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Maximizing System Throughput Using Cooperative Sensing in Multi-Channel Cognitive Radio Networks

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***Abstract*— In Cognitive Radio Networks (CRNs), unlicensed users are allowed to access the licensed spectrum when it is not currently being used by primary users (PUs). To guarantee a high system throughput in CRNs, the channel state of PUs needs to be accurately detected to reduce conflict. To this end, cooperative spectrum sensing has been proposed to improve sensing accuracy by exploiting the spatial diversity of secondary users (SUs). However, existing works either focus on a single-channel setting, or make certain restrictive assumptions for multi-channel scenarios. In particular, most works on multi-channel CRNs place no limit on the number of channels that an SU can sense, which is impractical due to hardware and sensing duration constraints. In this paper, we study the throughput maximization problem for a multi-channel CRN where each SU can only sense a limited number of channels. We show that this problem is strongly NP-hard, and propose an approximation algorithm with a factor of $\frac{1}{2}(1+\frac{1}{2\sqrt{\sum_{i=1}^{N} l_i}})$, where l_i is the number of channels that SU i can sense and N is the total number of SUs. This performance guarantee is achieved by exploiting a nice structural property, the *subadditivity*, of the objective function. Our numerical results demonstrate the advantage of our algorithm compared with both a random and a greedy sensing assignment algorithms.**

I. INTRODUCTION

In the past decade, cognitive radio networks (CRNs) have emerged as a promising solution for achieving better utilization of the frequency spectrum to satisfy the increasing demand of wireless communication resources. In CRNs, secondary users (SUs) are offered the opportunity of accessing the licensed channel when their activities do not cause disruptions for primary user (PU) transmissions. To this end, the Federal Communications Commission (FCC) [4] has opened the broadcast TV frequency bands for unlicensed users such as WLAN and WiFi. Most recently, congressional negotiators have reached the compromise to allow the auction of TV broadcast spectrum to wireless Internet providers [13]. IEEE has announced the IEEE 802.22 wireless network standard [12] that specifies how to utilize the unused resources between channels in the TV frequency spectrum.

To guarantee a high system throughput in a CRN, the main challenge is for the SUs to accurately detect the channel state of PUs while exploiting transmission opportunities over the white space. Sensing inaccuracies may lead to either a *false alarm*, where a channel is detected to be occupied when it is actually idle, or a *misdetection*, where a channel is detected to be idle when it is actually occupied. While the former hurts SU throughput, the latter hurts both PU and SU throughput. To improve sensing accuracy, **cooperative spectrum sensing** schemes [6], [9], [10] have been recently developed, where a joint decision is derived from individual observations made by multiple SUs, which effectively alleviates the impact of incorrect individual decisions on throughput by exploiting the spatial diversity of the SUs.

While cooperative sensing improves sensing accuracy, it also incurs sensing and reporting overhead at the SU side, especially when an SU senses multiple channels in a multi-channel CRN. In particular, requiring each SU to sense all the channels in a CRN may lead to long sensing durations, especially when the number of channels is large, which in turn reduces the average throughput of SUs. It is therefore reasonable to put a limit on the maximum sensing duration that an SU can afford, which translates to a budget on the number of channels that an SU can sense. Due to the hardware constraints, this budget could be different for different SUs. In this paper, we study the throughput optimization problem for a multi-channel CRN subject to this sensing constraint.

Various cooperative sensing protocols have been proposed for maximizing system-wide performance metrics such as sensing accuracy [9] and system throughput [8], [16]. However, these works either focus on a single-channel setting [9], [8] or allow each SU to sense all the channels [7], [3], [16]. In particular, an optimal Bayesian decision rule that maps a vector of local binary decisions made at SUs to a global decision on PU activity has been found for maximizing system throughput in a single channel setting [8], which achieves significantly better performance than linear rules such as AND, OR, and majority rules. However, a direct extension of the result in [8] to the multi-channel setting would require each SU to sense all the channels and incur high sensing duration. On the other hand, most works on multi-channel cooperative sensing put no explicit constraint on sensing duration of SUs. Furthermore, these works either use a simple linear decision rule [16] or require the transmission of the entire local sensing samples or sensing statistics at each SU. In our work, we choose to use a binary decision rule to avoid the high overhead involved in reporting complete local sensing results. However, instead of using a suboptimal linear rule as in [16], we use the optimal decision rule proposed in [8] for each channel.

