# (CORN)$^2$: Correlation-Based Cooperative Spectrum Sensing in Cognitive Radio Networks

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*Abstract*—In this paper, (CORN)$^2$, a correlation-based, optimal sensing scheduling algorithm is developed for cognitive radio networks to minimize energy consumption. A sensing quality metric is defined as a measure of the correctness of spectral availability information. The optimal scheduling algorithm is shown to minimize the cost of sensing (e.g., energy consumption, sensing duration) while meeting the sensing quality requirements. To this end, (CORN)$^2$ utilizes a novel sensing deficiency virtual queue concept and exploits the correlation between spectrum measurements of a particular secondary user and its collaborating neighbors. The proposed algorithm is further proved to achieve a distributed and optimal solution under certain, easily satisfied assumptions. In addition to the theoretically proved performance guarantees, the proposed algorithm is also evaluated through simulations.

## I. Introduction

Today's wireless networks are characterized by a fixed spectrum assignment policy. However, a large portion of the assigned spectrum is used sporadically and geographical variations in the utilization of assigned spectrum range from 15% to 85% with a high variance in time. The limited available spectrum and the inefficiency in the spectrum usage necessitate a new communication paradigm to exploit the existing wireless spectrum opportunistically. This new networking paradigm is referred to as cognitive radio networks (CRNs) [2]. Based on the ambient spectrum information, cognitive radio users (or secondary users (SUs)) communicate via available channels without disrupting the communication of spectrum owners (or primary user (PUs)).

To assess the spectral availability while maintaining efficient operation of CRNs, effective spectrum sensing solutions are required [2]. Recently, spectrum sensing solutions have been developed to provide high detection probability and minimize false alarm rates, where mainly physical layer metrics are considered. In general, spectrum sensing solutions can be classified as *cooperative* and *non-cooperative* [2]. More specifically, cooperative solutions rely on multiple SUs to exchange spectrum occupancy information through individual local measurements. This can be achieved through cluster-based architectures [5], wherein the CRN is divided into clusters and each cluster-head makes a decision on the availability of channels. Spectrum availability is assessed by leveraging spectrum utilization information from different cluster heads

that receive the local observations of SUs in their clusters. Cooperative sensing schemes are also utilized to estimate the maximum transmit power in cognitive networks so that the interference constraints are satisfied [9].

The existing studies indicate that collaboration among SUs improves the efficiency of spectrum utilization, and allows relaxation of the constraints at individual SUs [12], [25]. However, **network-wide** effects of spatio-temporal sensing have not been formally analyzed except for heuristics in [25], [26]. While collaboration is shown to improve sensing efficiency at the physical layer, two major tradeoffs exist in terms of network-wide considerations: (1) Cooperative spectrum sensing introduces communication overhead for the dissemination of local observations between SUs. Accordingly, a large number of SUs used for cooperation results in a higher communication cost irrespective of whether a cluster-based or a flat topology is employed. Consequently, energy consumption associated with such communication overhead increases with increasing cooperation. (2) Spectrum utilization observed by closely located SUs is highly correlated due to the inherent spatial correlation in the received PU signals and correlated shadow fading. In addition to the spatial correlation between SUs, the observed information is also correlated in the time domain. More specifically, spectrum information gathered at a particular time represents the spectrum activity at a later time, where the certainty decreases with time difference due to the temporally-correlated nature of the PU activity. If an SU performs local sensing, the additional use of highly correlated spectrum information has minimal effect on improving sensing accuracy [12], [25], [31]. On the other hand, the communication overhead increases regardless. These tradeoffs are generally exploited to limit the number of sensors that collaborate for a spectrum sensing task [30].

The existing work, however, considers local sensing is performed all the time. Instead, we argue that, *spectrum sensing information at a given space and time can* **represent** *spectrum information at a different point in the space-time space*. Accordingly, an SU can improve its sensing quality at a particular time by using spectrum information observed by a different SU at a different time instead of local sensing.

Identifying the main objective of sensing as maintaining a given minimum sensing quality, in this paper, we explore

cooperative methods that will minimize the ***cost of sensing***. The cost can flexibly be defined as a means to represent the resources spent (such as energy) or opportunities sacrificed for sensing (such as sensing duration). Since cooperation requires information exchange, these costs will also explicitly incorporate communication activities. Accordingly, we develop a ***provably optimal cooperative algorithm*** through a novel *sensing deficiency virtual queue* concept and exploit the correlation between SUs. The optimal algorithm further leads to a ***distributed solution*** when correlation weights are appropriately upper-bounded, which holds especially in low SNR environments with a high level of temporal correlation of spectrum sensing information.

The rest of the paper is organized as follows: In Section II, related works in spectrum sensing correlation and utility-optimal scheduling in wireless networks are discussed. The problem description and the models used to represent spectrum sensing are introduced in Section III. We introduce (CORN)$^2$, an optimal correlation-based cooperative spectrum sensing scheduling algorithm, in Section IV and analyze its theoretical performance in Section V. A numerical evaluation of (CORN)$^2$ is presented in Section VI. The paper is concluded in Section VII.

