max}$. This assumption is based on the fact that most real wireless systems use a fixed number of power levels [8], [24]. Given this assumption, (P.1) can be reformulated as an Linear Integer Program (LIP):

$$\begin{aligned}
&\max_{x_{ijk} \in \{0,1\}} \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{K} x_{ijk} r_{ijk} \\
&\textbf{s.t.: } C1: \sum_{i=1}^{N} \sum_{k=1}^{K} x_{ijk} p_k G_i^{BS} \leq \beta_j, \quad \forall j, \\
&\qquad C2: \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{K} C_{jl} x_{ijk} p_k \leq P_{max}, \quad \forall l, \\
&\qquad C3: \sum_{j=1}^{M} \sum_{k=1}^{K} x_{ijk} \leq 1, \quad \forall i, \\
&\qquad C4: \sum_{i=1}^{N} \sum_{k=1}^{K} x_{ijk} \leq 1, \quad \forall j,
\end{aligned} \tag{P.6}$$

where $r_{ijk} = \frac{1}{T} A_i \bar{T}_j B_j \log_2 \left(1 + \frac{p_k G_{ii}}{p_j^c G_{ji} + N_0}\right)$.

Algorithm 2 CSP-Based Method

Require: Normalize constraints to the form: $a\vec{x} \leq \vec{1}$
1: Solve LP relaxation of (P.6), and obtain fractional $\vec{x}^*$
2: Select item j independently with probability $\frac{x_j^*}{\alpha(L+3)}$, and let C_{old} be the set of selected items
3: **for** $j = 1$ **to** n **do**
4: Delete item j if any of the two events happen:
- Event E_j^1: there exists an item $l \in C_{old} \backslash \{j\}$, which is "big" for constraint i, or
- Event E_j^2: sum of sizes of items in C_{old} that are "small" for i is larger than 1.

5: **end for**
6: **return** Left items in C_{old} (denoted by C_{new})

Since LIP problems are usually NP-hard, (P.6) cannot be solved directly in polynomial time. Hence, we first solve LP relaxation of (P.6) to acquire a fractional solution. Then, we propose two rounding algorithms to round the fractional solution in this section and Section V, respectively.

A. CSP Problems

Before presenting the first rounding algorithm, we show that (P.6) belongs to column-sparse packing (CSP) problems. Packing problems are usually formulated as:

$$\max_{\vec{x} \in \{0,1\}^n} \vec{w}^T \vec{x} \quad \textbf{s.t.: } A\vec{x} \leq \vec{b}, \ A \in R^{m \times n} \tag{P.7}$$

where all vectors and matrices are non-negative. A k-CSP problem is defined as follows.

Definition 2 (CSP Problem): (P.7) is a k-column-sparse packing problem if every variable $x_i \in \vec{x}$ appears in at most k constraints.

Next, we show that (P.6) is an $(L+3)$-CSP problem.

Lemma 3: (P.6) is a an $(L+3)$-CSP problem.

Proof: Recall that $C1$ denotes the interference limit constraints of "orthogonal" bursts. Here "orthogonal" means that a vehicle can be scheduled to at most one burst. Hence, a variable x_{ijk} only appears in the j-th constraint of $C1$. The variable also only appears in the j-th constraint of $C4$ for the same reason. Similarly, $C3$ denotes assignment constraints of "orthogonal" vehicles. Here "orthogonal" means that a vehicle can be assigned to at most one vehicle. Hence, the variable x_{ijk} only appears in the i-th constraint of $C1$. Since $C2$ contains L constraints, x_{ijk} can participate in at most L constraints in $C2$. Therefore, x_{ijk} can participate in at most $L+3$ constraints. In particular, all Type 1 bursts belong to all burst intervals, and thus all variables containing Type 1 bursts particulate in exactly $L+3$ constraints. □

B. Proposed Randomized Rounding Algorithm

Next, we develop a rounding algorithm based on the idea of "randomized rounding with alteration" [25]. Let $n = M \cdot N \cdot K$ and $m = 2M + N + L$ be the number of variables and constraints in (P.6), respectively. In this section, we refer to every variable x_{ijk} in (P.6) as an "item". Hence, selecting/deleting an item simply means setting its corresponding variable x_{ijk}

to one/zero. Moreover, let α and β be constant values to be optimized, and we normalize constraints of (P.6) to the form $a\vec{x} \le \vec{1}$, where $a := \{a_{ij} | a_{ij} \in [0,1], \forall i, j\}$. An item j is called "big" for constraint i if $a_{ij} > \beta$, and "small" if $a_{ij} \le \beta$.

Given these definitions, we present our randomized rounding scheme in Algorithm 2. Although our rounding idea is similar to the one in [25], our work is a generalization of [25]. More specifically, Bansal *et al.* [25] set the boundary between "big" and "small" items to 1/2 without studying its impact on the performance of the algorithm. In our work, we consider a more general case, where we set the boundary to β, which is a random variable to be optimized. The trade-off of deciding β is that if we increase β, the probability that item j is deleted due to E_j^1 decreases (E_j^1, E_j^2 are defined in Step 4 of Algorithm 2), while the probability that item j is deleted due to E_j^2 increases, and vice versa. Feasibility analysis of Algorithm 2 is similar to [25] and thus is omitted here. Instead, we focus on studying the impact of β on the rounding performance. Given the definition of E_j^1, E_j^2 in Algorithm 2, we show that the probabilities that the two events happen are upper bounded. Let B_i, S_i be the set of items that are big and small for constraint i respectively, $F_i := \sum_{l \in B_i} x_l^*$, and we show the bounds of the two probabilities in Lemma 4 and 5.

Lemma 4: $Pr[E_j^1 | j \in C_{old}] \le \frac{F_i}{\alpha(L+3)}$, i.e., the probability that item j is deleted from C_{old} due to E_j^1 is upper bounded.

Proof: Recall that for any item that is big for i, we have $a_{ij} > \beta$. Therefore,

$$F_i = \sum_{l \in B_i} x_l^* < \sum_{l \in B_i} \frac{a_{il}}{\beta} x_l^* = \frac{1}{\beta} \sum_{l \in B_i} a_{il} x_l^* \le \frac{1}{\beta}, \tag{7}$$

where x_l^* is defined in Step 1 and the last inequality is due to $a\vec{x} \le \vec{1}$. Moreover, we have

$$\begin{aligned} &Pr[E_j^1 | j \in C_{old}] = Pr[\exists l \in B_i \backslash \{j\} | j \in C_{old}] \\ &\quad \le \sum_{l \in B_i \backslash \{j\}} Pr[l \in C_{old} | j \in C_{old}] = \sum_{l \in B_i \backslash \{j\}} Pr[l \in C_{old}] \\ &\quad = \sum_{l \in B_i \backslash \{j\}} \frac{x_l^*}{\alpha(L+3)} \le \frac{F_i}{\alpha(L+3)}, \end{aligned} \tag{8}$$

where the first inequality is due to union bound, the second equality is due to mutual independence of choosing items, and the last inequality is due to definition of F_i. □

Lemma 5: $Pr[E_j^2 | j \in C_{old}] \le \frac{(1-\beta F_i)}{\alpha(L+3)(1-\beta)}$, i.e., the probability that item j is deleted due to E_j^2 is bounded.