In this paper, we study the problem of maximizing the

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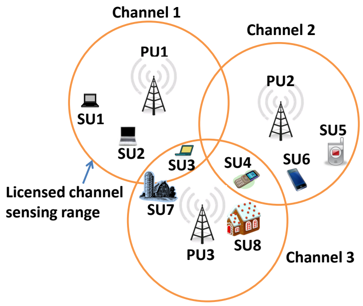


Fig. 1. System model of an SU network overlayed with three licensed channels. SUs in each circle are capable of sensing the corresponding channel(s). SUs outside the sensing range, if selected for sensing, report random sensing results.

system throughput in a multi-channel CRN, by deciding for each channel, a subset of SUs to sense the channel, subject to the sensing budget constraint at each SU. Our main contributions can be summarized as follows:

- We show that the throughput maximization problem is NP-hard in the strong sense and hence does not have a pseudo-polynomial time algorithm unless P = NP.
- We prove that the system throughput function satisfies subadditivity, and based on this property, we propose a matching-based algorithm, which achieves an approximation factor of $\frac{1}{2}(1+\frac{1}{2\sqrt{\sum_{i=1}^{N} l_i}})$, where l_i is the maximum number of channels that SU i can sense and N is the total number of SUs.

This paper is organized as follows. The system model and the problem formulation are introduced in Section II. In Section III, we prove that the optimization problem is NP-hard in the strong sense. We then prove the subadditivity of the system throughput function, and propose an approximation algorithm in Section IV. In Section V, numerical results illustrate the performance of our algorithms. The paper is concluded in Section VI.

II. System Model and Problem Formulation

In this section, we present the system model in two parts: communication model and cooperative sensing model. Based on the models, we formulate our overall objective, which is to decide the channel sensing assignment to maximize the overall system throughput.

A. Communication Model

We consider a time-slotted cognitive radio network composed of M orthogonal channels (each corresponding to a PU)[1] and N SUs (see Figure 1). In the figure, SUs in each circle are capable of sensing the corresponding PU (or channel). An SU may sense multiple PUs depending on its location. For example, SU 3 can sense both channels 1 and 3. When the channel is idle, SUs that do not interfere with each other can transmit over it. Since scheduling and channel assignment for SU transmission are not the focus of this paper, we employ a simple policy: an SU is randomly selected for transmission over each available channel. Our model can readily be extended to practical models where conflict sets for a given interference model are known. We denote the set of SUs by $S=\{s_1,...,s_N\}$ with $|S|=N$, and the set of channels by $C=\{c_1,...,c_M\}$ with $|C|=M$.

[1]Our model can be generalized to the scenario where multiple PUs access the same channel.

B. Cooperative Sensing Model

We assume that a binary decision is made at an SU for each channel it senses. Let $P_f^i(k)$ represent the **probability of false alarm**, i.e., the probability that a SU s_i senses channel k to be occupied when in fact it is idle. Similarly, $P_m^i(k)$ represents the **probability of mis-detection**, i.e., the probability that s_i senses channel k to be idle when it is actually occupied. When mis-detection happens, both PU and SU are transmitting which causes a collision. No packet will be successfully received. Note that SUs outside the sensing range, if selected for sensing, report random sensing results. For instance, in Figure 1, $P_m^1(3)=\frac{1}{2}$ and $P_f^1(3)=\frac{1}{2}$ since SU 1 is outside the sensing range of PU 3. We assume that these probabilities can be learned using historical data [3], [6], [7]. For instance, given the location information of SUs and hardware parameters such as energy detection threshold and time bandwidth product, etc., $P_m^i(k)$ and $P_f^i(k)$ can be calculated accordingly (see Section V-A for an example).

Multi-Channel Cooperative Sensing: SUs may sense the licensed channels cooperatively to reduce sensing errors. The sensing results of individual SUs are assumed to be independent. As mentioned earlier, due to practical constraints, SUs can sense a limited number of channels. We denote l_i as the maximum number of channels that SU s_i can sense in a time slot, $0 \le l_i \le M$, for all $i=1,\cdots,N$ and let $l_{max}=\max_{i=1}^{N} l_i$. Note that $l_i=0$ means that the SU is in not in the sensing range of any channel, thus it cannot do any sensing and only guess the PU state randomly. To encourage cooperative sensing, we assume that $\sum_i l_i \ge M$, which is common in cooperative sensing models [16], thus the expected number of SUs that sense a certain channel is at least 1. In cooperative sensing under the multi-channel setting, multiple SUs choose to sense different channels and predict channel availability subject to the budget constraint, and different sensing set assignments lead to different system throughput across channels. We consider a centralized system model, where a central controller is responsible for (1) maintaining system parameters for PUs and SUs (2) in each time slot, deciding for each channel, a subset of SUs to sense the channel, and (3) making a global decision on channel availability based on the local binary decisions of SUs. Let S_k denote the set of SUs that cooperatively sense channel k. The set of all feasible channel sensing assignment policies are denoted by $\mathcal{P}$, and defined as follows.