## II. RELATED WORK

Spectrum management in CRNs has recently been investigated in the literature [3]. Most notably, optimal spectrum sensing schemes have been developed for non-cooperative and cooperative spectrum sensing solutions [6], [7], [10], [11], [13]. To this end, recently, *spectral* correlation is exploited to minimize the spectrum sensing cost without hampering the spectrum sensing accuracy [4], [8]. However, these solutions focus only on the spectral content of the signal received by a single user and do not address *spatial* correlation between SUs. In addition, spatial correlation between signals received at closely located SUs have been investigated [12], [25], [31]. Furthermore, due to primary signal characteristics, the spectrum activity is also *temporally* correlated. To the best of our knowledge, existing methods that are developed to address the spectrum management problem do not consider the system-wide effects of spatio-temporal correlation in spectrum sensing. More specifically, correlation has not been explicitly leveraged to improve system throughput and energy efficiency.

The sensing problem investigated in this paper is a utility optimal scheduling problem with energy minimization. Optimum scheduling for wireless communication networks has been studied in the past in detail. The seminal work on back-pressure-based scheduling [14] and its extensions have been widely employed in developing optimal scheduling in wireless networks. Throughput/utility-optimal routing and scheduling algorithms have been developed in [16], [17], [18]. These optimal scheduling algorithms are generally computationally prohibitive and impractical for distributed implementations. Distributed algorithms are proposed in [19], [20], [21], [22] at the sacrifice of throughput/utility optimality. In our work, we show that when correlation weights are appropriately bounded

above, the proposed algorithm can achieve both distributed implementation and optimality of cost of sensing.

## III. PRELIMINARIES

### A. Objective and Motivation

Information about spectral availability is the key for a healthy CRN. We define the ***Sensing Quality*** as a measure of the correctness of spectral availability information. An instance of sensing quality can be formulated as a combination of false alarm and mis-detection probabilities of a particular sensing algorithm. Under our proposed framework, the sensing quality is a dynamic measure of spectral availability information. It varies not only based on the inherent limitations of individual sensors (e.g., ability to distinguish PU channel usage from noise), but also as a function of time and the origin of the information. For the sake of simplicity, consider a perfectly accurate spectral sensing result generated by a given node. Traditionally, the rigid period of validity is assumed for such information. However, the validity of information can be represented as a non-increasing function in time as long as it is not augmented with new measurements. Moreover, if the sensing information is shared in a neighborhood, its ability to represent the correct channel availability information generally diminishes with distance as well as time. As such, we envision that every cognitive node will strive to maintain a minimum sensing quality level to ensure proper operation. In the following, we introduce the models to represent spectrum sensing.

### B. Spectrum Sensing Quality

Consider a network with a PU and a set of SUs $\mathcal{N}$, where $N_s = |\mathcal{N}|$, and the wireless spectrum $\mathcal{S}$ divided into spectrum bands $c$, where $c \in \mathcal{S}$ and $N_c$ is the number of spectrum bands. Any one of these spectrum bands can be occupied by a PU at any time. Accordingly, the activity of a PU in a spectrum band $c$, at location $\mathbf{s}_p = \{x_p, y_p\}$ and time $t$ is denoted as a binary variable $A_p(c, \mathbf{s}_p, t)$, which is equal to 1 if the PU is transmitting.

We consider a spectrum sensing mechanism, where each SU $i$ samples the energy of a set of bands $Ch(i) \subseteq \mathcal{S}$ using energy detection. Accordingly, the SNR sampled by an SU $i$ at location $\mathbf{s}_i = \{x_i, y_i\}$ is denoted by $Y_i[c_i, n_i] = y(c_i, \mathbf{s}_i, t_{n_i})$ for a band $c_i$ at discrete time $t_{n_i} = n_i T_u$, where $T_u$ is unit time. Assuming all powers are normalized according to the transmit power of the primary transmitter and the PU resides at the center of the coordinate system, the received energy sampled by the SU $i$ can be modeled as follows:

$$y(c_i, \mathbf{s}_i, t_{n_i}) = A_s(c_i, \mathbf{s}_i, t_{n_i}) + W, \qquad (1)$$
$$A_s(c_i, \mathbf{s}_i, t_{n_i}) = \alpha(c_i, \mathbf{s}_i, t_{n_i}) A_p(c_i, \mathbf{0}, t_{n_i} - t_{pr_i}) \quad (2)$$

where $A_s(c_i, \mathbf{s}_i, t_{n_i})$ is the attenuated and delayed version of the PU signal, $A_p(c, \mathbf{0}, t)$, at the location of the SU $i$, $\mathbf{s}_i$, at time $t_{n_i}$; $\alpha(c_i, \mathbf{s}_i, t_{n_i})$ models the attenuation and fading in the channel; $\mathbf{0} = \{0, 0\}$ is the location of the PU; $W$ is the detection noise at the SU, and $t_{pr_i}$ is the time delay due to

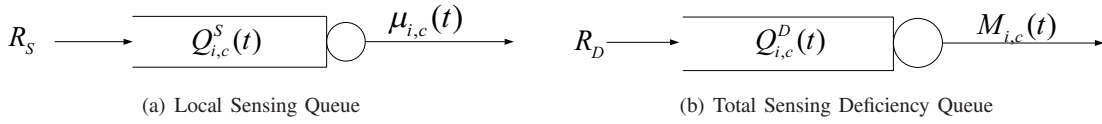(a) Local Sensing Queue      (b) Total Sensing Deficiency Queue

Fig. 1. Virtual Queue Structures for Sensing Quality Control.

propagation. In the following, we assume that the propagation delay is negligible and $t_{pr_i} = 0$.