Proof: In this case, if item j is big for constraint i, then we have $S_i = S_i \backslash \{j\}$, and total size of items in S_i must be larger than 1. Furthermore, we have

$$\begin{aligned} Pr[E_j^2 | j \in C_{old}] &= Pr\left[\sum_{l \in S_i} a_{il} > 1 | j \in C_{old}\right] \\ &= Pr\left[\sum_{l \in S_i \backslash \{j\}} a_{il} > 1 | j \in C_{old}\right] \\ &\le Pr\left[\sum_{l \in S_i \backslash \{j\}} a_{il} > 1-\beta | j \in C_{old}\right], \end{aligned} \tag{9}$$

where the second equality is due to $S_i = S_i \backslash \{j\}$, and the last inequality is because $\beta \in [0,1]$. In contrast, if j is small for constraint i, then E_j^2 occurs only if the total size of items in S_i is larger than 1. Since $a_{ij} \le \beta$, the total size of items in $S_i \backslash \{j\}$ must be larger than $(1-\beta)$, Equation (9) still holds. Hence, we can conclude that

$$Pr[E_j^2 | j \in C_{old}] \le Pr\left[\sum_{l \in S_i \backslash \{j\}} a_{il} > 1-\beta | j \in C_{old}\right]. \tag{10}$$

Furthermore, the expected total size of items in $S_i \backslash \{j\}$ is

$$\begin{aligned} E\left[\sum_{l \in S_i \backslash \{j\}} a_{il} | j \in C_{old}\right] &= \sum_{l \in S_i \backslash \{j\}} a_{il} \frac{x_j^*}{\alpha(L+3)} \\ &\le \frac{1}{\alpha(L+3)}\left(1 - \sum_{l \in B_i} a_{il} x_j^*\right) \le \frac{1-\beta F_i}{\alpha(L+3)}, \end{aligned} \tag{11}$$

where the last inequality is due to Equation (7). We can further extend Equation (10) as

$$\begin{aligned} Pr[E_j^2 | j \in C_{old}] &\le Pr\left[\sum_{l \in S_i \backslash \{j\}} a_{il} > 1-\beta | j \in C_{old}\right] \\ &\le \frac{E\left[\sum_{l \in S_i \backslash \{j\}} a_{il} | j \in C_{old}\right]}{1-\beta} \le \frac{1-\beta F_i}{\alpha(L+3)(1-\beta)}, \end{aligned} \tag{12}$$

where the second inequality is due to Markov Inequality, and the third inequality is due to Equation (11). □

Finally, we show performance of the Algorithm 2 as follows.

Theorem 6: Algorithm 2 is a $\frac{1}{8}\left(1 - \frac{1}{L+3} + \frac{1}{(L+3)^2}\right)$-approximation algorithm to (P.6).

Proof: Given Lemma 4 and 5, the total probability that item j is deleted in Step 4 is

$$\begin{aligned} &Pr[j \notin C_{new} | j \in C_{old}] \\ &\quad = Pr[E_j^1 | j \in C_{old}] + Pr[E_j^2 | j \in C_{old}] \\ &\quad \le \frac{F_i}{\alpha(L+3)} + \frac{1-\beta F_i}{\alpha(L+3)(1-\beta)} \\ &\quad = \frac{1}{\alpha(L+3)}\left(\frac{1-2\beta}{1-\beta} F_i + \frac{1}{1-\beta}\right). \end{aligned} \tag{13}$$

To improve the approximation ratio, we need to delete as few items as possible on condition that all constraints are satisfied. In other words, we should minimize $Pr[j \notin C_{new} | j \in C_{old}]$ by choosing β. Since $\beta \in [0,1]$, we consider two cases: (1) $\beta \in [0, 1/2]$ and (2) $\beta \in [1/2, 1]$.

Case 1: $\beta \in [0, 1/2]$. Recall that $F_i \in [0, \frac{1}{\beta}]$. Hence $(1-2\beta) \ge 0$, and Equation (13) implies

$$\begin{aligned} Pr[j \notin C_{new} | j \in C_{old}] &\le \frac{1}{\alpha(L+3)}\left(\frac{1-2\beta}{1-\beta} F_i + \frac{1}{1-\beta}\right) \\ &\le \frac{1}{\alpha(L+3)}\left(\frac{1-2\beta}{1-\beta} \cdot \frac{1}{\beta} + \frac{1}{1-\beta}\right) = \frac{1}{\alpha(L+3)} \cdot \frac{1}{\beta}, \end{aligned} \tag{14}$$

where the right hand side is minimized for $\beta = 1/2$.

Case 2: $\beta \in [1/2, 1]$. In this case, we have $(1-2\beta) \leq 0$, and Equation (13) implies that

$$\begin{aligned} Pr[j \notin C_{new} | j \in C_{old}] &\leq \frac{1}{\alpha(L+3)}\left(\frac{1-2\beta}{1-\beta}F_i + \frac{1}{1-\beta}\right) \\ &\leq \frac{1}{\alpha(L+3)}\left(\frac{1-2\beta}{1-\beta}\cdot 0 + \frac{1}{1-\beta}\right) = \frac{1}{\alpha(L+3)}\cdot\frac{1}{(1-\beta)}, \end{aligned} \tag{15}$$

where the right hand side is also minimized for $\beta = 1/2$. Hence $\beta^* = 1/2$, and we have $Pr[j \notin C_{new}|j \in C_{old}] \leq \frac{2}{\alpha(L+3)}$. Finally, the probability of choosing set C_{new} is

$$\begin{aligned} Pr[j \in C_{new}] &= Pr[j \in C_{old}] \cdot Pr[j \in C_{new}|j \in C_{old}] \\ &= Pr[j \in C_{old}] \cdot (1 - Pr[j \notin C_{new}|j \in C_{old}]) \\ &\geq \frac{x_j^*}{\alpha(L+3)}\left(1 - \frac{2}{\alpha}\right). \end{aligned} \tag{16}$$

It is easy to see that the right hand side is maximized for $\alpha = 4$, and thus $Pr[j \in C_{new}] \geq \frac{x_j^*}{8(L+3)}$. Let $\vec{x}^{OPT}$ be the optimal solution to (P.6), and Equation (16) implies

$$\begin{aligned} \sum_{j \in C_{new}} w_j x_j^* &\geq \frac{1}{8(L+3)} \sum_{j \in C_{old}} w_j x_j^* \\ &\geq \frac{1}{8(L+3)}\left(L+3-1+\frac{1}{L+3}\right)\sum_{j=1}^{n} w_j x_j^{OPT} \\ &= \frac{1}{8}\left(1 - \frac{1}{L+3} + \frac{1}{(L+3)^2}\right)\sum_{j=1}^{n} w_j x_j^{OPT} \end{aligned} \tag{17}$$

where the second inequality is because LP relaxation of the k-CSP problem has an integrality gap of $k - 1 + \frac{1}{k}$ [25]. □

Since there are totally n iterations and each iteration has $(m+n)$ operations, complexity of the proposed algorithm is merely $O(n(m+n))$.

