Multiple Access Game with a Cognitive Jammer

Karim Khalil and Eylem Ekici
Department of Electrical and Computer Engineering
Ohio State University, Columbus, Ohio
Email: {khalilk, ekici}@ece.osu.edu

***Abstract*—We consider a two-user multiple access game in which one player (primary user) is interested in maximizing its data rate at the minimum possible transmission power and the other player (secondary, cognitive user) can either jam the primary traffic or coordinate with the primary user and send its own message to the common destination. The cognitive user employs noise forwarding as a leverage to maximize its own data rate by forcing the primary user to decrease its power level. First, the unique Nash equilibrium of the non-cooperative static game is derived and shown to be inefficient for certain ranges of channel gains and cost parameters. Then, a Stackelberg game formulation is considered in which the primary user is the leader. Here, interestingly, it is shown that the secondary accepts to play as the follower where the Stackelberg equilibrium dominates the Nash equilibrium and hence lose-lose situations are eliminated.**

I. Introduction

Cognitive Radio Networks (CRNs) have been proposed to improve the utilization of scarce spectral resources to meet increasing demand for bandwidth over wireless channels. Last decade has witnessed significant research activity on CRNs [1].

Security issues have been studied for wireless networks in general and for cognitive radio networks in particular [2], [3], [4] (and references therein). For example, in [2], the authors develop a verification scheme to counter an attack where a malicious cognitive user emulates primary transmission to reduce channel resources available to other cognitive users. In [3], an anti-jamming game is considered where attackers inject interference to interrupt secondary transmissions.

However, in all these works, the sole objective of the attacker is to cause damage to the attacked system. Deviating from this attack model, we use adversarial activity to achieve other objectives. Specifically, secondary users can use the attack as a leverage to maximize their own performance. In our model, secondary users gain access to the spectrum by leveraging noise forwarding. In a system where both primary users (PUs) and secondary users (SUs) transmit to a common destination (e.g., base station or access point), SUs threaten the primary system by sending noise symbols and hence affecting the achieved utility of PUs. This threat possibly forces PUs to lower their transmission power levels to motivate SUs to coordinate their transmission. Consequently, SUs can then achieve higher rates since interference from primary to secondary transmission will be lowered, as well.

Game theory has been employed as an important mathematical tool in the wireless network research [5], [6] (and the references therein). Game theory provides an analytical framework to analyze situations of conflict and coordination between multiple decision makers that are ***rational, intelligent and selfish***. These attributes also accurately characterize wireless devices designed to optimize their performance. In many cases (e.g., in heterogeneous networks), it is unrealistic to assume that users will cooperate to reach the optimal performance of the network, unless the global optimal is beneficial for the users on an individual level. A good example of game theoretic modeling is given in [7], where a power allocation algorithm is designed for multiple access channels with selfish users. Recently, game theory has been employed as a mathematical tool to analyze and design protocols for future generation wireless networks like CRN. Examples can be found in [4] and references therein.

In our model, PU is interested in maximizing its data rate while transmitting at the minimum possible power. SU wishes to transmit at a minimum data rate to a common destination D and is willing to coordinate with PU (on the codebook level) if its minimum data rate constraint can be satisfied. To this end, SU divides the time between transmitting its own information (with coordinated codebook) and sending noise symbols. First, the Nash Equilibrium (NE) is characterized for the static game and shown to be unique. For certain ranges of the channel coefficients, the equilibrium point is inefficient and results in a lose-lose situation. A leader-follower game is then formulated in which PU is the leader who specifies its strategy and then announces it to the follower (SU). The follower then reacts to this strategy. In this case, Stackelberg Equilibrium (SE) is shown to dominate the NE and hence the follower is forced to comply with this strategy.

The rest of the paper is organized as follows. Section II presents the required background and results from game theory and information theory. In Section III, the game setup is given and then NE is characterized and interpreted. Stackelberg formulation is then considered in Section IV where the SE is shown to dominate the NE for all values of channel coefficients only when PU is the leader. Finally, in Section V, we conclude the paper and present future research directions.

II. Preliminaries

In this section, we review basic information theoretical results about the multiple access channel that we use in our game formulation. In addition, we present definitions from non-cooperative game theory that are essential in our analysis.