Now consider another SU $j$ at location $\mathbf{s}_j$ that performs spectrum sensing for the band $c_i$ at time $t_{n_j}$. Through collaboration, the SU $j$ can use the sensing information of SU $i$ at a different time and location for spectrum sensing. Due to the correlation in spatial and temporal dimensions, the sensed information at SUs $i$ and $j$ are correlated. More specifically, the spatio-temporal correlation function $\rho(\cdot)$ for two channel samples taken in channel $c_i$; at locations $\mathbf{s}_i$ and $\mathbf{s}_j$; and at times $t_{n_i}$ and $t_{n_j}$ is given by

$$\rho(\Delta_{i,j}^s, \Delta_{i,j}^t) \triangleq \frac{E\left[Y_i[c_i, n_i] Y_j[c_i, n_j]\right]}{\sigma_Y^2} = \rho_s(\Delta_{i,j}^s) \cdot \rho_t(\Delta_{i,j}^t) \tag{3}$$

where $\sigma_Y^2$ is the variance of the signal, $\Delta_{i,j}^s = ||\mathbf{s}_i, \mathbf{s}_j||$, and $\Delta_{i,j}^t = |t_{n_i} - t_{n_j}|$ are the differences in spatial and temporal dimensions. Without loss of generality, we model the correlation function with spatial ($\rho_s(\cdot)$) and temporal ($\rho_t(\cdot)$) components.

Note that recent long-term and large-scale spectrum sensing experiments also confirm the existence of correlation in space and time [35]. Small-scale temporal correlation is generally exploited within spectrum sensing algorithms in terms of sample averaging or cyclostationary feature analysis as a means to improve sensing accuracy. This correlation is negligible when the time difference between different spectrum sensing attempts are considered. Consequently, we model the spatial and temporal correlation in spectrum sensing in (3) as follows [32], [33], [34]:

$$\rho_s(\Delta_{i,j}^s) = e^{-\frac{\Delta_{i,j}^s}{D_{corr}}}; \quad \rho_t(\Delta_{i,j}^t) = e^{-\frac{\Delta_{i,j}^t v_i}{D_{corr}}} \tag{4}$$

where $D_{corr}$ is the decorrelation distance and $v_i$ is the speed of the SU.

Now, consider a multi-user spectrum sensing setting, where a node $j$ performs spectrum sensing according to the following observation definition:

$$T_j(c_j, n_j) = \sum_{i \in \mathcal{I}_j} w_{i,j}(c_j, n_j) Y_i[c_i, n_i], \tag{5}$$

where $\mathcal{I}_j$ is the set of neighbors of $j$, including the node $j$, within which cooperative sensing can be performed. If non-cooperative sensing is employed, $w_{i,j}(c_j, n_j) = 0$, $\forall i \neq j$ and $w_{j,j}(c_j, n_j) = 1$. For cooperative sensing, the weight factor, $w_{i,j}(c_j, n_j)$, can be selected according to the correlation between spectrum samples as modeled in (3) and (4).

It is clear that it takes a relatively long time to sense the *whole* spectrum in a wide band cognitive radio network. Instead, it might be more efficient to share channels to be sensed among neighbors, which have correlated observations. Intuitively, if the local observation of node $j$ on a particular channel is highly correlated with one of its neighbors, the node may use the neighbor's information only without any local sensing on that channel. Such cooperation decreases the cost for spectrum sensing. On the other hand, if node $j$ performs local sensing on a channel, any information from its closest neighbors would be highly correlated and will not improve sensing accuracy [28]. Thus, the weight in (5) is modeled such that $w_{i,j}(c_j, n_j) = w'_{i,j}(c_j, n_j) / \sum_i w'_{i,j}(c_j, n_j)$, where $w'_{i,j}(c_j, n_j)$ is found based on whether node $j$ performs local sensing in a channel or not.

$$w'_{i,j}(c_j, n_j) = \begin{cases} \rho & \text{if } \mu_{j,c_j} = 0 \\ \rho\sqrt{1 - \rho^2} & \text{if } \mu_{j,c_j} = 1 \end{cases} \tag{6}$$

where $\mu_{j,c_j}$ is an indicator which is set to 1 if node $j$ performs local sensing on channel $c_j$. If local sensing is not performed ($\mu_{j,c_j} = 0$), the sensing weights for neighbors' observations are proportional to their correlation. If local sensing is performed ($\mu_{j,c_j} = 1$), then the weights are scaled by $\sqrt{1 - \rho^2}$, which corresponds to the variance of conditional pdf of node $j$ observation given node $i$'s observation [33]. Accordingly, if the neighboring node is too close, its observation is very similar to node $j$ and provides no additional *information* about the spectrum. Furthermore, if the node is too far away from node $j$, then it may be observing a completely different channel and hence, its contribution is also limited.

Existing work so far has assumed all the information is available at an SU or a fusion center and the correlation aspects are considered accordingly. However, communication overhead increases to exchange local observations. With the notion of spectrum representativeness, this overhead can be mitigated. Next, we propose a novel model to capture this tradeoff in correlated sensing performance.

## IV. CORRELATION-BASED COOPERATIVE SPECTRUM SENSING SCHEDULING

In this section, we model the CRN from the perspective of individual SUs and their requirements of sensing information quality. Accordingly, we develop an optimal cooperative sensing scheduling algorithm and its distributed implementation through a novel *sensing deficiency virtual queue* concept.