V. Dependent Rounding Based Method

Although Algorithm 2 has more favorable complexity than Algorithm 1, its approximation ratio is still low. In this section, we develop a more efficient algorithm to round the fractional solution obtained from (P.8) (i.e., LP relaxation of (P.6)) into an integer solution to (P.6).

$$\begin{aligned} &\max_{x_{ijk} \in [0,1]} \sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k=1}^{K} x_{ijk} r_{ijk} \\ &\textbf{s.t.:}\ C1,\ C2,\ C3,\ C4\ of\ (P.6) \end{aligned} \tag{P.8}$$

A. Basics of Dependent Rounding

As discussed in [26]–[28], dependent rounding algorithms iteratively round fractional variables[1] into binary values without violating linear constraints $A\vec{x} \leq \vec{b}$. The rounding idea in a single iteration t is as follows. After treating variables with binary values as constants, one can obtain an n-dimensional fractional solution $\vec{x}(t)$ that satisfies the following constraints: (1) regular tight (i.e., equality) constraints: $A_1\vec{x}(t) = \vec{b_1}$, where $A_1 \in \mathbb{R}^{m_1 \times n}, \vec{b_1} \in \mathbb{R}^{m_1}$; (2) regular untight (i.e., inequality) constraints: $A_2\vec{x}(t) < \vec{b_2}$, where $A_2 \in \mathbb{R}^{m_2 \times n}, \vec{b_2} \in \mathbb{R}^{m_2}$; (3) domain constraints: $\vec{x}(t) \in (0,1)^n$. Then, a new solution $\vec{x}(t+1)$ is found such that: (1) $E[\vec{x}(t+1)] = E[\vec{x}(t)]$; (2) tight constraints are still satisfied, i.e., $A_1\vec{x}(t+1) = \vec{b_1}$; (3) at least one entry of $\vec{x}(t)$ is rounded into a binary value or at least one regular untight constraint becomes tight.

The three guarantees are accomplished as follows. Firstly, assume that null space of A_1 is nontrivial, and thus $S = \{\vec{s} \in \mathbb{R}^n | A_1\vec{s} = \vec{0}\}$ contains at least one nonzero element. Then, randomly choose one nonzero vector from S, e.g., $\vec{s} \in S, \vec{s} \neq \vec{0}$. Afterwards, we must be able to find a positive scalar θ_1 such that: (1) all entries of $\vec{x}(t) + \theta_1\vec{s}$ lie in [0, 1]; (2) at least one entry of $\vec{x}(t) + \theta_1\vec{s}$ becomes integer (i.e., a fractional variable is rounded to an integer) or at least one entry of $A_2(\vec{x}(t)+\theta_1\vec{s}) - \vec{b_2}$ becomes zero (i.e., an untight constraint becomes tight). Similarly, we must be able to find another positive scalar θ_2 such that: (1) all entries of $\vec{x}(t) - \theta_2\vec{s}$ lie in [0, 1]; (2) at least one entry of $\vec{x}(t) - \theta_2\vec{s}$ becomes integer or at least one entry of $A_2(\vec{x}(t) - \theta_2\vec{s}) - \vec{b_2}$ becomes zero. Finally, the new solution is obtained as follows:

$$\vec{x}(t+1) = \begin{cases} \vec{x}(t) + \theta_1\vec{s}, & \text{with probability } \dfrac{\theta_2}{\theta_1+\theta_2}, \\ \vec{x}(t) - \theta_2\vec{s}, & \text{with probability } \dfrac{\theta_1}{\theta_1+\theta_2}. \end{cases} \tag{18}$$

The three aforementioned guarantees can be verified as follows. Firstly, $E[\vec{x}(t+1)] = E[\vec{x}(t)]$ will be proved in Lemma 8. Secondly, since $\vec{s}$ is taken from null-space of A_1, we have $A_1\vec{s} = \vec{0}$, and thus all tight constraints are still tight (i.e., $A_1(\vec{x}(t) + \theta_1\vec{s}) = A_1\vec{x}(t) = \vec{b_1}$, or $A_1(\vec{x}(t) - \theta_2\vec{s}) = A_1\vec{x}(t) = \vec{b_1}$). The third guarantee is accomplished through the criteria of choosing θ_1, θ_2 as described above. We can see that the dependent rounding algorithm rounds at least one fractional variable into an integer or tightens at least one constraint in every iteration.

However, the necessary condition for the success of the rounding algorithm is that the linear system consisting of the tight constraints is underdetermined (i.e., null-space of A_1 is nontrivial). If not, some constraints must be dropped or combined, which can result in violations of these constraints. More importantly, no common procedure exists for dropping or combining constraints to guarantee the success of the dependent rounding algorithm. Instead, the criteria of constraint dropping and combining depend on specific problem structures. Next, we propose a dependent rounding algorithm with an efficient constraint dropping procedure for (P.8). Although our dependent rounding algorithm can also violate some constraints, we prove that the worst constraint violations are upper bounded.

B. Proposed Dependent Rounding Algorithm

The proposed algorithm rounds the fractional variables iteratively with the following invariants.

1) Once a variable is rounded into 0 or 1 , it will never change in remaining iterations.

[1]In this section, a fractional variable x means $0 < x < 1$, and an integer variable x means $x \in \{0, 1\}$.

Algorithm 3 Dependent Rounding Algorithm

Input: Fractional solution obtained by solving (P.8): $X^* = \{x^*_{ijk} | 0 \le x^*_{ijk} \le 1, \forall i, j, k\}$
Output: Integer solution: $X = \{x_{ijk} | x_{ijk} \in \{0, 1\}, \forall i, j, k\}$
Initialization : $\vec{x}(0) = X^*$
1: **while** there exist fractional variables in $\vec{x}(t)$ **do**
2: treat all integer variables in $\vec{x}(t)$ as constants and update all constraints by moving constant items to the right hand side of equalities
3: **if** $\Phi(t)$ is determined **then**
4: **if** $|F(t)| > 2L$ **then**
5: for bursts in $F_1(t), F_2(t)$, drop their corresponding $C1(t)$ and $C4(t)$ constraints from $\Phi(t)$ (if any); for bursts in $F_3(t), F_4(t)$, drop their corresponding $C1(t)$ constraints from $\Phi(t)$ (if any)
6: **else**
7: drop all $C2(t)$ constraints from $\Phi(t)$ (if any); for bursts in $F_1(t), F_2(t)$, drop their corresponding $C1(t)$ and $C4(t)$ constraints from $\Phi(t)$ (if any); for bursts in $F_3(t), F_4(t)$, drop their corresponding $C1(t)$ constraints from $\Phi(t)$ (if any)
8: **end if**
9: **end if**
10: find the null-space S of the new $A(t)$
11: choose a nonzero vector $\vec{s} \in S$
12: find scalars θ_1, θ_2 as described above Equation (18).
13: compute $\vec{x}(t+1)$ using Equation (18)
14: **end while**
15: **return** $\vec{x}(t+1)$

2) Once a constraint becomes tight, it remains tight in the remaining iterations.
3) Once a constraint is dropped, it will never be reinstated.

Firstly, we define the set of remaining fractional variables at iteration t as $X(t) := \{x_{ijk} | 0 < x_{ijk}(t) < 1, \forall i, j, k\}$. Furthermore, we define $F_m(t)$ to be the set of bursts with m fractional variables in $X(t)$, i.e., for any burst $j \in F_m(t)$, there are exactly m vehicle-power pairs (i, k) such that $0 < x_{ijk} < 1$. Since a burst can be assigned to at most $N \times K$ vehicle-power pairs, the set of remaining fractionally assigned bursts can be defined as $F(t) := F_1(t) \cup F_2(t) \cup F_3(t) \cdots F_{N \times K}(t)$. Let $|F_m(t)|$ be the number of bursts in $F_m(t)$, and the number of fractionally assigned bursts is

$$|F(t)| = \sum_{m=1}^{N \times K} |F_m(t)|. \tag{19}$$

Hence, $X(t)$ being nonempty implies $|F(t)| > 0$. Similar to the definition in Section V-A, let

$$\Phi(t) : A(t)\vec{x}(t) = \vec{b}(t) \tag{20}$$

be the linear system defined by the tight constraints in (P.8) with fractional variables at t.