A. Multiple Access Channel

The two-user multiple access channel is a well known channel model in the network information theory [8]. Let the channel capacity function be defined as $C(x) = 0.5\log(1+x)$, where all logarithms in the paper are taken to the base 2. The capacity region of a channel defines the maximum transmission rate transmitter(s) can use so that receiver(s) can decode the information reliably, i.e., with an arbitrarily small probability of decoding error. For a two-user Additive White Gaussian Noise (AWGN) Multiple Access Channel (MAC), the capacity region is a pentagon and is given by

$$R_1(P_1) \le C(aP_1),\ R_2(P_2) \le C(bP_2),$$
$$R_1(P_1) + R_2(P_2) \le C(aP_1 + bP_2), \quad (1)$$

where $R_1(\cdot), R_2(\cdot)$ are the achievable rates for transmitters 1 and 2, respectively, P_1, P_2 are the transmission power levels, and $a > 0$, $b > 0$ are the (constant) channel power gains when noise has unit variance. In the following, we assign user 1 to be PU and user 2 to be SU. To achieve all points within the capacity region (1), it is required that both transmitters *coordinate the codebook* used in the channel coding [8]. The two corner points of the capacity region are achieved by successive interference cancellation where the order of decoding at the destination determines the corner point [8]. In this paper, we employ rate expressions on the boundary of the region (1). Specifically, we assume that the destination always decodes SU first in the interference cancellation decoder and hence gives priority to PU. Consequently, the achievable rates at the destination for PU and SU are

$$R_1(P_1) = C(aP_1),\ R_2(P_2) = C(\frac{cP_2}{1+aP_1}). \quad (2)$$

B. Game Theory Basics

Now, we borrow definitions from [9] and [10] that are needed for the analysis of the games developed in the following sections. Specifically, we introduce two types of non-cooperative game formulations; the Nash game and the Stackelberg game. Let the utility of player i be given by $u_i(s_i, s_{-i})$ where $s_i \in \mathcal{S}_i$ is the (pure) strategy of player i chosen from the set of available strategies $\mathcal{S}_i$ and s_{-i} is the strategy profile of all other players except player i chosen from $\times_{j\in\mathcal{N}-\{i\}}\mathcal{S}_j$, where $\mathcal{N}$ is the set of players in the game. Formally, a strategic game is any $\mathcal{G}$ of the form $\mathcal{G} = (\mathcal{N}, (\mathcal{S}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$. In the following definitions and in the rest of the paper, we focus on two-player games, i.e., $\mathcal{N} = \{1, 2\}$.

Definition 1: An NE is a strategy pair (s_1^*, s_2^*) such that

$$u_1(s_1^*, s_2^*) \ge u_1(s_1, s_2^*), \forall s_1 \in \mathcal{S}_1,$$
$$u_2(s_1^*, s_2^*) \ge u_2(s_1^*, s_2), \forall s_2 \in \mathcal{S}_2. \quad (3)$$

This definition implies that at an NE point, no user has an incentive to unilaterally deviate. Assume there exists two well defined unique mappings $T_1 : \mathcal{S}_2 \to \mathcal{S}_1$ and $T_2 : \mathcal{S}_1 \to \mathcal{S}_2$ such that for any fixed $s_2 \in \mathcal{S}_2$

$$u_1(T_1(s_2), s_2) \ge u_1(s_1, s_2), \forall s_1 \in \mathcal{S}_1, \quad (4)$$

and for any fixed $s_1 \in \mathcal{S}_1$

$$u_2(s_1, T_2(s_1)) \ge u_2(s_1, s_2), \forall s_2 \in \mathcal{S}_2, \quad (5)$$

i.e., T_i defines strategies that are best response to each strategy chosen by the other player. Let the set

$$D_i = \{(s_1, s_2) \in \mathcal{S}_1 \times \mathcal{S}_2 : s_i = T_i(s_j)\}, \quad (6)$$

for $i = 1, j = 2$ and $i = 2, j = 1$ be called the rational reaction set of player i and let $D_i(s_j) = \{s_i \in \mathcal{S}_i : (s_i, s_j) \in D_i\}$. Note that any pair in the set $D_1 \cap D_2$ is an NE according to Definition 1.