### A. Problem Definition

Let an SU $i$ have access to a set of communication channels $Ch(i) \subseteq \mathcal{S}$. For sensing accuracy, we associate each pair of SU $i$ and channel $c$, $c \in Ch(i)$, with a minimum *rate* of information quality $R_D$ that needs to be maintained at all times. This minimum level can be achieved by sensing

the channel locally and/or by exchanging spectrum sensing reports between other SUs in the vicinity. The sensing quality is assumed to decay in time at a constant rate, and needs to be supplemented with additional sensing data. The cooperative nature of this framework stems from sharing of the sensing information within $\mathcal{I}_i$ neighborhoods. In addition to cooperation, each SU $i$ must also sense a channel $c$ locally at a rate of $R_S$. This requirement forces each SU $i$ to participate in sensing above a minimum rate and not rely solely on other nodes' observations. For analytical simplicity, we assume $R_D$ and $R_S$ are constant. The analysis can be easily extended to the case where $R_D$ and $R_S$ change over SUs and channels (i.e., $R_D$ and $R_S$ can be replaced with $R_D(i,c)$ and $R_S(i,c)$, $i \in \mathcal{N}$, $c \in Ch(i)$).

When an SU $i$ senses a channel $c$ at a discrete time $t$, this event contributes to its sensing quality by $\mu_{i,c}(t)$[1], where $\mu_{i,c}(t) \in \{0,1\}$ is defined an indicator of the sensing event of SU $i$ over channel $c$ at time $t$. $\mu_{i,c}(t)$ can also be considered as an integer value corresponding to the <u>normalized</u> quality of sensing. On the other hand, when the sensing information of another SU $j$ is used, its contribution is scaled by a factor of $w^c_{j,i}(t)$, the **correlation weight**, which captures the representativeness of $j$'s sensing data about channel $c$ at $i$. It is computed using methods outlined in Section III-B. At any given discrete point in time $t$, a node $i$ improves its sensing information quality about a channel $c$ by $M_{i,c}(t) \triangleq \min\left(\sum_{j \in \mathcal{I}_i} \mu_{j,c}(t) w^c_{j,i}(t), M^{\max}_{i,c}\right)$, where $M^{\max}_{i,c}$ represents the normalized maximum level of information that $i$ can obtain about $c$'s state, and $w^c_{i,i}(t) \triangleq 1$. The linear combination approximation of sensing quality holds especially in low SNR environments, where the PU signal is potentially sensed from sources located far from SUs, such as TV transmitters, and potential PU receivers are nearby, such as TV receivers. We also associate each local sensing event with a fixed cost $P_S$. While, in general, a node is not required to broadcast its sensing data, in a cooperative setting, we assume that an SU always broadcasts the result to its neighboring nodes at the cost of $P_{Tx}$. An SU receives this information at the cost of $P_{Rx}$. The parameters, $P_S$, $P_{Tx}$, and $P_{Rx}$, can readily be associated with energy consumption for the respective activities. In every discrete time instant, the cost $G_i(t)$ is computed as

$$G_i(t) = \sum_{c \in Ch(i)} \left[ (P_S + P_{Tx}) \mu_{i,c}(t) + \sum_{j \in \mathcal{I}_i \setminus \{i\}} P_{Rx} \mu_{j,c}(t) \right].$$

We would like to find the best scheduling policy $\Omega^* = \{\mu_{i,c}(t)\}_{i,c,t}$ to minimize the cost while satisfying the sensing information quality requirements.

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[1]For the simplicity of notation, we limit the discussion to temporal and spatial dimensions. A generalization of the approach also involves accounting for the additional information obtained for a channel $c'$ when a channel $c$ is sensed.

**Problem Formulation:**
Consider a CRN with SUs $i \in \mathcal{N}$. Find an optimal sensing scheduling policy $\Omega^*$ such that the network-wide cost is minimized while all individual sensing quality requirements are satisfied:

$$\text{minimize} \quad \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{i \in \mathcal{N}} G_i(t) \qquad (7)$$

$$\text{subject to} \quad \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mu_{i,c}(t) \geq R_S \qquad (8)$$

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} M_{i,c}(t) \geq R_D \qquad (9)$$

$$\forall i \in \mathcal{N}, \ c \in Ch(i),$$

where $\sum_{c \in Ch(i)} \mu_{i,c}(t) \leq K$ for a given $i$ and $t$, i.e., an SU can sense up to $K$ channels at any point in time.

Note that the above problem formulation aims to achieve minimum sensing quality rates asymptotically. While the first constraint in (8) forces each node to perform local sensing, the second constraint in (9) encourages collaboration. The problem can further be refined by including the decision to share or withhold sensing information, as well. However, for the sake of tractability, we limit the discussion to always sharing cases. Similarly, we assume that the broadcasting of information can be performed in a lossless manner with the help of the scheduling algorithm.

### B. Optimal Sensing Scheduling Algorithm

To solve the minimum-cost sensing scheduling problem, we present a novel **virtual deficiency queue** concept, where the *sensing dynamics are represented with virtual queuing structures*, operating in discrete time domain. We define two types of *per-node, per-channel virtual queues* that track the dynamics of sensing quality as shown in Fig. 1. The first virtual **local sensing queue** $Q^S_{i,c}(t)$ represents the local sensing events for channel $c$ in SU $i$ with a periodic arrival of "packets" at rate $R_S$ and an instantaneous service rate of $\mu_{i,c}(t)$. The evolution of the local sensing queue follows

$$Q^S_{i,c}(t+1) = [Q^S_{i,c}(t) + R_S - \mu_{i,c}(t)]^+, \qquad (10)$$

where $[a]^+ \triangleq \max\{a, 0\}$. A "packet" arrival to the local sensing queue represents an increase in need for local sensing, which is satisfied when the packet is "served" and departs from the queue, i.e., node performs local sensing.