Given the above definitions, we propose Algorithm 3 to round the fractional solution obtained from (P.8) into an integer solution to (P.6). Remainder of this section concentrates on analyzing feasibility and performance of Algorithm 3. We first show feasibility of Algorithm 3.

Lemma 7: Algorithm 3 rounds the fractional solution from (P.8) to an integer solution to (P.6).

Proof: As discussed in Section V-A, to show the success of a dependent rounding algorithm, we only need to prove that the linear system consisting of tight constraints is always underdetermined. Hence, in Algorithm 3, we must prove that the new linear system $\Phi(t)$ in Equation (20) is underdetermined after constraint dropping. Let $N_v(t), N_c(t)$ be the number of remaining fractional variables and the number of linearly independent tight constraints with fractional variables at t, respectively, and we must prove that $N_c(t) < N_v(t)$ after dropping constraints.

Firstly, we define $|C1(t)|$, $|C2(t)|$, $|C3(t)|$, and $|C4(t)|$ as the number of remaining tight constraints in $C1(t)$, $C2(t)$, $C3(t)$, and $C4(t)$ with fractional variables at iteration t. From (P.8), we can see that any burst j has one corresponding constraint in both $C1(t)$ and $C4(t)$ (not necessarily tight). Therefore, any remaining tight constraint j in $C1(t)$ (or $C4(t)$) with fractional variables corresponds to a fractionally assigned burst j in $F(t)$. However, a fractionally assigned burst j may not necessarily have a corresponding tight constraint in $C1(t)$ (or $C4(t)$) because (1) the corresponding constraint j can be untight, or (2) the corresponding constraint j has been dropped in previous iterations. Therefore, we must have

$$|C1(t)| \le |F(t)|, \quad |C4(t)| \le |F(t)|. \tag{21}$$

Next, based on Step 4 of Algorithm 3, we prove this lemma by considering two cases:

Case 1: $|F(t)| > 2L$. Notice that in this case, only constraints in $C1(t)$ and $C4(t)$ can be dropped. Hence, we have

$$\begin{aligned} N_c(t) &\le |C1(t)| + |C2(t)| + |C3(t)| + |C4(t)| \\ &= |C1(t)| + L + |C3(t)| + |C4(t)|, \end{aligned} \tag{22}$$

where the inequality is because remaining tight constraints may not necessarily be linearly independent, and the equality is because the number of $C2(t)$ constraints is L. Since $F_m(t)$ denotes the set of bursts with m vehicle-power pairs (i.e., m fractional variables), we have

$$N_v(t) = \textstyle\sum_{m=1}^{N \times K} m \times |F_m(t)|. \tag{23}$$

Moreover, each remaining tight $C(3)$ constraint with fractional variables in $\Phi(t)$ contains at least two fractional variables because all remaining variables in this constraint are between $(0, 1)$, and sum of these variables equals to 1. Therefore, we have

$$N_v(t) \ge 2 \times |C_3(t)|. \tag{24}$$

Equation (23) and (24) imply that

$$N_v(t) \ge |C_3(t)| + \sum_{m=1}^{N \times K} \frac{m}{2} \times |F_m(t)|. \tag{25}$$

Given the above equations, we can obtain Equation (26), as shown at the top of the next page, where the first

$$
\begin{aligned}
N_c(t) &\leq |C1(t)| + L + |C3(t)| + |C4(t)| \\
&\leq |F(t)| + L + |C3(t)| + |F(t)| \\
&= |C_3(t)| + \frac{\sum_{m=1}^{N\times K}|F_m(t)|}{|F(t)|}L + |F_3(t)| + |F_4(t)| + \sum_{m=5}^{N\times K} 2\times|F_m(t)| \\
&= |C_3(t)| + \frac{L}{|F(t)|}(|F_1(t)| + |F_2(t)|) + \sum_{m=3}^{4}\frac{|F(t)|+L}{|F(t)|}|F_m(t)| + \sum_{m=5}^{N\times K}\frac{2|F(t)|+L}{|F(t)|}|F_m(t)| \\
&< |C_3(t)| + \frac{1}{2}|F_1(t)| + |F_2(t)| + \frac{3}{2}|F_3(t) + 2|F_4(t)| + \sum_{m=5}^{N\times K}\frac{m}{2}|F_m(t)| = |C_3(t)| + \sum_{m=1}^{N\times K}\frac{m}{2}|F_m(t)| \leq N_v(t) \qquad (26)
\end{aligned}
$$

$$
\begin{aligned}
N_c(t) &\leq |C1(t)| + |C3(t)| + |C4(t)| \leq |F(t)| + |C3(t)| + |F(t)| = |C3(t)| + |F_3(t)| + |F_4(t)| + \sum_{m=5}^{N\times K} 2|F_m(t)| \\
&< |C3(t)| + \frac{1}{2}|F_1(t)| + |F_2(t)| + \frac{3}{2}|F_3(t)| + 2|F_4(t)| + \sum_{m=5}^{N\times K}\frac{m}{2}|F_m(t)| \leq N_v(t) \qquad (27)
\end{aligned}
$$

inequality is due to Equation (22), the second inequality is due to Equation (21), the first equality is due to Equation (19) and constraint dropping in Step 5 of Algorithm 3, the strict inequality is due to $|F(t)| > 2L$, and the last inequality is due to Equation (25).

Case 2: $|F(t)| \leq 2L$. In this case, Equation (25) still holds because no $C3(t)$ constraints are dropped, and thus we can obtain Equation (27), as shown at the top of this page, where the first equality is because not all remaining tight constraints are linearly independent after constraint dropping in Step 7, the second inequality is due to Equation (21), the equality is due to the constraint dropping in Step 7, the strict inequality is due to $|F(t)| > 0$, and the last inequality is due to Equation (25).

Therefore, we have $N_v(t) > N_c(t)$ in both the two cases, which finishes the proof. □

Next, we show performance of Algorithm 3.

Lemma 8: Let X^* be the fractional solution obtained from (P.8), and X be the integer solution returned by Algorithm 3. Then, we have $E[X] = X^*$.

Proof: We prove this lemma through induction. Firstly, Step 13 of Algorithm 3 implies

$$
\begin{aligned}
E[x_{ijk}(t+1)|x_{ijk}(t)] &= \frac{\theta_2}{\theta_1+\theta_2}(x_{ijk}(t) + \theta_1 s_{ijk}) \\
&\quad + \frac{\theta_1}{\theta_1+\theta_2}(x_{ijk}(t) - \theta_2 s_{ijk}) = x_{ijk}(t),
\end{aligned} \tag{28}
$$

where s_{ijk} is the corresponding element of x_{ijk} in $\vec{s}$ obtained in Step 11 of Algorithm 3. Hence,

$$E[x_{ijk}(t+1)] = E[E[x_{ijk}(t+1)|x_{ijk}(t)]] = E[x_{ijk}(t)]. \tag{29}$$

Since $\vec{x}(0) = X^*$ as shown in the *Initialization* Step of Algorithm 3, we have

$$E[x_{ijk}] = E[x_{ijk}(t+1)] \cdots = E[x_{ijk}(0)] = x^*_{ijk}, \tag{30}$$

which finishes the proof. □

Next, we first define constraint violation ratio in Definition 9 and then show the constraint violation ratios in Algorithm 3 in Lemma 10.