The second type of non-cooperative game formulation we employ in the paper is the Stackelberg game. In a Stackelberg game, we have a leader that makes a decision about its own strategy and followers that choose their strategies accordingly. In the following definitions, we fix player 1 as the leader and player 2 as the follower. Since the game structure is common knowledge, the leader chooses the strategy that maximizes its utility from the rational reaction set of the follower.

Definition 2: A strategy $\bar{s}_1 \in \mathcal{S}_1$ is a Stackelberg equilibrium strategy for the leader if

$$\inf_{s_2\in D_2(\bar{s}_1)} u_1(\bar{s}_1, s_2) \ge \inf_{s_2\in D_2(s_1)} u_1(s_1, s_2), \quad \forall s_1 \in \mathcal{S}_1. \quad (7)$$

In the rest of the paper, we sometimes use the shorthand Stackelberg Equilibrium (SE) to mean Stackelberg equilibrium strategy. We also use the shorthand SEP (respectively SES) to indicate an SE with the PU (respectively SU) as the leader. One property of Stackelberg games is that the utility of the leader is a well defined quantity [9] and is given by

$$\bar{u}_1 = \sup_{s_1\in\mathcal{S}_1} \inf_{s_2\in D_2(s_1)} u_1(s_1, s_2). \quad (8)$$

A Stackelberg equilibrium strategy for the leader may not exist in general. In this case, however, an ϵ-SE can possibly exist in which the leader achieves utility ϵ close to $\bar{u}_1$.

Definition 3: Let $\epsilon > 0$ be a given real number. Then, a strategy $\bar{s}_{1\epsilon} \in \mathcal{S}_1$ is called an ϵ-Stackelberg equilibrium strategy for the leader if

$$\inf_{s_2\in D_2(\bar{s}_{1\epsilon})} u_1(\bar{s}_{1\epsilon}, s_2) \ge \bar{u}_1 - \epsilon. \quad (9)$$

An important property is that an ϵ-SE exists if $\bar{u}_1$ is finite [9].

From Definitions 1, 2, it can be seen that the utility achieved by a user in a Stackelberg game under its leadership is always at least as good as the utility achieved under an NE for the same game [9]. This fact motivates the following definition.

Definition 4: A Stackelberg strategy $(\bar{s}_1, \bar{s}_2)$ is said to dominate an NE (s_1^*, s_2^*) if

$$u_2(\bar{s}_1, \bar{s}_2) \ge u_2(s_1^*, s_2^*). \quad (10)$$

That is, when player 2 (follower) achieves a utility in the Stackelberg game that is at least as good as the utility achieved in an NE, then this SE is said to dominate the NE. In this case, both players (leader and follower) would better choose to play the Stackelberg game under the leadership of player 1. In Section IV, this property will be vital to show that SU accepts to be a follower in a Stackelberg game that leads to better performance for both players.

We note that our model is appropriate for CRNs for multiple reasons. First, when SU is transmitting information to the common destination D, it is given a lower priority than PU. This is clear from the achievable rates (2). In addition, SU actually uses noise forwarding to access unlicensed spectrum to transmit its own information. Through this threat, PU may be forced to decrease its transmission power P_1 and hence SU achieves higher data rate, as will be discussed in the next section. Finally, as discussed in Section IV, the analysis reveals that SU is forced to follow PU in a leader-follower game.

III. Cognitive Threat Nash Game

We consider a two-player static non-cooperative game $\mathcal{G}$ where the players are the primary and the secondary users. In this game, the strategy of the PU is to select the power level $s_1 = P_1 \in [0, P_1^{\max}]$, while the strategy of the secondary is to choose a fraction $s_2 = \alpha \in [0, 1]$ by which it divides the total available time T into message transmission time αT and noise transmission time $(1-\alpha)T$. Without loss of generality, we assume $T = 1$.

PU is interested in maximizing its achievable data rate to the destination at the minimum cost for power, i.e., the utility function of PU is given by

$$u_1(P_1, \alpha) = \alpha C(aP_1) + (1-\alpha) C\left(\frac{aP_1}{1+bP_2}\right) - \gamma P_1, \tag{11}$$

where $\gamma > 0$ is the unit power cost. The first term in (11) reflects the portion of the rate achieved when SU is sending its message and the second term reflects the rate achieved when SU sends jamming signal.

We assume that SU is bounded by a maximum power constraint $P_2^{\max}$. SU uses $P_2^{\max}$ as its fixed transmission power level over the entire transmission period since its achievable data rate is increasing with power as in (2). In addition, we assume that SU selects $\alpha = 1$ only if its achievable rate is above certain threshold β. The utility function of SU is given by

$$u_2(P_1, \alpha) = \alpha \left(C\left(\frac{bP_2^{\max}}{1+aP_1}\right) - \beta \right). \tag{12}$$

Here, β can be also interpreted as the cost for coordination. We assume that $a, b, \gamma, \beta, P_1^{\max}, P_2^{\max}$ are common knowledge. The goal of each user is to maximize its own utility by selecting the appropriate strategy given the knowledge of the other user's utility function.

In the following, we focus our analysis only on pure strategies. This is made possible without any loss of generality since the strategy sets are convex and the utility functions are concave in the corresponding variables [9] as will be shown in Theorem 1. Now we present notation that will be useful in our analysis of the game $\mathcal{G}$.

We define $P_1^*(\alpha)$ as the power level that maximizes $u_1(P_1, \alpha)$ for a given α. Also we define the threshold power level Q as

$$Q = \frac{1}{a}\left(\frac{bP_2^{\max}}{2^{2\beta}-1} - 1\right), \tag{13}$$

where the slope of the function $u_2(P_1, \alpha)$ is zero at $P_1 = Q$ and is negative if $P_1 > Q$. The following Theorem characterizes the unique NE of the game $\mathcal{G}$.

Theorem 1: For the game $\mathcal{G}$, the unique NE point is given by

$$(s_1^*, s_2^*) = \begin{cases} \left(P_1^*(0), 0\right); & \text{if } Q < P_1^*(0) \\ (Q, \alpha_Q); & \text{if } P_1^*(0) \le Q \le P_1^*(1) \\ \left(P_1^*(1), 1\right); & \text{if } P_1^*(1) < Q, \end{cases} \tag{14}$$

where $P_1^*(\alpha)$ is given by

$$P_1^*(\alpha) = \min\left\{P_1^{\max}, \left[\frac{X + \sqrt{X^2 + Y(\alpha)}}{2a^2\bar{\gamma}}\right]^+\right\}, \tag{15}$$
$$X = a^2 - abP_2^{\max}\bar{\gamma} - 2a\bar{\gamma},$$
$$Y(\alpha) = 4a^2\bar{\gamma}\left[a + abP_2^{\max}\alpha - \bar{\gamma}(1 + bP_2^{\max})\right],$$

and α_Q is the time fraction of SU that solves the equation $P_1^*(\alpha) = Q$ and $\bar{\gamma} = \gamma \ln(4)$.

Proof: We show that the intersection of the best response curves of PU and SU are exactly the points in the Theorem. Therefore, no user has incentive to deviate unilaterally from such points and the conditions of Definition 1 are satisfied at these given points.

Consider the function $u_1(P_1, \alpha)$. Given some strategy α of SU, the best response for PU, i.e., $P_1^*(\alpha)$ is given as

$$P_1^*(\alpha) = \underset{P_1 \in [0, P_1^{\max}]}{\arg\max}\; u_1(P_1, \alpha). \tag{16}$$

Using the second derivative test, it can be shown that $u_1(P_1, \alpha)$ is strictly concave in $P_1 \in [0, P_1^{\max}]$ for any fixed $\alpha \in [0, 1]$. Thus, it can be easily shown that the solution of (16) is the one given in (15) by setting the first derivative of $u_1(\cdot)$ with respect to P_1 to zero and solving for P_1.

The utility of SU $u_2(P_1, \alpha)$ is linear in α given P_1. Given P_1, the slope of $u_2(P_1, \alpha)$ is negative for $P_1 > Q$. Consider the case $Q < P_1^*(0)$ and assume NE is at $\alpha = 0$. Then, the best response for the primary player is $P_1 = P_1^*(0) > Q$ implying that $(P_1^*(0), 0)$ is an NE in this case. This proves the first case

Now assume $\alpha^* = 1$. Then, the best response for the primary is $P_1 = P_1^*(1)$. When $P_1^*(1) < Q$, the best response for SU when $P_1 = P_1^*(1)$ is $\alpha = 1$ and hence $(P_1^*(1), 1)$ is an NE in this case.

For the remaining case, note that SU is indifferent to the choice of α when PU chooses $P_1 = Q$ since the slope of $u_2(Q, \alpha)$ is zero in this case. The intersection of the best response sets for PU and SU is at $\alpha^* = \alpha_Q$. The solution α_Q to the equation $P_1^*(\alpha) = Q$ is given by

$$\alpha_Q = \frac{\bar{\gamma}\left[aQ(aQ + bP_2^{\max}Q + 2) + 1 + bP_2^{\max}\right] - a(1 + aQ)}{abP_2^{\max}}.$$

Note that $\alpha_Q \in [0,1]$ if and only if $P_1^*(0) \le Q \le P_1^*(1)$ implying that it is the only NE in this case.

Finally, note that $P_1^*(\alpha)$ is an increasing function in α. Therefore, the relation $P_1^*(0) < P_1^*(1)$ always holds and we do not need to consider other cases. This concludes the proof. ■

We observe that, in the two cases where $Q \le P_1^*(1)$, the NE point is inefficient: there may be other operating points where both users can achieve better utility values. Assume $0 < Q$. In the first case where $Q < P_1^*(0)$, if PU chooses a power level less than but arbitrarily close to Q, then SU is willing to coordinate its transmission all time, i.e., chooses $\alpha = 1$. In this case, SU achieves a strictly positive utility rather than zero utility achieved at the NE. In addition, since the interference from SU vanishes in this case, PU can achieve a better utility if $u_1(Q,1) > u_1(P_1^*(0),0)$. The same argument is valid for the second case where $P_1^*(0) \le Q \le P_1^*(1)$.

To elaborate on our observation, consider the following numerical example. Let $a = 2, b = 0.8, P_1^{\max} = P_2^{\max} = 2, \beta = \bar{\gamma} = 0.5$. According to (13) and (15), these values imply that $Q < P_1^*(0)$. In this case, the utility of PU at the NE is $u_1(P_1^*(0),0) = 0.0583$ while its utility at $P_1 = Q$ and $\alpha = 1$ is $u_1(Q,1) = 0.2308$ which is four times better. When b is changed to $b = 1.2$ while keeping the rest of parameters the same, we have $P_1^*(0) < Q < P_1^*(1)$. Here, $\tilde{\alpha} = 0.2$, $u_1(Q,\tilde{\alpha}) = 0.0728$ and $u_1(Q,1) = 0.379$, which is more than five times better than the primary utility achieved at the NE point. The utility of PU is sketched in Figure 1 for different values of α and the points maximizing each case are marked in addition to the point $u_1(Q,1)$.

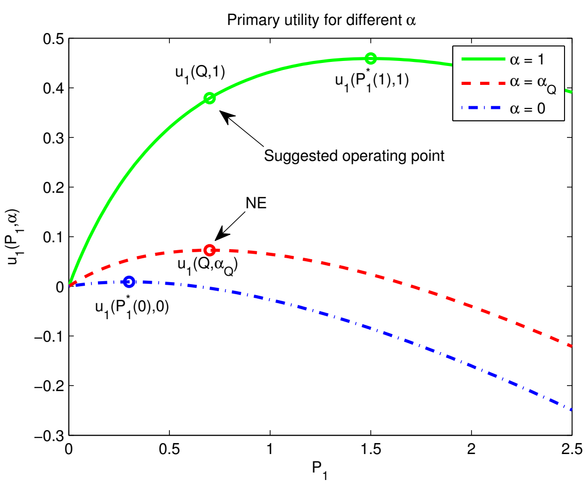

Fig. 1. Primary utility for the case $P_1^*(0) \le Q \le P_1^*(1)$.

The operating point $(Q-\epsilon, 1)$ is not an equilibrium point of the non-cooperative strategic game. For instance, in the case $Q < P_1^*(0)$, if SU chooses $\alpha = 1$, PU can take advantage of this choice and select $P_1 = P_1^*(1)$ which will cause SU to achieve negative utility. Therefore, without communication and contracts between users, there is no guarantee that both users will play the strategy profile $(Q-\epsilon, 1)$ in this case for an arbitrarily small $\epsilon > 0$. However, if both users agree to play the game with some order and not play simultaneously, better equilibrium points can be reached. In the following section, we formulate a leader-follower game in which the inefficient NE points of $\mathcal{G}$ are alleviated.

IV. Cognitive Threat Stackelberg Game

In this section, we show the existence of a Stackelberg strategy under the leadership of PU that results in better payoff values for both players in $\mathcal{G}$. In a Nash game, each player chooses its strategy independently of the *actual* choice of other players. This property of a strategic game can be viewed as if players decide their choices simultaneously. It can also be viewed as a sequential decision process where each player has no information about the decisions of other players.

On the other hand, in a Stackelberg game, the leader of the game chooses its strategy first and then announces it to other players in the game (followers). Then the followers react to the strategy of the leader to maximize their own utilities. This leader-follower scenario can model the situation where a player has the power to enforce other players to be followers. In addition, a rational player is willing to play the Stackelberg game as a follower if this implies a better utility than that achieved at the NE of the game. If all players in the game achieve higher utility in the Stackelberg game with the leadership of some player compared to that achieved at the NE of the game, then this SE is said to dominate the NE according to Definition 4. For our game $\mathcal{G}$, we have two possible Stackelberg games: In the first, PU is the leader and in the second, SU is the leader. Here, we show that an SE with PU as the leader (i.e., SEP) dominates the NE derived in Section III. Moreover, we show that any SE with SU as the leader (i.e., SES) does not dominate the NE of $\mathcal{G}$. This implies that SU is willing to be a follower in a Stackelberg game in order to achieve better utility values.

To check the existence of an SE for our game, we start by computing the rational reaction set for SU, D_2. Recalling the definition from (6), it can be seen that for $\alpha \in [0,1]$

$$D_2 = \{(P_1,0) : P_1 > Q\} \cup \{(P_1,1) : P_1 < Q\} \cup \{(Q,\alpha)\}. \tag{17}$$

The following lemma establishes the existence of an SEP and proves its dominance over the NE in Theorem 1.

Lemma 1: For the game $\mathcal{G}$ and $\forall \epsilon > 0$, if the channel gains a, b and cost parameters γ, β and power constraints $P_1^{\max}, P_2^{\max}$ are finite, then there exists an ϵ-SEP. Moreover, if ϵ is sufficiently small, then ϵ-SEP dominates the NE of $\mathcal{G}$.

Proof: For the existence part, it suffices to show finiteness of $\bar{u}_1$, the utility achieved by the leader, as in [9]. From Definition 3 and from (17), the utility of the PU for a Stackelberg game $\mathcal{G}$ under its leadership can be calculated as

$$\bar{u}_1 = \begin{cases} \max\{u_1(Q,1), u_1(P_1^*(0),0)\}; & \text{if } Q < P_1^*(0) \\ u_1(Q,1); & \text{if } P_1^*(0) \le Q \le P_1^*(1) \\ u_1(P_1^*(1),1); & \text{if } P_1^*(1) < Q. \end{cases}$$

If a, b and $\gamma, \beta, P_1^{\max}, P_2^{\max}$ are finite, then Q and $P_1^*(\alpha)$ are finite for all $\alpha \in [0,1]$. Then, $\bar{u}_1$ is finite in all cases and existence follows from Property 4.2 in [9]. Now, fix some $\epsilon > 0$ and consider the following strategy for PU.

$$s_{1\epsilon} = \begin{cases} \underset{P_1 \in \{Q-\epsilon, P_1^*(0)\}}{\arg\max} \quad u_1(P_1, D_2(P_1)); & \text{if } Q < P_1^*(0) \\ Q - \epsilon; & \text{if } P_1(0)^* \leq Q \leq P_1^*(1) \\ P_1^*(1); & \text{if } P_1^*(1) < Q. \end{cases} \tag{18}$$

For the last case in (18), it can be seen that $u_1(s_{1\epsilon}, D_2(s_{1\epsilon})) = \bar{u}_1$. In addition, for the other two cases, since $u_1(\cdot)$ is uniformly continuous in P_1, it can be seen that we have $u_1(s_{1\epsilon}, D_2(s_{1\epsilon})) \geq \bar{u}_1 - \epsilon$. This implies that $s_{1\epsilon}$ is in fact an ϵ SEP by definition. Finally, to show dominance of the above SEP over NE of $\mathcal{G}$, note that $\bar{u}_1 \geq u_1(s_1^*, s_2^*)$. Then, for ϵ sufficiently small, $u_1(s_{1\epsilon}, D_2(s_{1\epsilon}))$ is sufficiently close to $\bar{u}_1$ and the result follows. ■

When the game starts and according to the channel conditions and the cost parameters, PU and SU choose their strategies. By Definition 4, since the SEP in Lemma 1 dominates NE of $\mathcal{G}$, then it can be seen that SU will prefer to be the follower in a Stackelberg game under leadership of PU than to play Nash. At the NE of $\mathcal{G}$, SU achieves zero utility for $Q \leq P_1^*(1)$. However, at the SEP in Lemma 1, SU achieves a strictly positive utility value in all cases.