The second virtual queue is the **total sensing deficiency queue** $Q^D_{i,c}(t)$ for channel $c$ at SU $i$ with a periodic arrival of "deficiency packets" at rate $R_D$ and instantaneous service rate of $M_{i,c}(t)$. Each deficiency packet arrival to $Q^D_{i,c}(t)$ represents a decay in sensing quality. The decay in sensing quality is countered by the departure of "deficiency packets", which corresponds to the improvement of the sensing information quality about a channel $c$. In general, a large value of $Q^D_{i,c}(t)$ corresponds to a large sensing deficiency, i.e., a low sensing information quality. With local sensing events and sensing reports gathered

from neighbors, deficiency packets are "served", which causes the $Q_{i,c}^D$ to shrink. This, in turn, corresponds to an improved sensing quality for the given SU-channel pair. The evolution of the total sensing deficiency queue follows

$$Q_{i,c}^D(t+1) = [Q_{i,c}^D(t) + R_D - M_{i,c}(t)]^+. \quad (11)$$

When a sensing scheduling algorithm stabilizes the virtual queues $Q_{i,c}^S(t)$ and $Q_{i,c}^D(t)$, then this means that their respective arrival rates are smaller than or equal to their average service rates, i.e.,

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mu_{i,c}(t) \geq R_S, \quad \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} M_{i,c}(t) \geq R_D,$$

which are the two conditions of the general problem formulation expressed in (8) and (9). Therefore, when a feasible algorithm stabilizes the system, the constraints of the optimal sensing scheduling problem are automatically satisfied. Consequently, the system parameters $R_S$ and $R_D$ appear in the system model as arrival rates to our virtual queue structures. Without loss of generality, we assume that arrivals to the virtual queues occur periodically with rates $R_S$ and $R_D$, respectively.

At this point, we note that the sensing scheduling algorithm also bears significant differences from earlier works on power allocation: In single-channel power-optimal scheduling problem [15], [23], the channel state is known *a priori*, whereas in the power optimal sensing scheduling algorithm, multiple neighboring nodes are required to sense channels and share the information among themselves. Moreover, while a packet transmission has a direct effect on the reduction of queues (namely, on the queues of the transmitting node), a sensing event reduces the deficiency queues of multiple nodes in the neighborhood. Since a service decision for a node affects more than one other neighbor, the solution structure is completely different than traditional scheduling solutions.

The optimal cooperative sensing scheduling algorithm that solves the target problem is given below, with its optimality discussed in Section V.

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**Optimal Cooperative Sensing Scheduling Algorithm:**
Consider a CRN with SUs $i \in \mathcal{N}$. For each time slot $t$,

$$\max_{(\mu_{i,c}(t))} \sum_{i \in \mathcal{N}} \sum_{c \in Ch(i)} [\mu_{i,c}(t)(Q_{i,c}^S(t) \quad (12)$$

$$- V(P_S + P_{Tx} + P_{Rx}(I_i^c - 1)))$$
$$+ (\min\{\sum_{j \in \mathcal{I}_i} \mu_{j,c}(t)w_{j,i}^c(t), M_{i,c}^{\max}\})Q_{i,c}^D(t)],$$

$$\text{s.t.} \sum_{c \in Ch(i)} \mu_{i,c}(t) \leq K, \forall i \in \mathcal{N}, \quad (13)$$

where $V$ is a control parameter to be discussed in Section V and $I_i^c \triangleq \sum_{j \in \mathcal{I}_i} \mathbf{1}_{c \in Ch(j)}$, i.e., the number of neighboring SUs that can contribute to SU $i$'s sensing quality of channel $c$, including the node $i$.

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This optimal algorithm requires network-wide information to compute the sensing schedules for each SU. It can be executed centrally by a cluster head if the SUs are organized into (multi-hop or single-hop) clusters. However, it is possible for individual SUs to compute their own sensing schedules using information only from their cooperation neighborhood $\mathcal{I}_i$ under certain conditions as will be discussed next.

### C. Distributed Optimal Solution under Bounded Contribution Assumption

The general optimal algorithm of (12) uses a cap on the maximum amount of information that can be obtained in a sensing cycle, which is represented as $M_{i,c}^{\max}$. While this quantity has practical significance by not allowing arbitrary levels of accuracy to "accumulate" within a sensing cycle, it also complicates the algorithm execution. This parameter can be removed from consideration when correlation weights are appropriately upper-bounded such that the *bounded contribution assumption* holds:

$$\sum_{j \in \mathcal{I}_i} \mu_{j,c}(t)w_{j,i}^c(t) \leq M_{i,c}^{\max}, \forall i \in \mathcal{N}, \forall c \in Ch(i), \forall t. \quad (14)$$