Definition 9 (Violation Ratio): For any inequality constraint $A_j\vec{x} \leq b_j$, its violation ratio is defined as $\delta_j = \frac{A_j X - b_j}{b_j}$, for an integer solution X returned by Algorithm 3.

Lemma 10: The integer solution returned by Algorithm 3 can violate constraints of (P.6) as follows: (1) violation ratio of $C1$ constraints is at most 2; (2) violation ratio of $C2$ constraints is at most $2L$, where recall that $L \in \{1, 2, 3, 4\}$ according to IEEE 802.22 [7]; (3) no $C3$ constraints are violated; (4) violation ratio of $C4$ constraints is at most 1.

Proof: Since constraints can be dropped in both Step 5 (when $|F(t)| > 2L$) and Step 7 (when $|F(t)| \leq 2L$) of Algorithm 3, we prove this lemma by considering the two cases.

Case 1: $|F(t)| > 2L$. In Step 5, for bursts in $F_1(t) - F_2(t)$, both their $C1(t)$ and $C4(t)$ constraints are dropped from $\Phi(t)$ (if any). However, we can see that $F_1(t)$ must be empty in $\Phi(t)$. Specifically, assume there exists a burst $j \in F_1(t)$ and its corresponding fractional variable is $x_{i^*jk^*} \in (0, 1)$, then we have the following $C4(t)$ constraint

$$x_{i^*jk^*} + \sum_{(i,k)\neq(i^*,k^*)} X_{ijk} = 1, \tag{31}$$

where $X_{ijk} \in \{0, 1\}, \forall (i, k) \neq (i^*, k^*)$ because they are already rounded into integers. Hence, Equation (31) cannot hold, and thus $F_1(t)$ must be empty. Next, we consider bursts in $F_2(t)$ and their dropped tight $C1(t), C4(t)$ constraints in Step 5. These constrains can be expressed as

$$C1(t): q_1x_1 + q_2x_2 + d = \beta_j, \quad C4(t): x_1 + x_2 = 1, \tag{32}$$

where $x_1, x_2 \in (0, 1)$ denote the two fractional variables associated with burst j, $q_1 = p_{k_1}G^{BS}_{i_1}$, $q_2 = p_{k_2}G^{BS}_{i_2}$ for some

$(i_1,k_1),(i_2,k_2)$, and $0 \le d < \beta_j$ is a constant. Let X_1, X_2 be the final integer values of x_1, x_2, and we have

$$C1(t): \delta_j = \frac{q_1X_1+q_2X_2+d-\beta_j}{\beta_j} \le \frac{q_1+q_2+d-\beta_j}{\beta_j} = \frac{q_1(1-x_1)+q_2(1-x_2)}{\beta_j} \le \frac{q_1+q_2}{\beta_j} \le \frac{\beta_j+\beta_j}{\beta_j} = 2, \tag{33}$$

where the third inequality is due to the assumption $\max_{(i,k)}\{p_kG_i^{BS}\} \le \beta_j$. In addition, we have

$$C4(t): \delta_j = X_1+X_2-1 \le 1+1-1 = 1. \tag{34}$$

According to Step 5, for bursts in $F_3(t), F_4(t)$, only $C1(t)$ constraints are dropped from $\Phi(t)$ (if any). Similar to Equation (32), we express tight $C1(t), C4(t)$ constraints of a burst $j \in F_3(t)$ as

$$C1(t): q_1x_1+q_2x_2+q_3x_3+d = \beta_j, \quad C4(t): x_1+x_2+x_3 = 1. \tag{35}$$

Similarly, we can also bound the violation as

$$C1(t): \delta_j = \frac{q_1X_1+q_2X_2+q_3X_3+d-\beta_j}{\beta_j} < \frac{q_1x_1+q_2x_2+q_3x_3+q^*+d-\beta_j}{\beta_j} = \frac{q^*}{\beta_j} \le 1, \tag{36}$$

where $q^* = \max_{1\le i\le 3} q_i$, and the inequality is due to $q_1X_1+q_2X_2+q_3X_3 \le q^* < q_1x_1+q_2x_2+q_3x_3+q^*$ [27]. The violation bound of $C1(t)$ constraints for bursts in $F_4(t)$ can be proved in the same way as Equation (36).

Case 2: $|F(t)| \le 2L$. Here, we consider the constraint dropping in Step 7. Similar to the constraint dropping in Step 5, $F_1(t)$ must be empty in $\Phi(t)$. For a burst $j \in F_2(t)$, its constraint violation ratios are the same as Equation (33) and (34). The violation ratio of $C1(t)$ constraints for bursts in $F_3(t), F_4(t)$ is the same as Equation (36).

Finally, we study the violation ratio of $C2(t)$ constraints. In Step 7, tight $C2(t)$ constraints in $\Phi(t)$ are dropped. For any burst interval l, we can rewrite its constraints in $\Phi(t)$ as

$$C2(t): \sum_{(i,j,k):x_{ijk}\in(0,1)} C_{jl}x_{ijk}p_k + d = P_{max},$$
$$C3(t): \sum_{(j,k):x_{ijk}\in(0,1)} x_{ijk} = 1, \quad \forall i,$$
$$C4(t): \sum_{(i,k):x_{ijk}\in(0,1)} x_{ijk} = 1, \quad \forall j, \tag{37}$$

where $0 \le d < P_{max}$ is a constant. Moreover, $|F(t)| \le 2L$ implies that

$$\sum_j C_{jl} \times 1_{\{\exists(i,k):x_{ijk}\in(0,1)\}} \le |F(t)| \le 2L, \quad \forall l, \tag{38}$$

where the first inequality is because no burst interval can contain more than $|F(t)|$ number of unassigned bursts. Finally, we can compute the violation of $C2(t)$ constraints as

$$C2(t): \delta_j = \frac{\sum_{(i,j,k):x_{ijk}\in(0,1)} C_{jl}X_{ijk}p_k + d - P_{max}}{P_{max}} = \frac{\sum_j C_{jl} \times 1_{\{\exists(i,k):x_{ijk}\in(0,1)\}} \sum_{i,k} X_{ijk}p_k + d - P_{max}}{P_{max}} \le \frac{2L \times q^* + d - P_{max}}{P_{max}} < \frac{2L \times P_{max}}{P_{max}} = 2L, \tag{39}$$

where $q^* := \max_{1\le k\le K} p_k \le P_{max}$, and the inequality is due to Equation (37), (38) as well as $\sum_{i,k} X_{ijk}p_k \le q^*$ (because $\sum_{i,k} X_{ijk} = \sum_{i,k} x_{ijk} = 1$).

Given the above analysis, we can conclude that the violation ratios of $C1, C2, C4$ constraints are at most $2, 2L$ and 1, respectively. Finally, since no $C3$ constraints are dropped and Algorithm 3 never violates remaining constraints, no $C3$ constraints are violated, which finishes the proof. □

Finally, we summarize the performance of Algorithm 3.

Theorem 11: Algorithm 3 rounds the fractional solution X^* from (P.8) into an integer solution X with time complexity of at most $O(n^4)$, where $n = M\cdot N\cdot K+2M+N+K$. Moreover, X is optimal (in expectation) to (P.6) with bounded constraint violations.