Definition 2.1: **Feasible assignment policy** $\mathcal{P}$: A set of sensing sets $\{S_1,\cdots,S_M\}$ is a feasible assignment policy if $\sum_{k=1}^{M} 1_{\{s_i\in S_k\}} \le l_i$ for all i, i.e., all SUs must be assigned to at most l_i channels to sense.

Let $x_i(k)$ denote the observation of channel k by SU $s_i \in S_k$. Further, $x_i(k)=1$ represents that s_i observes channel k to be active, while $x_i(k)=0$ represents that s_i observes channel k to be idle. We let $\boldsymbol{x}(S_k)$ denote the vector of observations for channel k. Let $\Omega = \{0,1\}$, and let $f_A : \Omega^{|A|} \rightarrow \Omega$ denote a general decision rule that maps the local observations made by a set of SUs, $A \subseteq S$, to global decision on channel activity. As the domain of f_A will be clear from the context, we drop the subscript and use f instead. This decision rule applies per channel. Let $B(k)$ denote the activity of channel k such that $B(k) = 1$ if channel k is occupied, and $B(k) = 0$ otherwise. According to the definitions of false alarm and mis-detection, we define the conditional probability of sensing channel k to be idle when it is indeed idle as follows, where vector $\boldsymbol{y}$ denotes a particular instance of an observation vector:

$$\begin{aligned} & P(f(\boldsymbol{x}(S_k)) = 0 | B(k) = 0) \\ = & \sum_{\boldsymbol{y}: f(\boldsymbol{y})=0} P(\boldsymbol{x}(S_k) = \boldsymbol{y} | B(k) = 0), \end{aligned} \tag{1}$$

where

$$\begin{aligned} & P(\boldsymbol{x}(S_k) = \boldsymbol{y} | B(k) = 0) \\ = & \prod_{y_i=1, s_i \in S_k} P_f^i(k) \prod_{y_j=0, s_j \in S_k} (1 - P_f^j(k)), \end{aligned}$$

Similarly, we define the conditional probability of sensing channel k to be occupied when it is indeed occupied:

$$\begin{aligned} & P(f(\boldsymbol{x}(S_k)) = 1 | B(k) = 1) \\ = & \sum_{\boldsymbol{y}: f(\boldsymbol{y})=1} P(\boldsymbol{x}(S_k) = \boldsymbol{y} | B(k) = 1), \end{aligned} \tag{2}$$

where

$$\begin{aligned} & P(\boldsymbol{x}(S_k) = \boldsymbol{y} | B(k) = 1) \\ = & \prod_{y_i=1, s_i \in S_k} (1 - P_m^i(k)) \prod_{y_j=0, s_j \in S_k} P_m^j(k). \end{aligned}$$

We assume that in each time slot, a control slot T_c is assigned for cooperative sensing, during which time a central controller collects $P_m^i(k)$ and $P_f^i(k)$ from SUs, determines the channel sensing assignment, collects sensing results from SUs, and notifies an SU per channel to transmit if that channel is cooperatively sensed to be "idle." Note that each SU i only needs to send updates to the central controller of $P_m^i(k)$, $P_f^i(k)$ when their values change, e.g, when the location of the SU changes. Furthermore, the central controller only needs to compute a new assignment only when $P_m^i(k)$, $P_f^i(k)$ change. We assume T_c to be a constant in the paper. We further assume that SUs can transmit at the same bit rate over each channel, and normalize this rate to 1. SUs are assumed to be always backlogged and only one of them is scheduled over channel k if sensed available in each time slot. Let $\pi_0(k)$ denote the probability that channel k is idle, which is assumed to be acquired accurately over time. The capacity of channel k is denoted by $\gamma(k)$ (after normalization), $k = 1, \cdots, M$. We define $\theta_1(k) = (1 - T_c)\pi_0(k)$ and $\theta_2(k) = \gamma(k)(1 - \pi_0(k))$. Following the logic in [8] and extending to the multi-channel case, we define the expected SU throughput over channel k sensed by S_k.