Nevertheless, as stated in Section II, the utility of a player in a Stackelberg game under its leadership is at least as good as its utility in a Nash game. Consequently, SU may prefer to play a Stackelberg game under its own leadership and not to follow PU. The following Lemma, however, shows that for the game $\mathcal{G}$, no SES dominates the NE. This result shows that PU can in fact enforce SU to be a follower in a Stackelberg game.

Lemma 2: For the game $\mathcal{G}$, there exists no SE under the leadership of SU that dominates the NE of the game.

Proof: We start be computing the rational reaction set D_1. It can be easily seen that $D_1 = \{(P_1, \alpha) : P_1 = P_1^*(\alpha), \alpha \in [0,1]\}$. Now we check the SE point and compare it to the NE point of $\mathcal{G}$. The SES is given by $(P_1^*(\bar{\alpha}), \bar{\alpha})$ where

$$\bar{\alpha} = \underset{\alpha \in [0,1]}{\arg\max}\, \alpha \; (C(\frac{cP_2^{\max}}{1 + aP_1^*(\alpha)}) - \beta). \tag{19}$$

We start by comparing to the last case in (14). Suppose the maximizer of (19) is $\bar{\alpha} = 1$. Then, we have $u_1(s_1^*, s_2^*) = u_1(\bar{s}_1, \bar{s}_2)$. If $\bar{\alpha} < 1$ and since $u_1(\cdot)$ can only increase by decreasing α, then $u_1(s_1^*, s_2^*) > u_1(\bar{s}_1, \bar{s}_2)$ and the SES is not dominant in this case. For the first case in (14), it is easy to see that $u_1(s_1^*, s_2^*) = u_1(\bar{s}_1, \bar{s}_2)$. Finally, for the middle case, it can be seen that $\bar{\alpha} \leq \alpha_Q$ implying that $u_1(s_1^*, s_2^*) \geq u_1(\bar{s}_1, \bar{s}_2)$. This concludes the proof. ■

As given in Lemma 2, at any SES of $\mathcal{G}$ and comparing to the interesting cases in (14) (the first two cases), SU can choose α that leads to a larger secondary utility. However, this choice can only degrade the primary utility $u_1(\cdot)$ and hence any SES does not dominate the NE point of $\mathcal{G}$ according to Definition 4.

The following theorem summarizes the results of this section where the proof follows from Lemmas 1,2 and the fact that PU can threaten SU to play Nash.

Theorem 2: For the game $\mathcal{G}$, SU accepts to play as the follower and the output of the game is the SEP point in Lemma 1.

Given that both players are rational and that both consider Nash and Stackelberg games, it is clear from Theorem 2 that both players will choose to play the Stackelberg game with PU as leader and SU as follower in all cases of channel conditions and energy cost parameters. It is interesting to note that since PU is the leader in this game, it specifies how much transmission is allowed to the SU (above its threshold) by choosing a strictly positive ϵ. No matter how small ϵ is chosen, SU is forced to comply with this specification.

V. Conclusion

In this paper, we analyzed a possible situation of conflict, where a primary user and a cognitive user communicate with a common destination. The primary transmitter is interested in maximizing its own data rate at the minimum possible power while the secondary transmitter is willing to coordinate its transmission if its achievable data rate is above a minimum rate requirement. Necessarily, the secondary user threatens the primary user if it is not allowed to access the spectrum and transmit its own information at the minimum rate. Using tools from non-cooperative game theory, it is shown that a Stackelberg equilibrium dominates the Nash equilibrium and hence alleviates the inefficient equilibrium cases. In these cases, the cognitive user is forced to follow the strategy dictated by the primary system. In our future investigations, we will consider the effect of fading, and systems with multiple cognitive users.

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