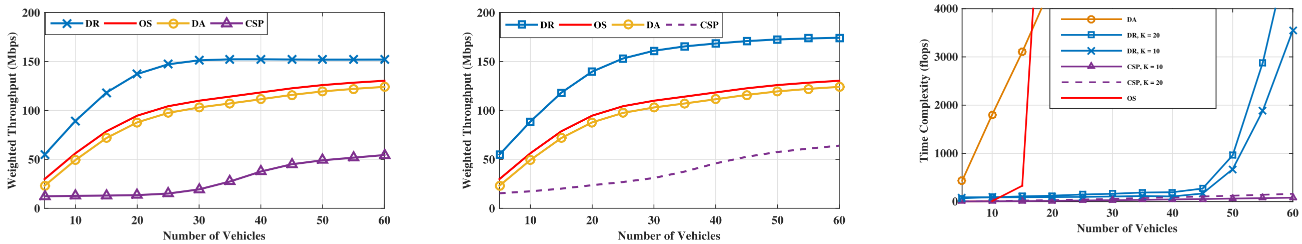
Proof: As mentioned in Section V-A, in every iteration, the proposed dependent rounding algorithm either rounds at least one fractional variable into an integer, or makes at least one untight constraint become tight. Hence, Algorithm 3 must terminate after at most n iterations, where n denotes the total number of variables and constraints in (P.8) [27]. Moreover, in each iteration, all operations have linear complexity over n except for computing null-space of matrix $A(t)$ in Step 10. Since computing the null-space takes at most $O(n^3)$ time using methods like QR decomposition, singular value decomposition and Gaussian elimination etc [29], the total complexity of Algorithm 3 is at most $O(n^4)$. Since (P.8) is linear programming relaxation of (P.6), utility achieved by X^* is an upper bound of the optimal utility in (P.6). Moreover, according to Lemma 8, $E[X] = X^*$. Therefore, X is optimal to (P.6) in expectation. Moreover, the worst case violation is upper bounded as shown in Lemma 10. □

VI. Numerical Results

We consider the coexistence of a CVN with an 802.22 network in a $5km \times 5km$ square region, and the BS is placed at the center of the region. CPEs are uniformly distributed in the whole region, and vehicles are in an $1km \times 10m$ road.

A. Simulation Setup

Channel gain between a transmitter and its receiver is determined by the product of power gains caused by path loss (denoted as G_{PL}), shadowing (denoted as G_{SD}), and multipath fading (denoted as G_{MF}) [30], i.e., $G = G_{PL}\cdot G_{SD}\cdot G_{MF}$. Specifically, the power gain caused by path loss



(a) K = 10 (b) K = 20 (c) Time Complexity

Fig. 2. Performance of the four algorithms.

can be approximated as $G_{PL} = A\frac{(h_t h_r)^2}{d^4}$, where $A = 10$ is assumed to antenna gain, $h_t = h_r = 1\ m$ are heights of the transmitter and receiver, and d is the distance between them. Shadowing loss is assumed to be log-normally distributed, i.e., $10\log G_{SD} \sim N(0,1)$. G_{MF} is assumed to be a constant number (e.g., 1) representing a large sacle fading factor [30].

A 6 MHz TVWS channel is used and return time of PUs on this channel (i.e., t_p) is assumed to follow from a Gamma distribution whose CDF is $F(t_p) = 1 - e^{-5t_p} - 5t_p e^{-5t_p}$. This channel is divided into 20 sub-carriers with equal bandwidth. Hence, each sub-carrier has a bandwidth of 300 kHz. Suppose there are $M = 44$ bursts. Bursts 1 to 12 are Type 1 bursts with length $T = 9\ ms$, and the other 32 bursts are uniformly distributed in $L = 4$ burst intervals with the length of $T/4$. Four priority factors (i.e., available A_p values) take values in $\{1, 2, 3, 4\}$. Transmit power of CPEs (i.e., p_j^c) takes value from 0 W to 4 W uniformly [5]. The maximum transmit power of vehicles on a channel (i.e., P_{max}) is set to 100 mW [7]. The noise power at both CPE and vehicular receivers (i.e., N_0) is set to $-100\ dBm$. The length of upstream sub-frames T is set to 9 milliseconds. The CPE interference limit at the BS for CPE j (i.e., β_j) is chosen based on the criterion that SINR at the BS is no less than 10 dB, i.e., $SINR_j = \frac{p_j^c G_j^{BS}}{\beta_j + N_0} \geq 10$, where G_j^{BS} denotes the channel gain between CPE j and the BS. N takes values from 5 to 60, and for each N we run the algorithms for 100 upstream sub-frames. Since Algorithm 2 and 3 dependes on the discretization of transmit power in (P.6), we consider two simulation scenarios: $K = 10$, and $K = 20$. We compare the performance of our proposed three algorithms with an optimal MINLP solver [31]. The MINLP solver uses the outer approximation algorithm to find the optimal solution to (P.1), which, has exponential time complexity [19], [31].

B. Simulation Results

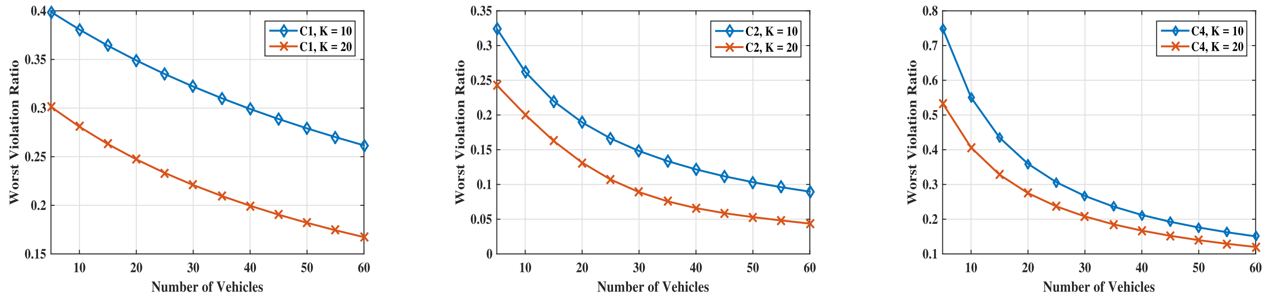
The main metrics we use to compare the proposed algorithms with the optimal MINLP solver are utility (i.e., the expectation of weighted throughput) of the whole vehicular network and time complexity. In addition, since Algorithm 3 can violate some constraints, we also study the average constraint violation ratio as well as the worst case (i.e., largest) constraint violation ratio. Note that in this section, we use OS, DA, CSP and DR to denote the Optimal (MINLP) Solver, Dual Algorithm (i.e., Algorithm 1), Column Sparse Packing-based algorithm (i.e., Algorithm 2) and Dependent Rounding-based algorithm (i.e., Algorithm 3), respectively.

a) Utility: Fig. (2a) and (2b) compare the utility achieved by the four algorithms. We can see that DR achieves higher utility than OS because DR can violate some constraints. Specifically, (P.8) can be viewed as a relaxed problem of the initial MINLP (P.1). Therefore, the optimum of (P.8) is no less than the optimum of (P.1). Since we have proved in Lemma 8 that DR is optimal in expectation, it can achieve higher utility than the optimal solver for (P.1). Comparing Fig. (2a) and Fig. (2b), we can see that larger K values result in higher utility in both CSP and DR because larger K values imply more accurate approximation of the real number transmit power values in (P.1). Fig. (2a) and (2b) also show that all the algorithms achieve higher utility with more vehicles.

b) Time complexity: In Fig. (2c), we compare the time complexity of the four algorithms. Although the practical running time of the four algorithms depends on the hardware platform, the comparison of running time is still meaningful because all the algorithms run on the same hardware platform. All our MATLAB simulation codes run on a Windows workstation with 8 GB memory. Fig. (2c) shows that the running time of DA is higher than CSP and DR.