This assumption especially holds in low SNR environments and when the temporal correlation levels of spectrum sensing information are high (i.e., $M_{i,c}^{\max}$ is sufficiently large). Under this assumption, the optimization (12) can be simplified as follows:

$$\max_{(\mu_{i,c}(t))} \sum_{i \in \mathcal{N}} \sum_{c \in Ch(i)} \mu_{i,c}(t)[Q_{i,c}^S(t) + \sum_{j \in \mathcal{I}_i} w_{i,j}^c(t)Q_{j,c}^D(t)$$
$$- V(P_S + P_{Tx} + P_{Rx}(I_i^c - 1))], \quad (15)$$

subject to the channel sensing constraints (13). (15) is a typical maximal weight matching problem on the bipartite graph formed by the node set $\mathcal{N}$ and the channel set $\mathcal{S}$. Since the matching weight $[Q_{i,c}^S(t) + \sum_{j \in \mathcal{I}_i} w_{i,j}^c(t)Q_{j,c}^D(t) - V(P_S + P_{Tx} + P_{Rx}(I_i^c - 1))]$ in (15) can be obtained locally for each SU $i$ and channel $c$, the optimization problem can be optimally solved in a greedy and distributed manner. Specifically, each SU $i \in \mathcal{N}$ chooses the first $\max\{Ch(i), K\}$ channels with largest matching weights over which SU $i$ performs sensing.

*Remark 1:* In each time slot, the optimization problem (15) can be solved ***locally and optimally*** by each SU using the sensing deficiency queue information and the correlation weight of its neighbors, without requiring network-wide information.

### V. OPTIMALITY OF THE COOPERATIVE SENSING SCHEDULING ALGORITHM

In this section, we present the main theorem that shows the optimality of our cooperative sensing scheduling algorithm and establish its performance measures. This theorem is based on the following assumptions:

*Assumption 1:* We assume that $w_{i,j}^c(t)$ is fixed over any time slot duration and i.i.d. over time slots, with values taking from a finite set.

*Assumption 2:* Without loss of generality, we assume that the cooperation between any two nodes is fair, i.e., for any channel

$c$, $\forall i, j \in \mathcal{N}$, we have: $c \in Ch(i)$ and $i \in \mathcal{I}_j$ *if and only if* $c \in Ch(j)$ and $j \in \mathcal{I}_i$.

*Assumption 3:* Without loss of generality, we assume that the sensing information quality is capped by some constant $M_{\max}$:

$$\sum_{j \in \mathcal{I}_i} \mu_{j,c}(t) w_{j,i}^c(t) \leq M_{\max}, \forall i \in \mathcal{N}, \forall c \in Ch(i).$$

Note that if the assumption (14) holds, we can let $M_{\max} \triangleq \max_{i,c} M_{i,c}^{\max}$.

We denote by $(g_i)_{i \in \mathcal{N}}$ the cost rate vector (i.e., time-average of $(G_i(t))_{i \in \mathcal{N}}$) induced by a generic sensing schedule $(\mu_{i,c}(t))_{i \in \mathcal{N}, c \in Ch(i)}$. Let the *feasible cost set*, $\mathcal{C}$, be defined as the closure of the set of all cost rate vectors of schedulers satisfying the channel assignment constraint (13). For some small $\epsilon > 0$, let $(g_{i,\epsilon}^*) \in \mathcal{C}$ be the minimum cost rate vector with a corresponding scheduler $(\mu_{i,c}(t))$ satisfying

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mu_{i,c}(t) \geq R_S + \epsilon, \quad \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} M_{i,c}(t) \geq R_D + \epsilon.$$

According to [15], $\lim_{\epsilon \to 0^+} \sum_{i \in \mathcal{N}} g_{i,\epsilon}^* = \sum_{i \in \mathcal{N}} g_i^*$ where $(g_i^*) \in \mathcal{C}$ is the minimum cost rate vector that satisfies the conditions (8) and (9).

The following theorem states the cost optimality and the stability of virtual queues, under the scheduler (12), and under the scheduler (15) if the assumption (14) holds:

*Theorem 1:* Given $\epsilon > 0$, the algorithm can achieve a time-averaged ensemble cost:

$$\limsup_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{G_i(t)\} \leq \sum_{i \in \mathcal{N}} g_{i,\epsilon}^* + \frac{B}{V}, \quad (16)$$

with all the virtual queues bounded by:

$$\limsup_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{i \in \mathcal{N}} \sum_{c \in Ch(i)} \mathbb{E}\{Q_{i,c}^D(t) + Q_{i,c}^S(t)\}$$
$$\leq \frac{V \sum_{i \in \mathcal{N}} g_{i,\epsilon}^* + B}{\epsilon}, \quad (17)$$

where $B \triangleq \frac{1}{2}(R_D^2 + R_S^2 + M_{\max} + 1) \sum_{i \in \mathcal{N}} Ch(i)$.

*Proof:* The proof of Theorem 1 is provided in [36]. ∎

The inequality (17) indicates that the virtual queues are stable, and hence the constraints (8) and (9) are satisfied. Since $B$ is independent of the control parameter $V$, the inequality (16) states that the cost of the proposed algorithm can be arbitrarily close to $\sum_{i \in \mathcal{N}} g_{i,\epsilon}^*$, when $V$ is chosen large enough. That is, the algorithm can approach arbitrarily to the optimal cost $\sum_{i \in \mathcal{N}} g_i^*$ when $\epsilon$ is chosen small enough and $V$ large enough. Note that a smaller value of $\epsilon$ and a larger value of $V$ increase the upper-bound of the virtual queues (17), and results in slower convergence of the algorithm.

## VI. NUMERICAL EVALUATION

The optimal sensing scheduling algorithm described in Section IV is evaluated in this section using numerical analyses in Matlab. For the evaluations, a CRN of $N$ SU nodes operating on a single channel is considered, where each node tries to estimate the spectrum occupancy on the channel using either non-cooperative or cooperative sensing employing the optimal scheduling algorithm. Non-cooperative sensing is modeled as a special case of the optimal scheduling algorithm, where the sensing deficiency queue is not considered ($R_D = 0$) and the spectrum sensing is scheduled according to only the local sensing queue. In non-cooperative sensing cases, the local sensing rate is set to the maximum of the $R_D$ and $R_S$ parameters used in the cooperative sensing case for fairness. In our numerical evaluations, the cost represents energy consumption associated with sensing. In all tests, we use the following parameters: $P_S = 3.5$mJ, $P_{Tx} = P_{Rx} = 0.1125$mJ, which are consistent with the values reported in [24], [27]. For cooperative sensing, all $N$ nodes are assumed to cooperate in sensing, i.e., $\mathcal{I}_i = \mathcal{N}$, $\forall i \in \mathcal{N}$. The maximum level of information that can be obtained in a single time slot is limited to 2, i.e., $M_{i,c}^{\max} = 2$, $\forall i \in \mathcal{N}$ and the channel $c$ considered. For notation simplicity, $w_{i,j}^c(t)$ is replaced by $w_{i,j}(t)$ in the following analysis. To further simplify the algorithm evaluation, we assume in this section that $w_{i,j}(t)$ is fixed over time $t$. Each data point represents the average values observed over 10000 simulated time slots.

In Fig. 2(a), the energy consumption per node per time slot is shown as a function of the number of nodes in the network. The energy consumption performance is investigated for different rate of information quality, $R_D$, values and $R_S = 0.55$. It can be observed that cooperative sensing with the optimal scheduling algorithm improves energy consumption compared to non-cooperative sensing when the number of nodes exceeds three, mainly owing to the fact that cost of cooperation is offset by its benefits for larger networks. For small clusters, $R_D$ is observed to have a negative impact on energy efficiency, where the energy consumption of cooperative sensing can be as much as 75% higher than that of non-cooperative sensing. On the other hand, as the number of cooperating nodes increases, $R_S$ dominates scheduling decisions since satisfying local sensing queue constraints becomes sufficient to satisfy any sensing deficiency queue constraints.

An important factor that affects the tradeoff between local sensing and cooperation is the cost for communication with respect to sensing. From energy consumption perspective, if communication is expensive, non-cooperative sensing may be more efficient. To investigate this tradeoff, we evaluate the optimal scheduling algorithm for different ratios of $P_{Tx}$ and $P_S$. In Fig. 2(b), the energy consumption per node per time is shown for different $P_{Tx}/P_S$ values, where sensing cost is fixed as $P_S = 3.5$mJ and $P_{Rx} = P_{Tx} = P_S \times \frac{P_t}{P_s}$. It can be observed that in cases, where communication is much cheaper than sensing ($P_t/P_s \ll 1$), cooperative sensing outperforms non-cooperative sensing with diminishing returns.

We next investigate the effect of the correlation weights among neighboring nodes on the energy optimality. In this investigation, we assume a uniform correlation $w_{i,j}(t)$, $\forall i, j \in \mathcal{N}$, $i \neq j$ among all nodes, which is varied. The local correlation values are assumed to be unity, i.e., $w_{i,i}(t) = 1$.

(a) $R_S = 0.55$, $w_{i,j}(t) = 0.50$

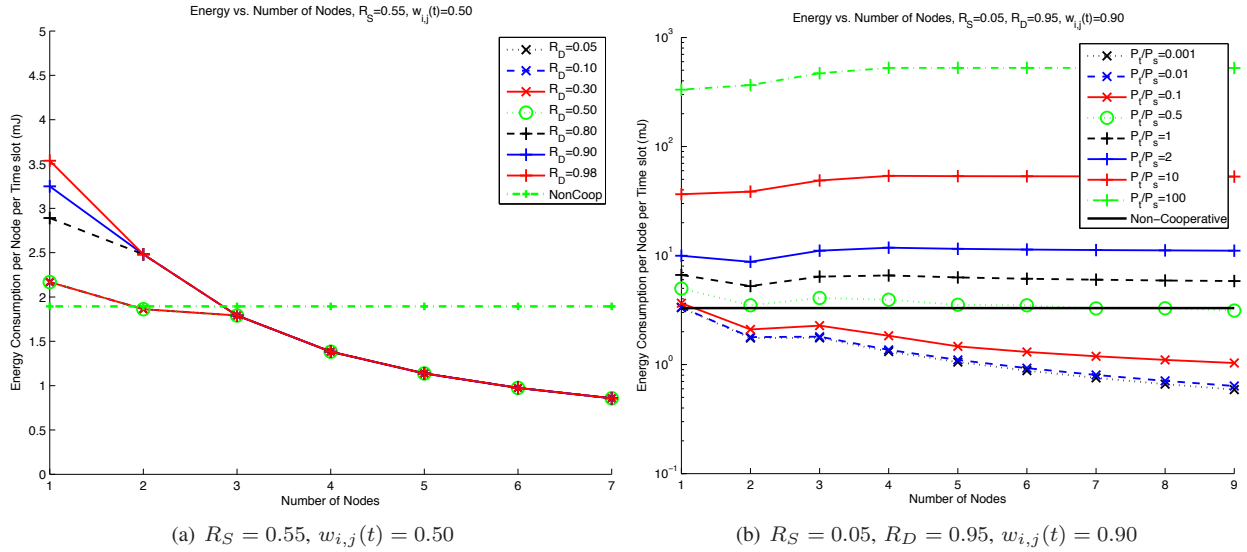(b) $R_S = 0.05$, $R_D = 0.95$, $w_{i,j}(t) = 0.90$

Fig. 2. Optimal energy consumption vs. number of nodes for (a) different information quality rates and (b) different ratios of communication and sensing power.



(a) $R_S = 0.05$, $R_D = 0.95$

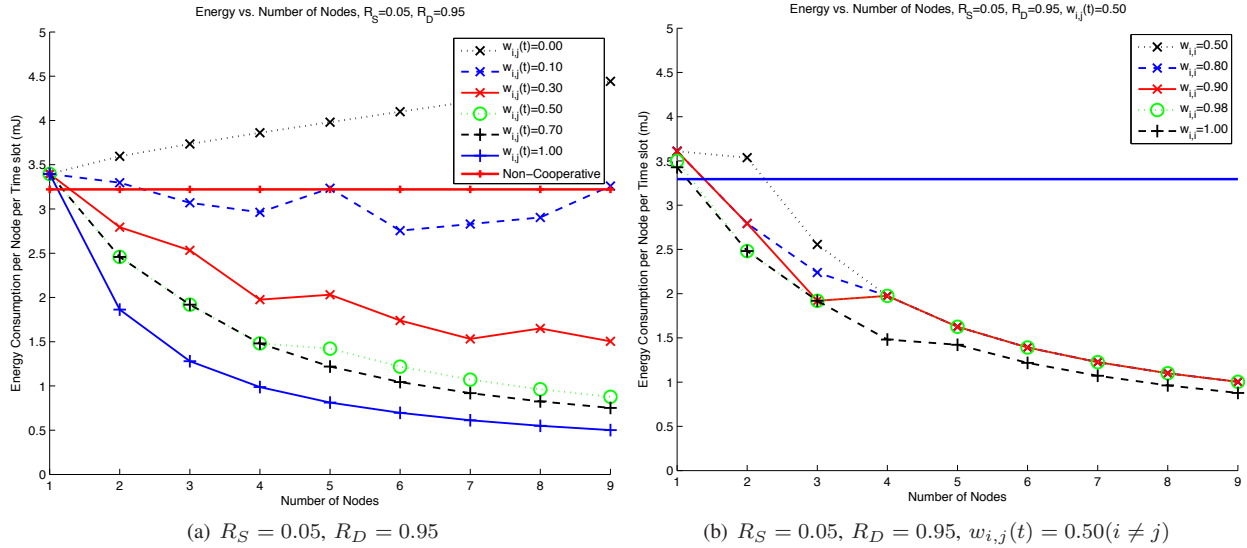(b) $R_S = 0.05$, $R_D = 0.95$, $w_{i,j}(t) = 0.50(i \neq j)$

Fig. 3. Optimal energy consumption vs. number of nodes for (a) different correlation weights among neighbors and (b) different local sensing correlation values.

The results presented in Fig. 3(a) indicate increasing cost benefits of growing correlation levels among nodes, perfect correlation resulting in the highest cost saving. When the correlation levels are sufficiently low, the cost of cooperation dominates benefits reaped. In the pathological case of no correlation, the cost increases with the number of nodes due to increased cost of sensing information reception, which does not benefit the receivers at all in terms of sensing information accuracy. These results emphasize the fact that cooperation is not beneficial in all cases, especially when the correlation between nodes is very limited *and* the energy consumption for communication is higher than that for sensing.

We also investigate the effect of local information sensing accuracy on energy consumption. This is modeled by varying the local correlation coefficient $w_{i,i}(t)$, which is ideally assumed as 1 in previous evaluations. This assumes that a local spectrum sensing value can accurately represent the spectrum occupancy at an SU location. Practically, this assumption may not hold true due to imperfect sensing hardware and random effects of the wireless channel. In Fig. 3(b), the energy consumption per node per time is shown for different values of $w_{i,i}(t)$. As expected, perfect local sensing accuracy leads to lowest energy consumption. For small clusters, lower local correlation leads to a higher number of neighbors needed for an energy efficient cooperation. Furthermore, for larger clusters, non-ideal local spectrum sensing ($w_{i,i}(t) \neq 1$) is always outperformed by ideal local spectrum sensing. This result highlights the importance of highly accurate sensing

mechanisms that can improve the performance of cooperative sensing solutions even further.

## VII. CONCLUSIONS

In this paper, we studied the problem of cooperative spectrum sensing, where we have leveraged the spatio-temporal correlations between spectral observations among different nodes and across different time points. Based on the notion of sensing information accuracy, which decays with time, we have developed virtual queue structures that represent the evolution of sensing information quality in a given node. These virtual queues form the basis of our novel sensing scheduling algorithms that minimize the total cost of spectrum sensing while guaranteeing given levels of average sensing information quality. The developed algorithm and its variants are theoretically shown to minimize the sensing cost and stabilizing all queues in the network, which in turn guarantees desired sensing information quality levels. Our numerical evaluations reveal other important properties of our algorithms, such as its ability to remedy inaccuracies of local sensing.

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