This paper was presented as part of the main technical program at IEEE INFOCOM 2011

Guaranteed Opportunistic Scheduling in Multi-Hop Cognitive Radio Networks

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***Abstract*—Cognitive radio networks enable opportunistic sharing of bandwidth/spectrum. In this paper, we introduce optimal control and scheduling algorithms for multi-hop cognitive radio networks to maximize the throughput of secondary users while stabilizing the cognitive radio network subject to collision rate constraints required by primary users. We show that by employing our proposed optimal algorithm, the achievable network throughput can be arbitrarily close to the optimal value. To reduce complexity, we propose a class of feasible suboptimal algorithms that can achieve at least a fraction of the optimal throughput. In addition, we also analyze the optimal algorithm in the fixed-routing scenario and deduce the corresponding lower-bound of average end-to-end delay across a link set.**

I. INTRODUCTION

In traditional static spectrum allocation policies, each spectrum band or channel is assigned specifically to one service type and its licensed users. However, the radio spectrum under-utilization problem in current networks adopting such policies was revealed by Federal Communication Commission (FCC) in [1]. Cognitive Radio Networks (CRNs) [2] have recently emerged as a new technology for unlicensed users to utilize the under-used spectrum opportunities.

In a typical CRN, licensed users are referred to as primary users (PUs) and secondary users (SUs) denote the users dynamically utilizing spectrum opportunities. The concept of CRN is simple, but the design of CRNs imposes challenges that are not present in conventional wireless networks [3]. Specific strategies should be developed for SUs to exploit channels in such a way that collisions between PUs and SUs transmission can be avoided.

Researchers have been working on improving the throughput of CRNs in single-hop scenarios in [18]-[22]. However, the applications of CRNs to a multi-hop setting pose a number of challenges for resource allocation problems, including *1) channel assignments:* the nature of CRNs force the multi-hop network to operate in a multi-channel setting where each node/link should dynamically select a channel to operate on; *2) PU constraints:* the scheduling policies should guarantee PU's requirements (collisions, power interference, etc.) while guaranteeing SU's performance, and it is often the case that SUs may only have partial knowledge of PUs' behavior (i.e., the channel states); *3) multi-hop scheduling and routing:* the multi-hop setting requires that scheduling policies take into consideration both exogenous and endogenous packet arrivals.

Recently, back-pressure algorithm and its extensions have been widely employed in developing throughput guaranteed dynamic resource allocation and scheduling algorithms for multi-hop wireless systems. In addition to the seminal work [4], a number of provably efficient and low-complexity algorithms are proposed in the literature [13]-[16], with delay issues addressed in [25]-[28] and time-varying channels considered in [7][11][12]. However, these works in multi-hop networks assume a single-channel setting with channel states known to nodes *a priori*, whereas in CRNs, SUs may not have knowledge of the exact current primary channel states. In addition, algorithms that work efficiently in a general single-channel network may not perform well in multi-channel CRNs, since PUs in different channels may have different requirements imposed on SUs such as collision rates and power interference constraints: these requirements are not addressed by a scheduling algorithm in the general single-channel network. Admission control and scheduling policies in *multi-channel single-hop* CRNs have been proposed in [6][8] to maximize throughput/utility subject to PU constraints.

In this paper, we propose a *multi-channel multi-hop* CRN overlayed with a PU network. With the constraints of collision rates observed by PUs, we propose an algorithm that can achieve the optimal throughput arbitrarily close in adaptive-routing scenarios, where routes for commodities are selected adaptively by the scheduling policy. A class of low-complexity suboptimal algorithms are proposed subsequently. We further modify and discuss our algorithm in fixed-routing scenarios, where the routes for commodities are known *a priori*.

Salient features of our work with respect to the literatures, especially [6][8], can be listed as follows: (1) To the best of our knowledge, our work is the first of its kinds to develop congestion controller and scheduling policy for multi-hop CRNs with provable throughput guarantees. We also analyze lower-bound and upper-bound for average end-to-end delay under our algorithm; (2) Our optimal algorithm can asymptotically achieve optimal throughput and we provide a *class* of suboptimal algorithms with lower complexity that achieve at least a fraction of optimal throughput; (3) We analyze both the adaptive-routing scenario and fixed-routing scenario, and their corresponding performances; (4) Our algorithm guarantees that the collision rates observed by PUs are below a given threshold, with the requirement of *partial*

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knowledge of current PU channel states;

The rest of the paper is organized as follows. Section II provides the network model for multi-hop cognitive radio networks. In Section III, we propose the optimal admission control and scheduling algorithm and analyze its performance. We provide a class of feasible suboptimal algorithms in Section IV. The impact of channel states on our algorithm and fixed routing scenarios are discussed in Section V. We present simulation results in section VI. Finally, we conclude our work in Section VII.

II. Network Model

A. PU and SU behaviors

We consider a time-slotted multi-hop CRN consisting of N secondary nodes and K commodities. Denote by $(m,n) \in \mathcal{L}$ a link from node m to node n, where $\mathcal{L}$ is the set of directed links in the CRN. The CRN is synchronized with a PU network with L primary channels orthogonal from point of view of SUs. From now on, we use 'channel' to represent primary channel for short.

Let $\mathbf{S}(t) = \{S_1(t), S_2(t), ..., S_L(t)\}$ be the channel state of primary user activities, where $S_l(t) = 1$ if the PU is idle in channel l in time slot t and $S_l(t) = 0$ if the PU is busy. We assume $\mathbf{S}(t)$ is a convergent Markovian process that takes value in a finite-state space and ergodic with time average probability of $\pi_\mathbf{S}$ for all $\mathbf{S}$. We define

$$P_l(t) \triangleq \mathbb{E}[\mathbf{1}_{S_l(t)=1}|\mathbf{S}(t-1)] \quad (1)$$

as the probability of channel l being idle in time slot t given the previous channel state history. Since SUs may not know the exact $\mathbf{S}(t)$ at the beginning of time slot t due to restrictions of sensing overheads or imperfect sensing, we assume $P_l(t)$ is the l-th channel state information *available* to secondary users. Denote $\psi(t) = (P_1(t), ..., P_L(t))$. Similarly, we assume $\psi(t)$ to be a convergent Markovian process taking values in a finite state space.