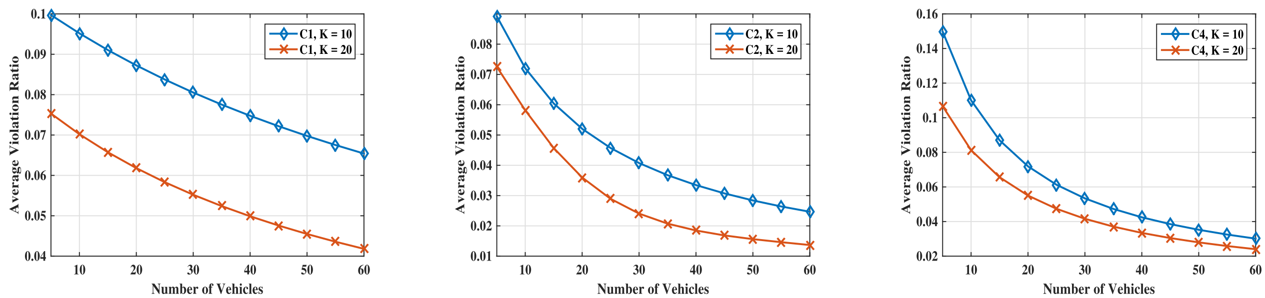
c) Constraint violation: The worst case constraint violation and average constraint violation in DR are shown in Fig. (3) and Fig. (4), respectively. Firstly, Fig. (3) demonstrates that the worst case constraint violations satisfy the upper bounds derived in Lemma 10. More specifically, in this lemma, the worst case violations for $C1$, $C2$, and $C4$ constraints are proved to be upper bounded by 2, $2 \times L = 8$, and 1, respectively. Fig. (3) shows that the worst case violations for $C1$, $C2$, and $C4$ constraints are all less than 1, which verifies Lemma 10. Secondly, Fig. (3) and (4) show that both larger N and K values result in lower worst case and average constraint violations. This is because larger N and K values introduce more random variables than constraints.

d) Overall comparison: The simulation results show that our proposed Algorithm 1 (DA) is able to achieve a near-optimal solution. However, its time complexity is much higher than the others. Therefore, it applies to scenarios where near-optimal solution is required while time complexity can be handled with sufficiently powerful hardware. Although Algorithm 2 achieves the lowest utility, it has the lowest



(a) C1 Constraints (b) C2 Constraints (c) C4 Constraints

Fig. 3. Worst case constraint violations in the DR algorithm.



(a) C1 Constraints (b) C2 Constraints (c) C4 Constraints

Fig. 4. Average constraint violations in the DR algorithm.

time complexity. Hence, it applies to scenarios where time complexity is the main concern. Finally, Algorithm 3 achieves the highest utility with moderate time complexity. Since it can violate some constraints, it applies to scenarios where both high utility and moderate time complexity are required while small constraint violations are tolerable.

VII. Conclusion

In this paper, we study the coexistence problem between a CVN and an 802.22 network. We formulate the coexistence problem as an MINLP resource allocation problem and propose three efficient centralized algorithms. The first algorithm converts the coexistence problem to a convex program and solves the convex program using near-optimal dual method. In addition, we convert the coexistence problem into an LIP by assuming that a fixed number of power levels are used. After solving the LP relaxation of the LIP, we propose two algorithms to round the obtained fractional solution into an integer solution. Finally, the proposed three algorithms are compared with an optimal MINLP solver using numerical examples. The design of distributed algorithms with provable performance guarantees is one of our future works.

References

[1] M. I. Hassan, H. Vu, and T. Sakurai, "Performance analysis of the IEEE 802.11 MAC protocol for DSRC safety applications," *IEEE Trans. Veh. Technol.*, vol. 60, no. 8, pp. 3882–3896, Oct. 2011.

[2] K. Fawaz, A. Ghandour, M. Olleik, and H. Artail, "Improving reliability of safety applications in vehicle ad hoc networks through the implementation of a cognitive network," in *Proc. IEEE 17th Int. Conf. Telecommun. (ICT)*, Apr. 2010, pp. 798–805.

[3] Z. Wang and M. Hassan, "How much of DSRC is available for non-safety use?" in *Proc. 5th ACM Int. Workshop Veh. Inter-Netw.*, New York, NY, USA, Sep. 2008, pp. 23–29.

[4] G. Karagiannis *et al.*, "Vehicular networking: A survey and tutorial on requirements, architectures, challenges, standards and solutions," *IEEE Commun. Surveys Tut.*, vol. 13, no. 4, pp. 584–616, 4th Quart., 2011.

[5] *Second Memorandum Opinion and Order In the Matter of Unlicensed Operation in the TV Broadcast Bands Additional Spectrum for Unlicensed Devices Below 900 MHz and in the 3 GHz Band*, Federal Commun. Commission, Washington, DC, USA, Sep. 2010, pp. 110–174.

[6] Y. Han, E. Ekici, H. Kremo, and O. Altintas, "A survey of MAC issues for TV white space access," *Ad Hoc Netw.*, vol. 27, pp. 195–218, Apr. 2015. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1570870514002431

[7] *IEEE Standard for Information Technology Telecommunications—Part 22: Cognitive Wireless RAN MAC and PHY Specifications: Policies and Procedures for Operation in the TV Bands*, IEEE Standard 802.22, 2011.

[8] T. Shu and M. Krunz, "Coordinated channel access in cognitive radio networks: A multi-level spectrum opportunity perspective," in *Proc. INFOCOM*, Apr. 2009, pp. 2976–2980.

[9] Y. Han, E. Ekici, H. Kremo, and O. Altintas, "Throughput-efficient channel allocation in multi-channel cognitive vehicular networks," in *Proc. IEEE Comput. Commun. (INFOCOM)*, Apr. 2014, pp. 2724–2732.

[10] X. Zhang and K. G. Shin, "Enabling coexistence of heterogeneous wireless systems: Case for ZigBee and WiFi," in *Proc. MobiHoc*, New York, NY, USA, May 2011, pp. 6:1–6:11.

[11] Y. Han, E. Ekici, H. Kremo, and O. Altintas, "Spectrum sharing methods for the coexistence of multiple RF systems: A survey," *Ad Hoc Netw.*, vol. 53, pp. 53–78, Dec. 2016. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1570870516302153

[12] Q. Xin and J. Xiang, "Joint QoS-aware admission control, channel assignment, and power allocation for cognitive radio cellular networks," in *Proc. IEEE 6th Int. Conf. Mobile Adhoc Sensor Syst.*, Oct. 2009, pp. 294–303.

[13] C. N. Hsu, H. J. Su, and P. H. Lin, "Joint subcarrier pairing and power allocation for OFDM transmission with decode-and-forward relaying," *IEEE Trans. Signal Process.*, vol. 59, no. 1, pp. 399–414, Jan. 2011.