$$\begin{aligned} U_k^1(S_k) &:= (1 - T_c) P(B(k) = 0, f(\boldsymbol{x}(S_k)) = 0) \\ &= \theta_1(k) P(f(\boldsymbol{x}(S_k)) = 0 | B(k) = 0) \\ & \quad \text{if } S_k \neq \emptyset; \\ U_k^1(S_k) &:= 0 \text{ if } S_k = \emptyset. \end{aligned} \tag{3}$$

where we assume that if $S_k = \emptyset$, no sensing is conducted for channel k and the channel is never accessed. Likewise, the expected PU throughput of channel k can be represented by

$$\begin{aligned} U_k^2(S_k) &:= \theta_2(k) P(f(\boldsymbol{x}(S_k)) = 1 | B(k) = 1) \\ & \quad \text{if } S_k \neq \emptyset; \\ U_k^2(S_k) &:= \theta_2(k) \text{ if } S_k = \emptyset. \end{aligned} \tag{4}$$

Definition 2.2: **System throughput**: For a channel assignment $\{S_1, \cdots, S_M\}$, we define the throughput over channel k to be the sum of SU and PU throughput over channel k, denoted as $U_k(S_k) = U_k^1(S_k) + U_k^2(S_k)$. The system throughput is defined as $\sum_{k=1}^{M} U_k(S_k)$.

Note that for a given channel sensing assignment, the achievable system throughput is determined by the decision rule f. In this paper, we apply the optimal Bayesian decision rule proposed in [8] to each channel respectively, to obtain the optimal expected system throughput. Formally, for each channel k and an observation vector $\boldsymbol{y}$ by S_k, if $\theta_2(k)P(\boldsymbol{x}(S_k) = \boldsymbol{y}|B(k) = 1) \geq \theta_1(k)P(\boldsymbol{x}(S_k) = \boldsymbol{y}|B(k) = 0)$, the decision on channel k is "occupied", and the contribution to throughput is $\theta_2(k)P(\boldsymbol{x}(S_k) = \boldsymbol{y}|B(k) = 1)$; otherwise, the decision on channel k is "idle" and the contribution is $\theta_1(k)P(\boldsymbol{x}(S_k) = \boldsymbol{y}|B(k) = 0)$.

C. Problem Formulation

We formulate the optimization problem to maximize the system throughput, including PUs and SUs on all channels, as follows:

Problem (A): $\max_{\{S_1, \cdots, S_M\} \in \mathcal{P}} \sum_{k=1}^{M} U_k(S_k)$,

where the Bayesian decision rule is implicit in the definition of $U_k(\cdot)$.

Our goal is to decide the optimal channel sensing assignment to maximize system throughput. We adopt a common assumption that PUs can tolerate interference to a certain extent, which may appear in the form of a constraint as in [3], [9] and our earlier paper [8] for the single channel setting. In the future, we plan to extend our solution presented in this paper to Problem (A) with explicit constraints on PU throughput.

We assume that the system is static and the optimization is done in a single time slot. Note that the solution of the static assignment would apply to multiple time slots if $P_m^i(k)$ and $P_f^i(k)$ do not change over time, or if changes occur over a much slower time scale.

III. Hardness of the Problem

In this section, we will show that Problem (A) is strongly NP-hard [14], by a reduction from Product Partition, which is NP-complete in the strong sense [1]. The Production Partition problem is defined as follows: Given N positive integers $a_1, a_2, \cdots, a_N$, is there a subset $X \subseteq \mathcal{N} := \{1, 2, \cdots, N\}$ such that $\prod_{i \in X} a_i = \prod_{i \in \mathcal{N} \setminus X} a_i$?

We reduce Product Partition to the following subproblem of Problem (A), with $M = 2$, $P_f^i(1) = P_f^i(2) = 0$ for all i, $P_m^i(1) = P_m^i(2) := P_m^i$ for all i, and $l_i = 1$ for all i,

$\gamma(1) = \gamma(2) := \gamma$, $\pi_0(1) = \pi_0(2) := \pi_0$, $(1 - T_c)\pi_0 := \theta_1$, $\gamma(1 - \pi_0) := \theta_2$, and $\theta_1 = \theta_2$.

Let (S_1, S_2) denote a solution to this subproblem. Without loss of optimality, we can assume S_1 and S_2 form a partition of the set of SUs, i.e., $S_1 \cup S_2 = S$ and $S_1 \cap S_2 = \emptyset$. The expected system throughput can then be easily determined using the Bayesian rule as $U_1(S_1) = \theta_1 + \theta_2(1 - \prod_{s_i \in S_1} P_m^i)$ and $U_2(S_2) = \theta_1 + \theta_2(1 - \prod_{s_i \in S_2} P_m^i)$. Problem (A) then becomes: $\max_{S_1 \subseteq S} \left[2\theta_1 + \theta_2(2 - (\prod_{s_i \in S_1} P_m^i + \prod_{s_i \in S \setminus S_1} P_m^i))\right]$, which is further equivalent to $\min_{S_1 \subseteq S} (\prod_{s_i \in S_1} P_m^i + \prod_{s_i \in S \setminus S_1} P_m^i)$ since $2\theta_1 + 2\theta_2$ is a constant. We then establish the strong NP-hardness of Problem (A) by showing that this new problem is strongly NP-hard.

Proposition 3.1: Problem (A) is strongly NP-hard.

Proof: By the above argument, it suffices to prove that the subproblem, $\min_{S_1 \subseteq S} (\prod_{s_i \in S_1} P_m^i + \prod_{s_i \in S \setminus S_1} P_m^i)$, is strongly NP-hard. Given an instance of Product Partition with parameters $a_1, \cdots, a_N$, we reduce it to an instance of this subproblem as follows: let $P_m^i = a_i / 10^r$, $i = 1, \cdots, N$, where r is the smallest integer such that $P_m^i \leq 1$ for all $i = 1, ..., N$. This reduction can clearly be done in polynomial time. Furthermore, if there is a subset $X \subseteq \mathcal{N}$, such that $\prod_{i \in X} a_i = \prod_{i \in \mathcal{N} \setminus X} a_i = \sqrt{\prod_{i \in \mathcal{N}} a_i}$, then the optimal solution to the subproblem is $2\sqrt{\prod_{s_i \in S} P_m^i}$, and vice-versa. Hence if there is polynomial time algorithm to the subproblem, the Product Partition problem can be determined in polynomial time as well, which contradicts the fact that Product Partition is strongly NP-complete. ■

Since Problem (A) is strongly NP-hard, no pseudo-polynomial time algorithms exist unless P = NP [14]. We will propose a matching-based approximation algorithm that has theoretical lower bound in Section IV.

IV. Approximate Solutions

In this section, we propose an efficient approximation algorithm for Problem (A). We first note that Problem (A) can be reviewed as a welfare maximization problem studied in the context of combinatorial auctions, where the set of SUs correspond to the set of items for sale, and the set of PUs are the bidders, and the throughput function $U_k(S_k)$ models the valuation for the k-th bidder when it obtains a subset of items S_k. Although the general welfare maximization problem is hard to approximate [2], it allows efficient approximations when the utility function (system throughput function in our scenario) satisfies some structural properties [2], [5].

In the following, we first prove that the system throughput function satisfies the *subadditivity* in Section IV-A. By exploiting this property, we then design a matching-based approximation algorithm with an approximation factor of $\frac{1}{2}(1 + \frac{1}{2\sqrt{\sum_i l_i}})$ in Section IV-B, which adapts the algorithm in [2] with two extensions. First, the algorithm in [2] allows each item to be purchased by at most one bidder, while we allow each SU to sense multiple channels. Second, a direct application of the algorithm in [2] leads to a factor $O(1/\sqrt{\sum_i l_i})$ approximation, while we improve it to a $> \frac{1}{2}$ factor approximation in our setting by subtracting a lower bound from the throughput function.

A. Subadditivity of the System Throughput

We first show that for any $k = 1, ..., M$, the utility function $U_k(\cdot)$ as a set function satisfies subadditivity. A set function $U : 2^S \rightarrow R^+$ is subadditive if for any $V, W \subseteq S$, $U(V \cup W) \leq U(V) + U(W)$, which models a complement-free property commonly seen in reality. A detailed proof of the following proposition can be found in our online technical report [17].

Proposition 4.1: $U_k(\cdot)$ is subadditive for all k.

Proof: (Proof sketch) For two sets V, W, we consider all observations made by V which may lead to the decision of 0 or 1, and all observations made by W which may lead to the decision of 0 or 1. The challenge in establishing the proposition lies in that even if the observations of a set of SUs V and the observations of another set of SUs W lead to the same decision on the status of channel k, the joint decision when combining the two sets of SUs could flip. The details are in [17]. ■

B. A Matching-Based Approximation Algorithm

In this section, we propose a maximum weighted matching (MWM) [15] based algorithm to Problem (A) by extending the algorithm in [2]. We first provide a detailed description of our algorithm (see Algorithm 1), and then establish its approximation factor.

The algorithm starts with constructing a complete and weighted bipartite graph (lines 2-4), where for each channel k, a vertex c_k is constructed, and for each SU s_i, l_i vertices are constructed corresponding to the l_i copies of the SU, denoted as $s_i^j, j = 1, ..., l_i$, and for any pair of vertices s_i^j and c_k, there is an edge connecting them. For any channel k, let Δ_k denote the minimum achievable throughput respecting to the channel k when sensed by a single SU, i.e., $\Delta_k = \min_{i=1,...,N} U_k(\{s_i\})$, which can be determined using the Bayesian decision rule. The weight of an edge (s_i^j, c_k) is then defined as $w(s_i^j, c_k) = U_k(\{s_i\}) - \Delta_k$ (line 6).

A maximum weight matching in the bipartite graph is then found (line 7). A matching in a graph is a set of pairwise non-adjacent edges and a maximum weight matching is then a matching of maximum weight [15]. For each edge (s_i^j, c_k) in the matching, SU s_i is assigned to sense channel c_k. A greedy heuristic is applied for determining the assignment of the remaining copies of SUs to channels (lines 9-13). Basically, the remaining copies are first sorted in an arbitrary order, and a copy of s_i is assigned to the channel that provides the maximum marginal improvement of the system throughput among all the channels not assigned to s_i yet. This scheme is then compared with another scheme for which all SUs are assigned to a single channel that gives maximum throughput (line 14). The algorithm outputs whichever scheme provides a larger system throughput.

Algorithm 1 A maximum weighted matching based algorithm for maximizing the system throughput across channels

Input: N, M, T_c, $\pi_0(k)$, $\gamma(k)$ for all k; l_i for all i; $P_m^i(k)$, $P_f^i(k)$ for all i and k

Output: U and S_k for all k

$S_k \leftarrow \emptyset$ for all k
$V \leftarrow \{s_1^1, \cdots, s_1^{l_1}, \cdots, s_N^1, \cdots, s_N^{l_N}\} \cup \{c_1, \cdots, c_M\}$
$E \leftarrow \bigcup_{i=1,\cdots,N;\ k=1,\cdots,M} \left\{ \bigcup_{j=1}^{l_i} (s_i^j, c_k) \right\}$
$G \leftarrow (V, E)$
$\Delta_k \leftarrow \min_{i=1,\cdots,N} U_k(\{s_i\})$
$w(s_i^j, c_k) \leftarrow U_k(\{s_i\}) - \Delta_k$, $\forall i = 1, \cdots, N$, $j = 1, \cdots, l_i$, $k = 1, \cdots, M$
$\mathcal{M} \leftarrow$ a maximum weight matching in G
$S_k \leftarrow \{s_i : (s_i^j, c_k) \in \mathcal{M}\}$, $\forall k$
$R \leftarrow \{s_i^j : s_i^j$ is not matched in $\mathcal{M}\}$
for all $s_i^j \in R$ **do**
  $k^* \leftarrow \arg\max_{k\in\{1,\cdots,M\}, s_i \notin S_k} \left[U_k(S_k \cup \{s_i\}) - U_k(S_k) \right]$
  $S_{k^*} \leftarrow S_{k^*} \cup \{s_i\}$
$U \leftarrow \sum_{k=1}^{M} U_k(S_k)$
$U_1 \leftarrow \max_{k=1}^{M} U_k(S)$
if $U_1 > U$ **then**
  $U \leftarrow U_1$
  $k^* \leftarrow \arg\max_{k=1}^{M} U_k(S)$
  $S_{k^*} \leftarrow S$, $S_k \leftarrow \emptyset\ \forall k \neq k^*$

We then analyze the complexity of Algorithm 1, which is dominated by computing the maximum weighted matching and evaluating the throughput function $U_k(\cdot)$. It is shown in [8] that for a given sensing set S_k, $U_k(S_k)$ can be evaluated using a dynamic programming algorithm in pseudo-polynomial time. Let Q denote the time complexity for one evaluation of $U_k(\cdot)$. Note that the total number of such evaluations is bounded by $Nl_{max}M$. Therefore, the time complexity of Algorithm 1 is $O(Nl_{max}MQ + (Nl_{max} + M)^3)$.

We then show that our algorithm closely approximates the optimal solution to Problem (A). To this end, we first present a lower bound for Δ_k.

Lemma 4.2: $\Delta_k \geq \frac{1}{2}(\theta_1(k) + \theta_2(k))$ for all k.

Proof: For any SU s_i and channel c_k, we have

$$\begin{aligned} U_k(\{s_i\}) &= \max\left\{\theta_1(k)(1 - P_f^j(k)), \theta_2(k)P_m^j(k)\right\} \\ &\quad + \max\left\{\theta_1(k)P_f^j(k), \theta_2(k)(1 - P_m^j(k))\right\} \\ &\geq \max\left\{\theta_1(k), \theta_2(k)\right\} \\ &\geq \frac{1}{2}(\theta_1(k) + \theta_2(k)). \end{aligned}$$

Hence

$$\Delta_k = \min_{i=1,\cdots,N} U_k(\{s_i\}) \geq \frac{1}{2}(\theta_1(k) + \theta_2(k)) \text{ for all } k.$$

■

We then establish the approximation factor of Algorithm 1 for Problem (A) in the following proposition.

Proposition 4.3: Algorithm 1 achieves at least a fraction of $\frac{1}{2}(1 + \frac{1}{2\sqrt{\sum_{i=1}^{N} l_i}})$ of the optimal system throughput for Problem (A).

Proof: We shift all the edge weights by Δ_k so that they are still non-negative by Lemma 4.2. Since U_k is subadditive as proved in Proposition 4.1, we can apply the proof of Theorem 2.2 in [2] and further show the fraction. The details of the proof is in our online technical report [17]. ■

The above approximation factor can be further improved by introducing an input-dependent factor defined as follows:

$$\beta := \min\{\beta' : \Delta_k \geq \frac{1}{\beta'}(\theta_1(k) + \theta_2(k)) \text{ for all k}\}. \quad (5)$$

We then have the following improved performance guarantee, the proof of which is in our online technical report [17].

Corollary 4.4: For β defined in (5), Algorithm 1 achieves at least a fraction $\frac{1}{\beta} + \frac{1}{2\sqrt{\sum_i l_i}}(1 - \frac{1}{\beta})$ of the optimal system throughput for Problem (A).

Remark 1: In the proof of Proposition 4.3, we have ignored the greedy heuristic applied to the copies of SUs not included in the matching. Hence the result established above only provides a lower bound on the performance of our algorithm. Proving a tighter bound for the algorithm that incorporates the greedy heuristic is part of our future work.

Remark 2: We note that a system throughput of $\sum_{k=1}^{M} \frac{1}{2}(\theta_1(k) + \theta_2(k))$ can be easily achieved even when a single worst SU with $P_m^i(k) = P_f^i(k) = \frac{1}{2}$ is sensing each channel. Thus it is the lower bound of the system throughput when each PU is assigned at least one SUs for sensing.

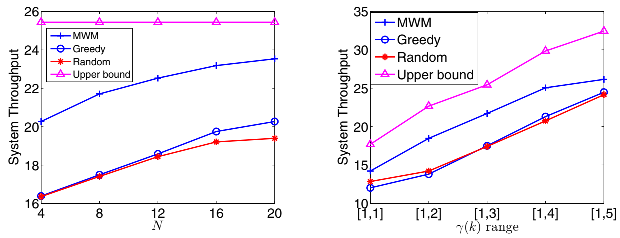
V. Simulations

In this section, we study the performance of our algorithm through simulations by comparing Algorithm 1 (MWM) with a random sensing assignment algorithm, and a greedy algorithm (defined next). In the random algorithm, the copies of SUs are randomly assigned to PUs. The greedy algorithm works as follows: for each PU k, the set of SUs are first sorted by $P_m^i(k) + P_f^i(k)$ in a non-decreasing order as its preference list. In each round, a random permutation of the set of PUs is applied. The algorithm then goes through the PU list, and for each PU k, a copy of the SU, say s_i, with the lowest $P_m^i(k) + P_f^i(k)$ among the remaining SUs, which has not been assigned to k before and has remaining copies, is assigned to k. Repeat this procedure till all copies of SUs have been assigned.

A. Simulation Setting

The following parameters are fixed throughout the simulations. We consider a 100×100 area, where the locations of M PUs are randomly generated. For each PU k, its maximum power level is randomly chosen between 1 and 10, and $\pi_0(k)$ are randomly generated in $[0, 1]$. We also set $T_c = 0.2$ fixed.

In each of the 100 runs of the simulation, the following parameters are varied independently. First, the channel status of PU k, either transmitting with the maximum power or idle, is randomly chosen according to $\pi_0(k)$. The locations of N SUs (each SU is capable of sensing any of the M channels) are then randomly generated. The details in generating $P_m^i(k)$ and $P_f^i(k)$ are in [17]. In each of the



(a) $M = 20, l_{max} = 3, \gamma(k) \sim U[1, 3]$. (b) $M = 20, N = 8, l_{max} = 3$.

Fig. 2. System throughput achieved by our algorithm, greedy algorithm, random algorithm and the upper bound.

100 runs, we repeat these steps to generate new $P_m^i(k)$ and $P_f^i(k)$.

B. Simulation Results

The simulation results are shown in Figure 2. Note that we do not restrict $\sum_i l_i \geq M$ in our simulations. If PU k is not assigned any SU for sensing ($S_k = \emptyset$), the system throughput on channel k is $\theta_2(k)$ (Definition 2.2). In all the figures, we plot $\sum_k \left[\theta_1(k) + \theta_2(k)\right]$ as the upper bound for the optimal solution.

In Figure 2(a), we fix $M = 20$, $l_{max} = 3$, and vary N from 4 to 20. For each PU k, $\gamma(k)$ in generated randomly in $[1, 3]$ and then fixed over all 100 runs. We choose this range since the average PU throughput is usually larger than the SU throughput, which is normalized to 1. For each SU i, l_i is randomly generated between 1 and l_{max} and fixed over all the runs. The simulations results are averaged over all 100 runs. We observe that Algorithm 1 achieves significant improvement over the random and the greedy algorithms for all N, although the gap shrinks as N increases. For instance, the system throughput of Algorithm 1 is 24% larger than that of the greedy algorithm when $N = 4$ and it decreases to 16% when $N = 20$. Note that the performance of Algorithm 1 reaches 95% of the upper bound of the optimal solution when $N = 20$. When more SUs join the network, the random and the greedy algorithms have more chance to choose "good" SUs. The greedy algorithm is comparable to the random algorithm when N is small. However, it wins over the latter when $N \geq 12$. This indicates that the sorting step in the greedy algorithm helps PUs pick the "right" SUs, which is more useful when N is large.

In Figure 2(b), we fix $M = 20$, $N = 8$, $l_{max} = 3$, and vary the range of the channel capacity $\gamma(k)$. For instance, $[1, 2]$ means all channel capacities are randomly generated between 1 and 2. Algorithm 1 is constantly better than the other two algorithms. The gap first increases as the channel capacity increases (from 18% to 34%) till $\gamma(k) \in [1, 2]$, and decreases thereafter (7% at $\gamma(k) \in [1, 5]$). This can be explained by: When the channel capacity is comparable to unit SU capacity, the choice of SUs for sensing does not affect the system throughput significantly; When the channel capacity dominates the system throughput, the choice of SUs again loses its leading role. Thus the largest gap appears in the middle.

VI. Conclusion

In this paper, we investigate the problem of throughput maximization using cooperative sensing in multi-channel CRNs, where each SU can only sense a limited number of channels with various sensing capabilities, due to time or energy constraints. We show that under the optimal Bayesian decision rule, the channel sensing assignment problem is strongly NP-hard. A matching based algorithm is then proposed with an approximation ratio of $\frac{1}{2}(1 + \frac{1}{2\sqrt{\sum_{i=1}^N l_i}})$, where l_i is the maximum number of channels SU i can sense and N is the total number of SUs. Our numerical results demonstrate that our algorithm performs significantly better than the random channel sensing assignment algorithm and a greedy algorithm. As part of our future work, we plan to establish a tighter performance bound for our algorithm enhanced with a greedy heuristic, and consider the system throughput maximization problem with extra constraints on the PU throughput.

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