We define h^l_{nm} as SU's accessibility, where $h^l_{nm} = 1$ denotes that link (n,m) can access channel l, and otherwise $h^l_{nm} = 0$. We assume that (h^l_{nm}) is fixed across time. [1] We call (m,n) and l form a *link-channel pair* if $h^l_{nm} = 1$. If a SU transmits in channel l where the PU is busy, there will be a collision, and we denote the collision as $C_l(t)$. Specifically, $C_l(t) = 1$ if there is a collision in channel l in the time slot t and $C_l(t) = 0$ otherwise. Let c_l denote the time average collision rate for the PU in channel l:

$$c_l \triangleq \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} C_l(\tau).$$

We also denote the maximum allowed collision rate as ρ_l for channel l, such that c_l must satisfy $c_l \leq \rho_l$. [2]

[1]Please note that our model can be readily extended where the accessibility is Markovian, i.e., $(h^l_{nm}(t))$ is Markovian with respect to time t.

[2]Note that according to our definition, the collision rate is different from PU's collision probability which can be defined as $\lim_{t\to\infty} \sum_{\tau=0}^{t-1} C_l(\tau)/\sum_{\tau=0}^{t-1}(1 - S_l(\tau))$.

Let $A^c_n(t)$ denote the arrival rate of commodity c packets at node n at time t at the transport layer. Note that the constraint on commodity is quite loose as each commodity may have multiple source nodes (but a single sink). Specifically a commodity consists of a number of flows with distinct sources and an identical sink. We assume that $A^c_n(t)$ is i.i.d. with respect to t with mean λ^c_n. We also assume there are no buffers at the transport layer and denote $R^c_n(t)$ as the rate of packets admitted to node n by a congestion controller *at the end of* time t. Let A_{max} be the maximum arrival rate at the transport layer, i.e., $\forall n, c$: $R^c_n(t) \leq A^c_n(t) \leq A_{max}$.

Furthermore, let the scheduling parameter $\mu^{cl}_{mn}(t)$ denote the link rate assignment of commodity c for link (m,n) in channel l at time t according to scheduling decisions. We assume only one packet can be transmitted in a link for any channel, so $\mu^{cl}_{mn}(t)$ takes value in $\{0,1\}$. Thus, a node can transmit successfully at most one commodity c packet over link (m,n) if $\mu^{cl}_{mn}(t)S_l(t) = 1$.

B. Network Constraints and Approaches

We now model queue dynamics and network constraints in the CRN. Let $U^c_n(t)$ be the backlog of the total amount of commodity c packets waiting for transmission at node n. Considering the above discussion, we formulate the link dynamics for a general multi-interface scenario, i.e., a node may have multiple transceivers. If the destination of commodity c is n, then we have $U^c_n(t) = 0$. If the destination of commodity c is not n, we have:

$$\begin{aligned} U^c_n(t+1) \quad &\leq [U^c_n(t) - \sum_{l=1}^{L} \sum_{i:(n,i)\in\mathcal{L}} \mu^{cl}_{ni}(t) S_l(t)]^+ \\ &+ R^c_n(t) + \sum_{l=1}^{L} \sum_{j:(j,n)\in\mathcal{L}} \mu^{cl}_{jn}(t) S_l(t) \end{aligned} \quad (2)$$

where the operator $[x]^+$ is defined as $[x]^+ = \max\{x, 0\}$ so that the number of packets transmitted for c from a node does not exceed the backlog at node n. The terms $\sum_{l=1}^{L} \sum_{i:(n,i)\in\mathcal{L}} \mu^{cl}_{ni}(t)S_l(t)$ and $\sum_{l=1}^{L} \sum_{j:(j,n)\in\mathcal{L}} \mu^{cl}_{jn}(t)S_l(t)$ represent, respectively, the scheduled departure rate from node n and the scheduled internal arrival rate into node n by the scheduling algorithm with respect to commodity c. Note that (2) is an inequality since the arrival rates from neighbor nodes may be less than $\sum_l \sum_j \mu^{cl}_{jn}(t)S_l(t)$ if some neighbor node has few or no packets to transmit. Even in the single-interface scenario, if a neighbor node j has $\mu^{cl}_{jn}(t) = 1$ according to a scheduling algorithm but has no packets in the queue backlog, then the inequality still holds. For simplicity of analysis, in the rest of our paper, we assume that each node in CRN is equipped with only one transceiver. Note that our model can be readily extended to the scenario where each node may have multiple transceivers and primary channels are not orthogonal with each other.

Following the above analysis, we now construct the following network constraints for our model:

$$\mu_{nm}^{cl}(t) \in \{0,1\} \quad \forall (n,m) \in \mathcal{L}, \forall c, \forall l. \tag{3}$$

$$0 \le \sum_{c=1}^{K} \mu_{nm}^{cl}(t) \le h_{nm}^{l} \quad \forall (n,m) \in \mathcal{L}, \forall l. \tag{4}$$

$$0 \le \sum_{c=1}^{K} \sum_{l=1}^{L} [\sum_{m:(n,m)\in\mathcal{L}} \mu_{nm}^{cl}(t) + \sum_{k:(k,n)\in\mathcal{L}} \mu_{kn}^{cl}(t)] \le 1 \quad \forall n. \tag{5}$$

$$0 \le \sum_{c=1}^{K} \sum_{(n,m)\in\mathcal{L}} \mu_{nm}^{cl}(t) \le 1 \quad \forall l. \tag{6}$$

$$\mu_{nm}^{cl}(t) = 0 \text{ if } c \text{ ends at } n \ \forall l, \forall m : (n,m) \in \mathcal{L}, \forall c. \tag{7}$$

$$0 \le R_n^c(t) \le A_n^c(t) \quad \forall n, c. \tag{8}$$

Inequality (4) is the commodity constraint that only one packet of a commodity can be transmitted for any link and any accessible channel in a time slot. Inequality (5) is the node constraint that each node can access at most one channel in one time slot. Inequality (6) is the channel constraint that each channel can only be allocated to one link in a time slot. (7) is flow constraint.

To model collision rate constraint of channels, we now construct a virtual interference queue $X_l(t)$ for any given channel l as follows:

$$X_l(t+1) = [X_l(t) - \rho_l]^+ + C_l(t), \tag{9}$$

where the initial $X_l(0) = 0$. Considering $C_l(t)$ as the arrival rate and ρ_l as the service rate, and according to queueing theory, $c_l \le \rho_l$ if queue $X_l(t)$ is stable, i.e.,

$$\limsup_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{X_l(\tau)\} < \infty, \quad \forall l.$$

Note here with the introduction of scheduling parameters, we have:

$$C_l(t) = \sum_{(m,n)\in\mathcal{L}} \sum_{c} \mu_{mn}^{cl}(t)(1 - S_l(t)) \mathbf{1}_{U_m^c(t)>0}.$$

Specifically, a collision occurs in channel l if PU in that channel is busy and for some node m with non-zero backlog for a commodity c, the scheduling parameter $\mu_{mn}^{cl}(t) = 1$, such that a secondary transmission is active on link (m,n) and in channel l.

From the above description, we can define the network to be *stable* under some scheduling algorithm with admitted arrival rate matrix $(R_n^c(t))$ if queues $U_n^c(t)$ are stable for any node and commodity, i.e.,

$$\limsup_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{U_n^c(\tau)\} < \infty, \quad \forall n, c$$

with the network constraints (3), (4), (5), (6), (7) and (8).

An arrival rate matrix (z_n^c) is called *admissible* if there is some scheduling algorithm (without congestion control) under which the system is stable and the channel collision rate constraints are satisfied. We denote Λ to be *the capacity region* consisting of all admissible (z_n^c).

Now we let (r_n^{c*}) denote an optimal solution to the following optimization:

$$\max_{(r_n^c)\in\Lambda} \sum_{n} \sum_{c} r_n^c. \tag{10}$$

Hence, $\sum_n \sum_c r_n^{c*}$ can represent the maximal network throughput.

III. Optimal Control Scheduling Algorithm for Cognitive Radio Networks

Now we propose an optimal control and scheduling algorithm ***ALG*** for the introduced CRN model so that the CRN is stable under ***ALG*** with collision rate constraints satisfied and that the achievable throughput is arbitrarily close to the optimal throughput $\sum_n \sum_c r_n^{c*}$. The algorithm can be applied to both adaptive-routing and fixed-routing scenarios. We first analyze the performance of ***ALG*** in adaptive-routing scenarios in this section and later analyze its performance in fixed-routing scenarios in Section V.

The optimal algorithm ***ALG*** consists of two parts: congestion controller and scheduling policy. Now we propose and analyze the algorithm in the following subsections.

A. Algorithm Description and Analysis

1) Congestion Controller:

$$\min_{0\le R_n^c(t)\le A_n^c(t)} R_n^c(t)[U_n^c(t) - V] \tag{11}$$

where $V > 0$ is a control parameter. We note that this congestion controller performs locally at each SU node. Specifically, when the queue backlog $U_n^c(t) > V$, the admitted arrival $R_n^c(t)$ is set to zero; Otherwise, all arrival packets are admitted, i.e., $R_n^c(t) = A_n^c(t)$.

2) Scheduling Policy: At each time slot, with constraints (3)(4)(5)(6)(7), the network solves the following optimization problem:

$$\max_{(\mu_{mn}^{cl}(t))} \sum_{l=1}^{L} \sum_{(m,n)\in\mathcal{L}} \mu_{mn}^{c_{mn}^*(t)l}(t)[w_{mn}(t)P_l(t) - (1-P_l(t))X_l(t)] \tag{12}$$

$$\text{s.t.} \quad \mu_{mn}^{cl}(t) = 0 \quad \text{if } c \ne c_{mn}^*(t), \quad \forall l, \forall (m,n) \in \mathcal{L}, \tag{13}$$

where $c_{mn}^*(t)$ and $w_{mn}(t)$ are defined as follows:

$$c_{mn}^*(t) \triangleq \arg\max_{c} [U_m^c(t) - U_n^c(t)], \quad \forall (m,n) \in \mathcal{L}, \tag{14}$$

$$w_{mn}(t) \triangleq [U_m^{c_{mn}^*(t)} - U_n^{c_{mn}^*(t)}]^+. \tag{15}$$

The optimization (12) is a typical Maximum Weight Matching (MWM) problem. We first decouple commodity scheduling from the MWM. Specifically, for each link $(m,n) \in \mathcal{L}$, the commodity $c_{mn}^*(t)$ is fixed as the candidate for transmission.

Note that $c^*_{mn}(t)$ and $w_{mn}(t)$ can be computed locally for each link. We then assign the weight for a link-channel pair as $(w_{mn}(t)P_l(t) - (1 - P_l(t))X_l(t))$. We can consider $w_{mn}(t)P_l(t)$ and $(1 - P_l(t))X_l(t)$ as the reward and the cost, respectively, of scheduling link (m, n) and channel l. Note that when $w_{mn}(t)P_l(t) - (1 - P_l(t))X_l(t) \leq 0$, without loss of optimality, we set $\mu^{cl}_{mn}(t) = 0$, $\forall c$ to maximize (12).

From the above algorithm, we have the following theorem as our main results:

Theorem 1: Employing ***ALG***, each queue in the CRN has a deterministic worst-case bound:

$$U^c_n(t) \leq U_{max} \triangleq A_{max} + 1 + V, \quad \forall t, n, c \tag{16}$$

$$X_l(t) \leq X_{max} \triangleq U_{max}\frac{1-\epsilon}{\epsilon} + 1, \quad \forall l, t \tag{17}$$

where $0 < \epsilon \leq 1 - P_l(t)$, for any l such that $P_l(t) \neq 1$. We know that such an ϵ exists since ψ takes value in a finite state space. In addition, the average throughput achievable by ***ALG*** is at most $\frac{\bar{B}}{V}$ away from the optimal throughput, i.e.:

$$\liminf_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1}\sum_{n=1}^{N}\sum_{c=1}^{K}\mathbb{E}\{R^c_n(\tau)\} \geq \sum_{n=1}^{N}\sum_{c=1}^{K} r^{c*}_n - \frac{\bar{B}}{V} \tag{18}$$

where $\bar{B} > 0$ is a constant which will be given later, and recall that (r^{c*}_n) is a solution of (10). Similar form is given in [6] for a single-hop setting.

Remark 1: As a direct result of Little's Theorem, the average end-to-end delay $\bar{D}$ for a *flow* [3] satisfies:

$$\bar{D} = \frac{\sum_n \sum_c \bar{U}^c_n}{\sum_n \sum_c \bar{r}^c_n},$$

where

$$\bar{U}^c_n \triangleq \limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\{U^c_n(\tau)\} \quad \forall n, \forall c;$$

$$\bar{r}^c_n \triangleq \liminf_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\{R^c_n(\tau)\} \quad \forall n, \forall c. \tag{19}$$

Hence, we have the following upper-bound on the average end-to-end delay according to Theorem 1:

$$\bar{D} \leq \frac{NK(A_{max} + 1 + V)}{\sum_{n=1}^{N}\sum_{c=1}^{K} r^{c*}_n - \frac{\bar{B}}{V}}. \tag{20}$$

Remark 2: By the queue bounds stated in Theorem 1, ***ALG*** stabilizes the CRN and collision rate constraints are met. The upper-bound U_{max} is increased for large V. Furthermore, according to (18), ***ALG*** can achieve a throughput arbitrarily close to optimality when V is sufficiently large. By adjusting parameter V, we can tradeoff between the throughput and the delay performance.

To assist the proof of Theorem 1, we establish the following Lemma from the structure of ***ALG***:

[3]Recall that a commodity may have a number of flows that have distinct sources and an identical sink.

Lemma 1: Given $\mathbf{Q}(t)$ and $\psi(t)$, ***ALG*** maximizes the following over all *feasible* scheduling algorithms:

$$\sum_l \sum_{(m,n)} \sum_{c=1}^{K} \mu^{cl}_{mn}(t)[(U^c_m(t) - U^c_n(t))P_l(t) - (1 - P_l(t))X_l(t)]. \tag{21}$$

Proof: Lemma 1 is proved in Appendix A. ■

Since Theorem 1 consists of three parts: upperbound of $U^c_n(t)$, upperbound of $X_l(t)$ and the throughput bound, we present the proof accordingly.

Proof of the upper-bound of queue backlogs ($U^c_n(t)$) in Theorem 1: see [17]. ■

Proof of Upper-Bound of $X_l(t)$ in Theorem 1: The proof is given in [17]. ■

Proof of Throughput Achievement in Theorem 1: The proof is given in Appendix B. ■

IV. Algorithm Complexity Reduction

It is known that MWM problem such as (12) is complex and hard to solve. In [6] it is claimed that in their single-hop scenario, MWM has polynomial complexity; In [10], the authors have shown that MWM can be NP-hard depending on the underlying interference model. In this section, we propose a class of algorithms that are easy to implement and can guarantee at least a fraction of optimal throughput. We refer to these algorithms as suboptimal algorithms.

Here, for convenience, we denote the scheduling decisions made by ***ALG*** at time t as $(\mu^{cl,ALG}_{mn}(t))$ and denote the scheduling decisions made by suboptimal algorithms as $(\mu^{cl,SUB}_{mn}(t))$. Algorithms are called *suboptimal* if:

$$\sum_l \sum_{(m,n)} \mu^{c^*_{mn}(t)l,SUB}_{mn}(t)[w_{mn}(t)P_l(t) - (1 - P_l(t))X_l(t)]$$
$$\geq \gamma \sum_l \sum_{(m,n)} \mu^{c^*_{mn}(t)l,ALG}_{mn}(t)[w_{mn}(t)P_l(t) - (1 - P_l(t))X_l(t)]$$

where $c^*_{mn}(t)$ and $w_{mn}(t)$ are given in (14)(15) by ***ALG*** and are obtained locally for links, and $\gamma \in (0, 1)$ is a parameter determined by the structure of a suboptimal algorithm. In addition, we require:

$$\mu^{cl,SUB}_{mn}(t) = 0, \text{ if } c \neq c^*_{mn}(t);$$
$$\mu^{c^*_{mn}(t)l,SUB}_{mn}(t) = 0, \text{ if } w_{mn}(t)P_l(t) - (1 - P_l(t))X_l(t) \leq 0.$$

Also note that the congestion controller of suboptimal algorithms is the same as ***ALG***.

Following the steps of the proof in Theorem 1, one can show that suboptimal algorithms have the same deterministic bounds on queue backlogs as in (16)(17), and we propose Theorem 2:

Theorem 2: The average throughput achievable by a suboptimal algorithm is:

$$\liminf_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1}\sum_{n=1}^{N}\sum_{c=1}^{K}\mathbb{E}\{R^c_n(t)\} \geq \gamma\sum_{n=1}^{N}\sum_{c=1}^{K} r^{c*}_n - \frac{B^{SUB}}{V}, \tag{22}$$

where $B^{SUB} > 0$ is a constant.

Proof: The proof is given in [17]. ■

Note that the suboptimal algorithm can *at least* achieve a fraction γ of the optimal throughput when V is large enough.

We now give two examples of suboptimal algorithms in the following subsections: Greedy Maximal Matching algorithm and GWMAX algorithm.

A. Greedy Maximal Matching Algorithm

In the Greedy Maximal Matching (GMM) algorithm, we first fix $c_{mn}^*(t)$ for each link $(m,n) \in \mathcal{L}$ so that $\mu_{mn}^{cl}(t) = 0$, $\forall c \neq c_{mn}^*(t)$, where $c_{mn}^*(t)$ is given as in ***ALG***. The GMM algorithm utilizes a bipartite graph $G = (V(G), E(G))$. The set of vertices $V(G) = \{(m,n) : (m,n) \in \mathcal{L}\} \cup \{l : 1 \leq l \leq L\}$, i.e., $V(G)$ consists of all secondary links and channels in the CRN. The set of edges is $E(G) = \{\{(m,n), l\} : (m,n) \in V(G), l \in V(G), h_{mn}^l = 1\}$, i.e., an edge is equivalent to a link-channel pair. The weight for each edge $\{(m,n), l\}$ is $(w_{mn}(t)P_l(t) - (1 - P_l(t))X_l(t))$[4].

For each $e = \{(m,n), l\} \in E(G)$, we define its *interference set* $N(e)$ as the edge set of all link-channel pairs that cannot be scheduled when $\mu_{mn}^{c_{mn}^*(t)l}(t) = 1$, and furthermore we let $e \in N(e)$. Let $G[V(G)\backslash N(e)]$ denote the subgraph induced by $V(G)\backslash N(e)$.

In addition, given a graph G', let $V(G')$ denote its vertex set and $E(G')$ denote its edge set. Now we give GMM algorithm in Figure 1. For each edge $\{(m,n), l\}$ selected by GMM, we set $\mu_{mn}^{c_{mn}^*(t)l,SUB}(t) = 1$; Otherwise $\mu_{mn}^{c_{mn}^*(t)l,SUB}(t) = 0$.

```
Algorithm GMM
INPUT: undirected weighted graph G
I := ∅; i := 0; G_i = G
while E(G_i) ≠ ∅ do
    Choose an edge e_i ∈ E(G_i) that has max weight;
    I = I ∪ e_i; G_{i+1} = G[V(G_i)\N(e_i)]
    i = i + 1;
end;
Output I;
```

Fig. 1. Algorithm GMM

Proposition 1: In GMM algorithm, $\gamma = \frac{1}{3}$.

Proof: The proof is given in [17]. ■

B. GWMAX algorithm

GWMAX algorithm, a greedy method for the maximum weighted independent set optimization problem, is proposed in [23].

In the implementation of GWMAX algorithm, again we fix $c_{mn}^*(t)$ for each link $(m,n) \in \mathcal{L}$ and $\mu_{mn}^{cl}(t) = 0$, $\forall c \neq c_{mn}^*(t)$. We construct an undirected graph $G = (V(G), E(G))$, where $V(G) = \{\{(m,n), l\} : (m,n) \in \mathcal{L}, h_{mn}^l = 1\}$, i.e., $V(G)$ consists of all link-channel pairs that are candidates for scheduling decisions. There is an edge between two vertices if the two link-channel pairs cannot be scheduled at the same time. We assign a weight $(w_{mn}(t)P_l(t) - (1 - P_l(t))X_l(t))$ to any given vertex $\{(m,n), l\}$[5].

In addition, given a graph G' and a vertex u, let $d_{G'}(u)$ denote the degree of u in G', and let $W(u)$ denote the weight for the vertex. Then we can give GWMAX algorithm in Figure 2 as proposed in [23]. For each vertex $\{(m,n), l\}$ selected by GWMAX, we set $\mu_{mn}^{c_{mn}^*(t)l,SUB}(t) = 1$; Otherwise $\mu_{mn}^{c_{mn}^*(t)l,SUB}(t) = 0$.

```
Algorithm GWMAX
INPUT: undirected weighted graph G
I := ∅; i := 0; G_i = G
while E(G_i) ≠ ∅ do
    Choose a vertex v_i s.t.:
        v_i = arg min_{u∈V(G_i)} W(u) / (d_{G_i}(u)(d_{G_i}(u)+1));
    G_{i+1} = G[V(G_i)\{v_i}];
    i = i + 1;
end;
Output V(G_i);
```

Fig. 2. Algorithm GWMAX

We have the following proposition according to [23]:

Proposition 2: In GWMAX algorithm, $\gamma = \frac{1}{\Delta}$, where Δ is the maximum degree of G.

V. Further Discussions

In this section, we further analyze the impact of channel states on our algorithm and apply our algorithm to the fixed-routing scenario.

A. Effects of Channel States

Recall that in our problem definition, the primary channel state vector $\mathbf{S}(t)$ is not available to SUs. Now we consider the case when $\mathbf{S}(t)$ is available to SUs (probably due to specific designs in PHY layer and MAC layer). Then we can discard the virtual queues $\{X_l(t)\}$ and modify the proposed scheduling policy in ***ALG*** as solving the following optimization problem instead of (12):

$$\max_{(\mu_{mn}^{cl}(t))} \sum_{l=1}^{L} \sum_{(m,n)\in\mathcal{L}} \mu_{mn}^{c_{mn}^*(t)l}(t) w_{mn}(t), \tag{23}$$

$$\text{s.t. } \mu_{mn}^{cl}(t) = 0 \text{ if } S_l(t) = 0,\ \forall l, \forall c, \forall (m,n) \in \mathcal{L},$$

with the same network constraints (3)(4)(5)(6)(7) and (13)(14)(15) in the original scheduling policy. With such modifications, ***ALG*** then becomes the traditional back-pressure algorithm as first described in [5] and can be implemented in a distributed way such as in [13][14].

When both $\mathbf{S}(t)$ and $\{P_l(t)\}$ are not directly available to SUs but the distribution of $\{P_l(t)\}$ is known *a priori* by SUs, then SU's choice of a channel can be modeled as a partially observable Markov decision process (POMDP). Single-hop solutions are proposed in [18] and [19], but are not readily extendable to multi-hop setting. Such discussion is out of the scope of our paper.

[4]We delete an edge from graph G if its weight is nonpositive.

[5]We delete a vertex from graph G if its weight is nonpositive.

B. Fixed-Routing Scenario and Delay Issues

We have analyzed the proposed algorithm ***ALG*** in an adaptive-routing setting in Section III and have obtained the upper-bound of average delay in (20) as an immediate result by applying Little's Theorem to the queue backlogs. In this subsection, we analyze ***ALG*** in the scenario of fixed routing, and correspondingly analyze both lower-bound and upper-bound of delay. In the fixed-routing scenario, we can immediately extend ***ALG*** by adding routing constraints to (12).

Here we introduce additional notations for cases when routes for *commodities* are fixed *a priori*. Let the loopless path of commodity c be denoted by a set of nodes $P_c = \{v_0^c, v_1^c, ..., v_{q_c}^c\}$, where v_i^c is the i-th hop for commodity c from v_0^c ($i > 0$). $v_{q_c}^c$ is the sink for commodity c. Thus, we must have $A_n^c(t) = 0$ and $U_n^c(t) = 0$ if $n \neq v_i^c \; \forall \; i < q_c$; We must have $\mu_{mn}^{cl}(t) = 0$ if $(m, n) \neq (v_i^c, v_{i+1}^c) \; \forall i < q_c$.

The *deterministic* upper-bound of queue backlog is the same as in Theorem 1, and similar as in Remark 1, the upper-bound of the *average* end-to-end delay in fixed-routing scenario for a flow is given as:

$$\bar{D}^{FIX} \leq \frac{(A_{max} + 1 + V)\sum_{n=1}^{N}\sum_{c=1}\mathbf{1}_{n\in P_c \backslash \{v_{q_c}^c\}}}{\sum_{n=1}^{N}\sum_{c=1}^{K} r_n^{c*} - \frac{\bar{B}'}{V}}.[6]$$

The queue dynamics for commodity c at $v_i^c \in P_c \backslash \{v_{q_c}^c\}$ can be rewritten from (2) as:

$$\begin{aligned} U_{v_i^c}^c(t+1) \leq & [U_{v_i^c}^c(t) - \sum_l \mu_{v_i^c v_{i+1}^c}^{cl}(t) S_l(t)]^+ \\ & + R_{v_i^c}^c(t) + \sum_l \mu_{v_{i-1}^c v_i^c}^{cl}(t) S_l(t) \mathbf{1}_{i\neq 0}. \end{aligned}$$

Following the description of ***ALG***, we have:

Proposition 3: The queue dynamics for commodity c at $v_i^c \in P_c \backslash \{v_{q_c}^c\}$ can be simplified as:

$$\begin{aligned} U_{v_i^c}^c(t+1) = & U_{v_i^c}^c(t) - \sum_l \mu_{v_i^c v_{i+1}^c}^{cl}(t) S_l(t) \\ & + R_{v_i^c}^c(t) + \sum_l \mu_{v_{i-1}^c v_i^c}^{cl}(t) S_l(t) \mathbf{1}_{i\neq 0} \quad . \end{aligned} \tag{24}$$

Proof: The proof is given in [17]. ■

Now let X be a link set of CRN. Define k_X such that no more than k_X link-channel pairs can be simultaneously scheduled with links in X, according to secondary network constraints. We denote $c \in X$ when the path of commodity c intersects the link set X. We denote a commodity c enters X at node $v_{k_c}^c$ and leave at $v_{l_c}^c$. For simplicity of analysis, we assume that $k_c < l_c$.

Denote D_X as the average end-to-end delay experienced by a *flow*[7] that crosses the link set X. Then we introduce the following theorem on the lower bound of D_X:

Theorem 3: Given a link set X, we have:

$$D_X \geq \frac{\lambda_{(X,1)}\bar{D}_X}{\lambda_{(X,2)} \max_{c\in X}(l_c - k_c)} + \frac{\lambda_{(X,3)}}{\lambda_{(X,2)}} \tag{25}$$

where

$$\lambda_{(X,1)} \triangleq \sum_{c=1}^{K} \mathbf{1}_{c\in X} \sum_{j=k_c}^{l_c - 1} \sum_{i=0}^{j} \bar{r}_{v_i^c}^c,$$

$$\lambda_{(X,2)} \triangleq \sum_{c=1}^{K} \mathbf{1}_{c\in X} \sum_{i=0}^{q_c - 1} \bar{r}_{v_i^c}^c,$$

$$\lambda_{(X,3)} \triangleq \sum_{c=1}^{K} \mathbf{1}_{c\in X} \sum_{j=l_c}^{q_c - 1} \sum_{i=0}^{j} \bar{r}_{v_i^c}^c,$$

and $\bar{r}_{v_i^c}^c$ follows (19). Here $\bar{D}_X$ is defined as the average delay of the G/D/1 system $W(t)$ with queue dynamics:

$$W(t+1) = [W(t) - k_X]^+ + A_X(t) \tag{26}$$

where $A_X(t) \triangleq \sum_{c=1}^{K} \mathbf{1}_{c\in X} \sum_{j=k_c}^{l_c-1} \sum_{i=0}^{j} R_{v_i^c}^c(t)$. Thus, the lower-bound of D_X depends only on the admitted arrival rates of commodities.

Proof: The proof is given in [17]. ■

In CRNs, the behavior of PUs usually affects a set (or a cluster) of links, so the delay of flows passing this link set is of great concern, which is our motivation in analyzing the lower-bound of end-to-end flow delay across a specific link set X. The lower-bounds deduced in this subsection and the upper-bounds deduced in Section III shed light on the quality of service (QoS) experienced by SUs in CRNs.

VI. Simulation Results

In this section, we present the simulation results for the proposed optimal algorithm ***ALG*** and one of the suboptimal algorithms, GMM. Simulations are run in Matlab 2009A.

In the network topology illustrated in Figure 3, we allow multiple PU activities to interfere with multiple secondary links. There are two source-destination pairs (A, H) and (D, E) with identical Poisson arrival rates, and we limit the maximum admitted rate by 10 (i.e., $A_{max} = 10$). The secondary links (B, F) and (B, C) coexist in a single channel that belongs to primary user PU1. Similarly, the secondary links (F, G) and (C, G) employ a single channel designated to primary users PU2. The other links use mutually non-interfering channels that are free of PU activity. The transition behavior of both PUs from one slot to the next is illustrated in Figure 4, where state 0 denotes a busy state and state 1 denotes an idle state. Thus, we ensure a statistical channel availability of 0.5 for secondary links that share the resource with PUs. The collision rate threshold is set to 0.1.

We present the simulation results of ***ALG*** and GMM algorithm with $V = 20$ in Figure 5, where the illustrated collision rate is the larger one of the collision rates experienced by PU1 and PU2. We observe that the collision rate threshold is not exceeded by either proposed algorithm. The throughput of the suboptimal GMM algorithm is close to the optimal algorithm.

[6]Here (r_n^{c*}) is the solution to (10) with the capacity region determined in the fixed routing scenario. V is the same control parameter as before and $\bar{B}'$ is a constant.

[7]Note again that a commodity can be composed of a number of flows that have distinct sources and a common sink.

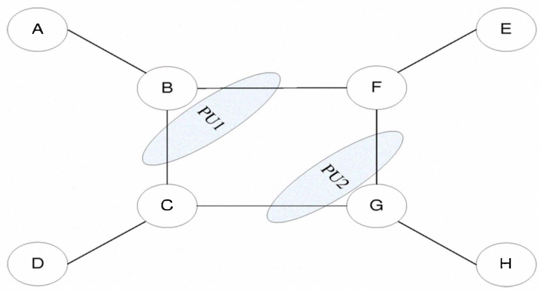

Fig. 3. Network topology

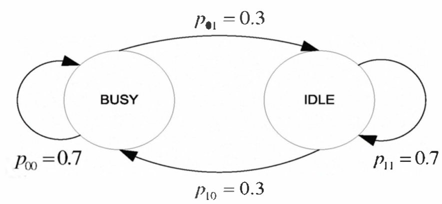

Fig. 4. Transition diagram of primary activity

In addition, for both algorithms, the throughput is increased at the expense of a higher congestion level measured as the time-averaged total number of packets in the secondary network.

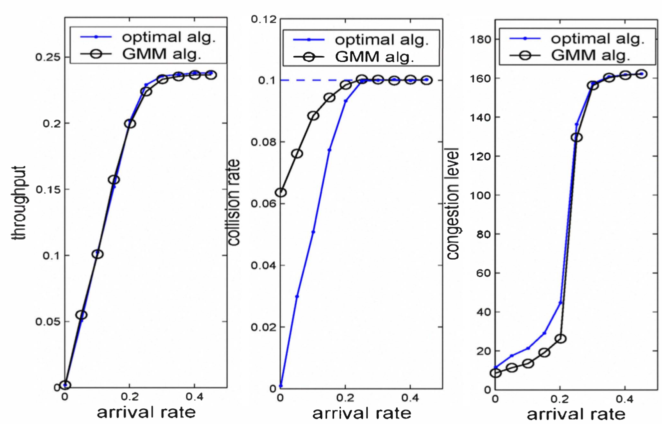

Fig. 5. Performances of the optimal and GMM algorithms

We also present results on the throughput optimality of ***ALG*** and the effect of control parameter V in [17].

VII. Conclusions

In this paper, we have proposed an optimal algorithm to maximize the throughput of a multi-hop CRN with a constraint on time-average collision rate experienced by PUs. We provide a deterministic upper-bound for queue backlogs under the optimal algorithm. We show that with a control parameter V, the throughput of the CRN under the proposed optimal algorithm can approach close to optimality with the tradeoff in queue backlog/delay. Next, we propose a class of suboptimal algorithms to reduce complexity while achieving at least a fraction of the optimal throughput. In addition, we apply the optimal algorithm to a fixed-routing setting and obtain the corresponding lower-bound of average end-to-end delay across a link set.

Our work aims at a better understanding of the fundamental properties and performance limits of dynamic control and scheduling in multi-hop cognitive radio networks. In the future, we will investigate distributed scheduling algorithms for multi-hop CRNs. Our future work will also involve 1) specific imperfect sensing schemes to estimate PU channel behavior and 2) possible delay reducing schemes for heavily loaded CRNs.

VIII. Acknowledgment

This work is supported by NSF under grant number CCF-0914912.

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APPENDIX A
PROOF OF LEMMA 1

We prove Lemma 1 by contradiction. Assume that ***ALG*** is not one of the optimal algorithms that maximize (21). We have two cases for a given time t:

(1) In the first case, we assume that an optimal scheduling algorithm chooses all-zero scheduling parameters, i.e., $\mu_{mn}^{cl}(t) = 0, \forall c, \forall l, \forall (m,n)$, and thus the value of (21) is zero. However under ***ALG***, the value of (21) is always nonnegative, so ***ALG*** performs better than or equal to the optimal algorithm.

(2) The second case is that an optimal algorithm does not choose all-zero parameters. We assume that such a scheduling algorithm chooses some $\mu_{mn}^{c_1 l}(t) = 1$ so that $c_1 \neq c_{mn}^*(t)$. Then, $\mu_{mn}^{c_{mn}^*(t) l}(t) = 0$ by (5) and we have

$$\begin{aligned}&\mu_{mn}^{c_1 l}(t)[(U_m^{c_1}(t) - U_n^{c_1}(t))P_l(t) - (1 - P_l(t))X_l(t)]\\ \leq& w_{mn}(t)P_l(t) - (1 - P_l(t))X_l(t)\end{aligned}$$

where $w_{mn}(t)$ is defined in (15).

Now for any c_1 and any link-channel pair $((m,n),l)$ in the optimal algorithm such that $\mu_{mn}^{c_1 l}(t) = 1$ and $c_1 \neq c_{mn}^*(t)$, by setting $\mu_{mn}^{c_1 l}(t) = 0$ and $\mu_{mn}^{c_{mn}^*(t) l}(t) = 1$, the value of (21) does not decrease while satisfying all other constraints (3)(4)(5)(6)(7). Then by such settings, there exists an optimal algorithm Γ that has $\mu_{mn}^{cl}(t) = 0, \forall c \neq c_{mn}^*(t)$. It is clear that Γ should be one solution of ***ALG***, which leads to contradiction.

Hence, we conclude that ***ALG*** maximizes (21) over all feasible algorithms.

APPENDIX B
PROOF OF THROUGHPUT ACHIEVEMENT IN THEOREM 1

Before we proceed, we present the following additional lemmas which will assist us in proving the throughput bound provided in Theorem 1.

Lemma 2: For nonnegative numbers $A_1, A_2, A_3, Q \in \mathbb{R}$ such that $Q \leq [A_1 - A_2]^+ + A_3$, then $Q^2 \leq A_1^2 + A_2^2 + A_3^2 - 2A_1(A_2 - A_3)$.

Proof of Lemma 2 is trivial and omitted for brevity. We will later use lemma 2 to simplify queue dynamics.

Lemma 3: There exists a stationary algorithm STAT that stabilizes the CRN with admitted arrival matrix $(R_n^{c,STAT}(t))$ and scheduling parameters $(\mu_{nm}^{cl,STAT}(t))$ depending only on $\psi(t)$, such that the expected admission rates are:

$$\mathbb{E}\{R_n^{c,STAT}(t)\} = r_n^{c*}, \forall t.$$

STAT also ensures the commodity is balanced at any given node n:

$$D_n^{c,STAT} \triangleq \lim_{t\to\infty} \frac{1}{t}\mathbb{E}\{\sum_{\tau=0}^{t-1} D_n^{c,STAT}(\tau)\} \geq r_n^{c*},$$

and ensures:

$$\bar{C}_l^{STAT} \triangleq \lim_{t\to\infty} \frac{1}{t}\mathbb{E}\{\sum_{\tau=0}^{t-1} \bar{C}_l^{STAT}(\tau)\} \leq \rho_l$$

where

$$D_n^{c,STAT}(t) \triangleq \sum_l S_l(t)(\sum_i \mu_{ni}^{cl,STAT}(t) - \sum_j \mu_{jn}^{cl,STAT}(t)),$$

$$\bar{C}_l^{STAT}(t) \triangleq \sum_{(m,n)\in\mathcal{L}} \sum_c \mu_{mn}^{cl,STAT}(t)(1 - S_l(t)).$$

Given commodity c at node n, recall that $(A_n^c(t))$ is i.i.d. with mean (λ_n^c), and we assume $(\lambda_n^c) > (r_n^{c*})$ element-wise. The congestion controller for STAT can be given as: Admit $R_n^{c,STAT}(t) = A_n^c(t)$ w.p. $\frac{r_n^{c*}}{\lambda_n^c}$; Otherwise, $R_n^{c,STAT}(t) = 0$. Then $\mathbb{E}\{R_n^{c,STAT}(t)\} = r_n^{c*}, \forall t$.

Let Θ be an optimal scheduling algorithm that can achieve (r_n^{c*}). Note that the probabilistic decisions of Θ may depend on time-varying information other than $\psi(t)$, e.g., arrival rates by time t, decisions made before t, etc.. Then STAT can be constructed according to Θ so that STAT makes probabilistic decisions independent of any time-varying information other than $\psi(t)$ (hence, independent of queue backlogs). Similar formulation of STAT and its proof have been given in [8] and [9], so we omit the proof of Lemma 3 for brevity.

Here, we let $\mathbf{Q}(t) = ((U_n^c(t)), (X_l(t)))$ and define the quadratic Lyapunov function $L(\mathbf{Q}(t))$ as the following:

$$L(\mathbf{Q}(t)) \triangleq \frac{1}{2}(\sum_{l=1}^{L} X_l(t)^2 + \sum_{n=1}^{N}\sum_{c=1}^{K}(U_n^c(t))^2).$$

It is obvious that $L(\mathbf{Q}(0)) = 0$. We define the Lyapunov drift by

$$\Delta(t) \triangleq \mathbb{E}\{L(\mathbf{Q}(t+1)) - L(\mathbf{Q}(t))|\mathbf{Q}(t)\}. \quad (27)$$

Now we prove the throughput bound of Theorem 1:

According to queue backlogs (2)(9) and Lemma 2, we have:

$$\Delta(t) \leq \frac{1}{2}\mathbb{E}\{\sum_{n=1}^{N}\sum_{c=1}^{K}[(\sum_{l=1}^{L}\sum_{i:(n,i)\in\mathcal{L}}\mu_{ni}^{cl}(t)S_l(t))^2 + (R_n^c(t)+\sum_{l=1}^{L}\sum_{j:(j,n)\in\mathcal{L}}\mu_{jn}^{cl}(t)S_l(t))^2 - 2U_n^c(t)(\sum_{l=1}^{L}\sum_{i:(n,i)\in\mathcal{L}}\mu_{ni}^{cl}(t)S_l(t) - R_n^c(t) - \sum_{l=1}^{L}\sum_{j:(j,n)\in\mathcal{L}}\mu_{jn}^{cl}(t)S_l(t))] + \sum_{l=1}^{L}(\rho_l^2 + C_l^2(t) - 2X_l(t)(\rho_l - C_l(t)))|\mathbf{Q}(t)\} \tag{28}$$

and from (28), we have

$$\Delta(t) - V\mathbb{E}\{\sum_{n=1}^{N}\sum_{c=1}^{K}R_n^c(t)|\mathbf{Q}(t)\} \leq B - V\mathbb{E}\{\sum_{n=1}^{N}\sum_{c=1}^{K}R_n^c(t)|\mathbf{Q}(t)\} - \mathbb{E}\{\sum_{n=1}^{N}\sum_{c=1}^{K}U_n^c(t)(\sum_{l=1}^{L}\sum_{i:(n,i)\in\mathcal{L}}\mu_{ni}^{cl}(t)S_l(t) - R_n^c(t) - \sum_{l=1}^{L}\sum_{j:(j,n)\in\mathcal{L}}\mu_{jn}^{cl}(t)S_l(t))|\mathbf{Q}(t)\} - \mathbb{E}\{\sum_{l=1}^{L}X_l(t)(\rho_l - \bar{C}_l(t))|\mathbf{Q}(t)\} \tag{29}$$

where constant $B \triangleq \frac{1}{2}(\sum_{l=1}^{L}\rho_l^2 + L + 2N + NKA_{max}^2 + 2NA_{max})$ and $\bar{C}_l(t) \geq C_l(t)$ is defined as

$$\bar{C}_l(t) \triangleq \sum_{(m,n)\in\mathcal{L}}\sum_{c=1}^{K}\mu_{mn}^{cl}(t)(1-S_l(t)). \tag{30}$$

By substituting (30), we rewrite (29) as:

$$\Delta(t) - V\mathbb{E}\{\sum_{n=1}^{N}\sum_{c=1}^{K}R_n^c(t)|\mathbf{Q}(t)\} \leq B - \sum_{l=1}^{L}\rho_l\mathbb{E}\{X_l(t)|\mathbf{Q}(t)\} + \mathbb{E}\{\sum_{n=1}^{N}\sum_{c=1}^{K}(U_n^c(t)-V)R_n^c(t)|\mathbf{Q}(t)\} - \mathbb{E}\{\sum_{l=1}^{L}\sum_{(m,n)\in\mathcal{L}}\sum_{c=1}^{K}\mu_{mn}^{cl}(t)[(U_m^c(t)-U_n^c(t))S_l(t) - (1-S_l(t))X_l(t)]|\mathbf{Q}(t)\} \tag{31}$$

where we have employed the equality:

$$\sum_n U_n^c(t)(\sum_i \mu_{ni}^{cl}(t) - \sum_j \mu_{jn}^{cl}(t)) = \sum_{m,n}\mu_{mn}^{cl}(U_m^c(t)-U_n^c(t)),$$

which can be obtained from simple algebra.

We now consider the last term on the RHS of (31). According to the fact that $\mathbf{S}(t-1)$ is independent of $\mathbf{Q}(t)$, by taking iterated conditional expectation on $\mathbf{S}(t-1)$, we have:

$$\mathbb{E}\{\sum_{l=1}^{L}\sum_{(m,n)\in\mathcal{L}}\sum_{c=1}^{K}\mu_{mn}^{cl}(t)[(U_m^c(t)-U_n^c(t))S_l(t) - (1-S_l(t))X_l(t)]|\mathbf{Q}(t)\} = \sum_{\mathbf{S}}\mathbb{E}\{\sum_l\sum_{(m,n)}\sum_c\mu_{mn}^{cl}(t)[(U_m^c(t)-U_n^c(t))S_l(t) - (1-S_l(t))X_l(t)]|\mathbf{Q}(t),\mathbf{S}(t-1)\}p_{\mathbf{S}} \tag{32}$$

where $p_{\mathbf{S}} \triangleq Pr\{\mathbf{S}(t-1)=\mathbf{S}\}$.

Note that $\psi(t)$ is known to SUs given $\mathbf{S}(t-1)$. Conditioned by $\mathbf{Q}(t)$ and $\mathbf{S}(t-1)$, $(\mu_{mn}^{cl}(t))$ and $(U_n^c(t))$ are fixed under ***ALG***. Thus, by the definition of $P_l(t)$ in (1), we can rewrite (32) in the form as shown in Lemma 1. We can see that our proposed ***ALG*** (12) maximizes (32) over all scheduling algorithms that choose scheduling parameters $(\mu_{mn}^{cl}(t))$ depending only on $\mathbf{Q}(t)$ and $\mathbf{S}(t-1)$. Thus, ***ALG*** minimizes the last term of RHS in (31) over a set of feasible algorithms including the STAT algorithm.

The second term of RHS in (31) is also minimized by congestion controller (11). Then, since (29) is equivalent to (31), ***ALG*** minimizes (29) over a set of algorithms including STAT. We can substitute STAT in the RHS of (29) and obtain:

$$\Delta(t) - V\mathbb{E}\{\sum_{n=1}^{N}\sum_{c=1}^{K}R_n^c(t)|\mathbf{Q}(t)\} \leq B - V\mathbb{E}\{\sum_n\sum_c R_n^{c,STAT}(t)\} - \mathbb{E}\{\sum_n\sum_c U_n^c(t)(D_n^{c,STAT}(t) - R_n^{c,STAT}(t))\} - \mathbb{E}\{\sum_{l=1}^{L}X_l(t)(\rho_l - \bar{C}_l^{STAT}(t))\}. \tag{33}$$

Following the analysis in [17] which employs properties of finite-state ergodic Markov chains and Lemma 3, we can obtain:

$$\mathbb{E}\{\Delta(t)\} - V\mathbb{E}\{\sum_{n=1}^{N}\sum_{c=1}^{K}R_n^c(t)\} \leq \bar{B} - V\sum_n\sum_c r_n^{c*}, \tag{34}$$

where $\bar{B}$ is also given in [17].

By taking the time average of both sides of (34) over $0,...,t-1$ and by taking the liminf, we can prove (18), which is the lower-bound on throughput in Theorem 1.