[14] Y. Tachwali, B. F. Lo, I. F. Akyildiz, and R. Agusti, "Ultiuser resource allocation optimization using bandwidth-power product in cognitive radio networks," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 3, pp. 451–463, Mar. 2013.

[15] Y. Han, E. Ekici, H. Kremo, and O. Altintas, "Enabling coexistence

of cognitive vehicular networks and IEEE 802.22 networks via optimal resource allocation," in *Proc. 13th Int. Symp. Modeling Optim. Mobile Ad Hoc Wireless Netw. (WiOpt)*, May 2015, pp. 451–458.

[16] S. Li, Z. Zheng, E. Ekici, and N. Shroff, "Maximizing system throughput by cooperative sensing in cognitive radio networks," *IEEE/ACM Trans. Netw.*, vol. 22, no. 4, pp. 1245–1256, Aug. 2014.

[17] Y.-C. Liang, Y. Zeng, E. C. Y. Peh, and A. T. Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 4, pp. 1326–1337, Apr. 2008.

[18] S. Senthuran, A. Anpalagan, and O. Das, "Throughput analysis of opportunistic access strategies in hybrid underlay—Overlay cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 11, no. 6, pp. 2024–2035, Jun. 2012.

[19] M. Köppe, "On the complexity of nonlinear mixed-integer optimization in mixed integer nonlinear programming," in *Mixed Integer Nonlinear Programming* (IMA Volumes in Mathematics and Its Applications), vol. 154, J. Lee and S. Leyffer, Eds. New York, NY, USA: Springer, 2012, pp. 533–557.

[20] M. Ito and M. Fukuda. (Mar. 2014). "A family of subgradient-based methods for convex optimization problems in a unifying framework." [Online]. Available: https://arxiv.org/abs/1403.6526

[21] D. Bertsekas, *Linear Network Optimization: Algorithms and Codes*. Cambridge, MA, USA: MIT Press, 1991.

[22] A. Beck and M. Teboulle, "Mirror descent and nonlinear projected subgradient methods for convex optimization," *Oper. Res. Lett.*, vol. 31, no. 3, pp. 167–175, 2003.

[23] Y. Nesterov, "Primal-dual subgradient methods for convex problems," *Math. Program.*, vol. 120, no. 1, pp. 221–259, 2009.

[24] Y. Shi and Y. Hou, "Optimal power control for multi-hop software defined radio networks," in *Proc. 26th IEEE Int. Conf. Comput. Commun. (INFOCOM)*, May 2007, pp. 1694–1702.

[25] N. Bansal, N. Korula, V. Nagarajan, and A. Srinivasan, "Solving packing integer programs via randomized rouding with alterations," *Theory Comput.*, vol. 8, no. 1, pp. 533–565, 2012.

[26] R. Gandhi, S. Khuller, S. Parthasarathy, and A. Srinivasan, "Dependent rounding and its applications to approximation algorithms," *J. ACM*, vol. 53, no. 3, pp. 324–360, 2006.

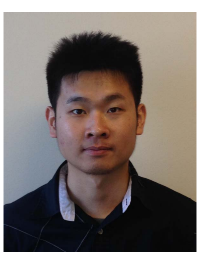
[27] B. Saha, "Approximation algorithms for resource allocation," Ph.D. dissertation, Dept. Comput. Sci., Univ. Maryland, College Park, MD, USA, 2011.

[28] Y. Han, E. Ekici, H. Kremo, and O. Altintas, "Optimal spectrum utilization in joint automotive radar and communication networks," in *Proc. 14th Int. Symp. Modeling Optim. Mobile Ad Hoc Wireless Netw. (WiOpt)*, May 2016, pp. 1–8.

[29] L. Hogben, Ed., *Handbook of Linear Algebra*. CRC Press, 2006.

[30] G. L. Stüber, *Principles of Mobile Communication*, 2nd ed. Norwell, MA, USA: Kluwer, 2001.

[31] P. Bonami *et al.*, "An algorithmic framework for convex mixed integer nonlinear programs," *Discrete Optim.*, vol. 5, no. 2, pp. 186–204, 2008,



You Han (S'14) received the B.E. degree in electrical engineering and automation from Zhejiang University, Hangzhou, China, in 2012. He is currently pursuing the Ph.D. degree with the Department of Electrical and Computer Engineering, The Ohio State University, Columbus, OH, USA. He was an intern with the Google X Lab (now X: The Moonshot Factory) as a Software Engineer from 2014 to 2015, respectively, and with the Google Access and Energy as a Software Engineer in 2016 where he was involved in network simulation, embedded systems, routing algorithm design, and frontier wireless technologies. His research interests include connected vehicles, cognitive radio, network optimization theory, and resource management in wireless networks.

Eylem Ekici (S'99–M'02–SM'11–F'17) received the B.S. and M.S. degrees in computer engineering from Boğaziçi University, Istanbul, Turkey, in 1997 and 1998, respectively, and the Ph.D. degree in electrical and computer engineering from the Georgia Institute of Technology, Atlanta, GA, USA, in 2002. He is currently a Professor with the Department of Electrical and Computer Engineering, The Ohio State University, Columbus, OH, USA. His current research interests include cognitive radio networks, vehicular communication systems, and next generation wireless systems, with a focus on algorithm design, medium access control protocols, resource management, and analysis of network architectures and protocols. He is also the TPC Co-Chair of the IEEE INFOCOM 2017. He is currently an Associate Editor of the IEEE TRANSACTIONS ON MOBILE COMPUTING and the Computer Networks Journal (Elsevier). He was an Associate Editor of the IEEE/ACM TRANSACTIONS ON NETWORKING.

Haris Kremo (M'12) received the Dipl.Ing. degree from the School of Electrical Engineering, University of Sarajevo, Bosnia and Herzegovina, in 2000, and the M.S. and Ph.D. degrees from Rutgers, The State University of New Jersey, in 2005 and 2010, respectively. From 2000 to 2002, he was a Research Engineer with Energoinvest, d.d.,-Sarajevo. From 2003 to 2010, he was a Graduate Assistant with the Wireless Information Networks Laboratory, where he was one of the engineers involved in design and maintenance of the ORBIT wireless testbed. In 2010, he was with the EDGE Lab, Princeton University, as a Post-Doctoral Research Associate. He was a Post-Doctoral Researcher with the Toyota InfoTechnology Center Co., Ltd., Tokyo. He is currently a Research Fellow with the CONNECT Center, Trinity College Dublin, Dublin, Ireland.

Onur Altintas (S'92–M'95) received the B.S. and M.S. degrees from Orta Dogu Teknik Universitesi, Ankara, Turkey, and the Ph.D. degree from the University of Tokyo, Japan, all in electrical engineering. He was a Research Scientist with the Ultra High Speed Network and Computer Technology Labs, Tokyo. From 1999 to 2001, he was with the Toyota Motor Corporation and a Visiting Researcher with Telcordia Technologies. From 2001 to 2004, he was with the Toyota InfoTechnology Center, U.S.A., Inc., Mountain View, CA, USA, where he is currently a Fellow with the Network Division. He is also an IEEE VTS Distinguished Lecturer. He has been the Co-Founder and the General Co-Chair of the IEEE Vehicular Networking Conference since 2009. He currently serves as an Associate Editor of the IEEE *ITS Magazine* and in the Editorial Board of Connected Vehicles Series of the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY.