

# Distributed Scheduling and Its Asymptotic Analysis for Cognitive Radio Networks Under the Many-Channel Regime

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**Abstract**—The design of efficient and distributed scheduling algorithms is essential to garner the full potential of cognitive radio networks (CRNs). In this paper, we propose a distributed orthogonal frequency-division multiplexing (OFDM)-based scheduling algorithm, which is called the *collision-queue-regulated algorithm*, that aims to limit the collision rate to a level imposed by primary users of a CRN. Via a novel equivalent queue system analysis, we prove that the proposed algorithm can achieve *at least* a constant fraction of the capacity region in the many-channel regime, and our numerical studies indicate that the proposed distributed collision-queue-regulated algorithm achieves a throughput very close to that achievable by a centralized throughput-optimal backpressure-based scheduling algorithm.

**Index Terms**—Asymptotic analysis, cognitive radio networks, distributed algorithm, OFDM.

## I. INTRODUCTION

COGNITIVE radio networks (CRNs) [1] allow unlicensed users, which are referred to as secondary users (SUs), to opportunistically exploit the unused spectrum allocated to licensed users, which are referred to as primary users (PUs). Different from traditional wireless networks, a typical design requirement for CRNs is that the collisions/interference between PU and SU transmission should be avoided or kept under a certain acceptable threshold.

Developing efficient scheduling algorithms for CRNs is essential to garner the full potential of CRNs. In recent years, centralized opportunistic scheduling algorithms have been developed for CRNs. Throughput-optimal cooperative scheduling has been studied in [2]–[4], where PUs are aware of SU activities and SUs cooperatively relay PU data. Non-cooperative scheduling has been studied in [5]–[7] to achieve optimal

SU throughput/utility. However, these algorithms, although throughput optimal, are centralized with high time complexity and, hence, are not suitable for practical implementations. In addition to computational complexity, these algorithms do not work when a global centralized component is not available (e.g., a scenario where SUs transmit peer to peer without centralized control and only local information is available). Therefore, low-complexity and distributed algorithms are needed to deploy efficient and high-performance CRNs. Few heuristic distributed solutions have been proposed in the literature (e.g., in [8]). In [8], a heuristic distributed algorithm is designed that aims to maximize the global scaling factor (with respect to the minimum rate requirement) by iteratively increasing the smallest scaling factor among all SU sessions in a CRN in each algorithm iteration. While it is shown through simulation results in [8] that the algorithm achieves a performance close to the optimal solution, the algorithm has no provable performance guarantees. Thus, the design of distributed algorithms for CRNs with provable properties (i.e., provable throughput/utility performance) remains an open research problem.

Distributed scheduling algorithms for traditional wireless networks have been proposed in the literature over the last decade. Earlier examples [9], [10] achieve at least certain fractions of the optimal throughput in single-channel wireless networks. More recently, throughput optimality has been achieved with distributed queue-length-based scheduling algorithms [11]–[15] for the same setting. Among the few attempts to design distributed scheduling algorithms for multichannel networks, the work in [16] guarantees at least a certain fraction (dependent on the network interference model) of the optimal throughput. However, these algorithms have been designed for wireless networks *with a nonfading channel capacity* and are thus not suitable for CRNs where channel states are modulated by PU activities.

In this paper, we propose a *distributed* scheduling algorithm, which is called *collision-queue-regulated algorithm*, for a fully connected CRN in the many-channel regime. We consider an orthogonal frequency-division multiplexing (OFDM) setting (which is the basis for IEEE 802.22 standard for CRNs), where spectrum is partitioned into tens or hundreds of orthogonal subchannels. With collision rate constraints imposed by PUs, the algorithm achieves *at least*  $1/e$  fraction of the capacity region as the number of channels grows, where  $e$  is the Euler number. We also show via simulation results that the throughput performance is close to the optimal.

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Salient contributions of this paper are summarized in the following.

- 1) Under the collision-queue-regulated algorithm, the collision rates observed by the PUs are upper bounded by an arbitrary threshold.
- 2) We design a novel equivalent queuing system such that the queues in the original CRN converge asymptotically (with respect to the number of channels) to this system.
- 3) By analyzing the equivalent queuing system, we show that the collision-queue-regulated algorithm achieves *at least* a  $1/e$  fraction of the capacity region in the many-channel regime.

In vehicular and transportation environments, CR-based communications can improve road traffic awareness/management, route planning, vehicular diagnostics, radio resource allocation, etc. [17]. It is further suggested in [17] that multicarrier OFDM-based modulation will be the most likely basic strategy for CR communications for vehicular technology. Thus, the OFDM-based distributed scheduling algorithm proposed in this paper also sheds light on optimal scheduling in CR-based vehicular networks.

A preliminary version of this paper [22] appeared in the *Proceedings of the 11th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks*. The major differences of this paper with respect to its conference version are as follows. First, the analysis is generalized from a single-PU case to a multiple-PU case, with each PU posting a maximum collision rate constraint. Accordingly, we show under the many-channel regime that the queue lengths in the considered CRN converge to an equivalent multiple-queue system. Second, a rigorous proof of Theorem 1 is provided, where we present the asymptotic behavior of the proposed algorithm in a multiple-PU scenario. Third, simulation results are provided to demonstrate the throughput performance of the proposed algorithm in a multiple-PU scenario.

The remainder of this paper is organized as follows. The network model and the distributed collision-queue-regulated algorithm are described in Section II. This is followed by an asymptotic analysis on the queuing behavior and the throughput performance in Section III. Numerical results are provided in Section IV, and this paper is concluded in Section V.

## II. NETWORK MODEL AND THE DISTRIBUTED ALGORITHM

### A. Network Elements

Consider a time-slotted CRN composed of a PU system and an SU system. We consider a fully connected SU system composed of a set  $\mathcal{L}$  of single-hop directional SU communication links, with  $|\mathcal{L}| = L$ , i.e., when the PU system is idle, the transmission of SU link  $i \in \mathcal{L}$  over a channel fails if and only if there is a simultaneous transmission of another link  $l \in \mathcal{L}$ ,  $l \neq i$ , over the same channel. Let  $A_i(t)$  be the amount of data (in unit of bits) arriving at SU link  $i \in \mathcal{L}$  at the beginning of time slot  $t$ , where  $t$  is the time slot variable taking integer values in  $\{0, 1, 2, \dots\}$ . For analytical simplicity, we assume the

arrivals are uniform and constant, i.e.,  $A_i(t) = \lambda$ ,  $\forall i \in \mathcal{L}$ ,  $\forall t$ , with arrival rate  $\lambda$ .

OFDM has proven to be one of the prime candidates for CRNs (e.g., IEEE 802.22 standard). The reason is twofold: 1) We have irregular openings in the PU spectrum, and OFDM helps with collectively utilizing noncontiguous PU channels in one SU transmission, and 2) intersymbol interference can be significantly reduced in an OFDM system by transmitting data in parallel over *a large number of low-rate subchannels* [18]. Thus, we consider an OFDM mechanism for SUs' channel access: An SU link can transmit its data opportunistically over multiple-PU channels in a time slot.

We consider a multiple-PU scenario. Specifically, the CRN is synchronized with a time-slotted PU system comprised of  $M$  PUs and  $N$  orthogonal PU subchannels, which we refer to as *channels* for short in the following. Each PU  $k \in \{1, 2, \dots, M\}$  is licensed to a set of  $\mathcal{I}_k$  with  $|\mathcal{I}_k| = n_k N$  channels, where  $n_k$  are constant fractional numbers satisfying  $\sum_{k=1}^M n_k = 1$ . We assume each channel has a uniform capacity (i.e., a maximum data rate in bits per time slot) equal to  $K/N$ , where we can consider  $K$  (bit per time slot) as the total capacity of the considered PU system. Note that the growing number of channels leads to diminishing bandwidth per channel, where the sum of all bands is constant  $K$ , and the sum of licensed bands for PU  $k$  is  $n_k K$ , which conforms to the setting of OFDM systems with a large number of low-rate channels [18].

We assume that PU activities are independent over PUs and that the activities of each PU  $k$  evolve according to an ON-OFF Markovian process  $C_k(t)$ : At time slot  $t$ , we let  $C_k(t) = 1$  if PU  $k$  is busy (PU  $k$  is in ON state and occupies the entire set of channels  $\mathcal{I}_k$ ) or let  $C_k(t) = 0$  if PU  $k$  is idle (PU  $k$  is in OFF state and the entire set of channels  $\mathcal{I}_k$  are available to SUs). For analytical simplicity, we assume the process  $C_k(t)$  starts with a steady-state distribution at  $t = 0$ ,  $\forall k$ . Similar to related CRN scheduling works [5]–[7], we assume that the exact knowledge of  $(C_k(t))$  may not be available to SUs due to time-varying PU activities or sensing overheads. We also note that, given  $C_k(t - 1)$ , the current channel state distribution is independent of past channel state information before  $(t - 2)$ . Thus, we denote by  $\mathcal{H}(t) = (C_k(t - 1))_{k=1}^M$  the channel availability information of SUs at time slot  $t$ . In addition

$$S_k(t) \triangleq \mathbb{E} \{1 - C_k(t) | C_k(t - 1)\} = \mathbb{E} \{1 - C_k(t) | \mathcal{H}(t)\}$$

defines the probability that PU  $k$  is OFF given  $\mathcal{H}(t)$ , which is known to the SUs at time slot  $t$ . We note that  $S_k(t)$  is simply the transition probability of  $C_k(t)$  and can be obtained by SUs via the knowledge or observation of PU data traffic statistics.

Let  $\mu_{ij}(t) \in \{0, 1\}$  denote the schedule of SU link  $i \in \mathcal{L}$  over channel  $j$  at time slot  $t$ , with  $j = 1, \dots, N$ . Specifically,  $\mu_{ij}(t) = 1$  if SU link  $i$  is scheduled over channel  $j$ ;  $\mu_{ij}(t) = 0$ , otherwise. For analytical simplicity, we let  $\mu_{ij}(0) = 0$ ,  $\forall i, j$ . Note that, when  $\mu_{ij}(t) = 1$ , SU link  $i$  is scheduled to transmit up to  $K/N$  bits over channel  $j$  in one time slot. We say a collision with PU  $k$  occurs if  $\mu_{ij}(t)C_k(t) = 1$ ,  $j \in \mathcal{I}_k$ , i.e., there is a scheduled SU data transmission when PU  $k$  is busy. We denote by  $q_i(t)$  the SU data queue backlog for link  $i$  at time slot  $t$ , i.e., the amount of data in bits in queue for SU link  $i$  at

time  $t$ . Thus, for each SU data queue  $q_i(t)$ ,  $i \in \mathcal{L}$ , we have the following queue dynamics:

$$q_i(t) = \left[ q_i(t-1) - \frac{K}{N} \sum_{k=1}^M \sum_{j \in \mathcal{I}_k} (\mu_{ij}(t-1) (1 - C_k(t-1))) + A_i(t-1) \right]^+ \quad (1)$$

with  $q_i(0) = 0$ ,  $\forall i \in \mathcal{L}$ , and  $[\cdot]^+ \triangleq \max\{\cdot, 0\}$ .

To constrain the potential interference caused by the SUs to the PU system, we require that the collision rate (caused by any SU link  $i$ ) observed by any given PU  $k$  be upper bounded by a maximum collision rate  $\rho_k$  (normalized by the number of channels licensed to PU  $k$ )

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \frac{1}{n_k N} \sum_{j \in \mathcal{I}_k} \mu_{ij}(t) C_k(t) \leq \rho_k \quad \forall i \in \mathcal{L}. \quad (2)$$

As suggested in [19], for the considered OFDM-based CRN, we assume the existence of an out-of-band common control channel (CCC), which is not interrupted by PU activities. In the collision-queue-regulated algorithm proposed in Section II-B, the exchange of local control information is performed over the CCC at the beginning of each time slot. Since the CCC is dedicated only to the transmission and reception of control messages, the CCC can utilize the small portions of the guard bands between the licensed channels [19].

### B. Collision-Queue-Regulated Algorithm

Here, we propose a distributed collision-queue-regulated algorithm. We will show in Section III that the proposed algorithm can achieve at least  $1/e$  fraction of the capacity region asymptotically with respect to  $N$ .

We maintain a virtual collision queue  $X_{ik}(t)$  at each SU link  $i \in \mathcal{L}$  corresponding to any given PU  $k$ , to assist the development of the proposed algorithm. Specifically, the queue dynamics of  $X_{ik}(t)$  is defined as  $\forall i \in \mathcal{L}, \forall k$

$$X_{ik}(t) = \left[ X_{ik}(t-1) - \rho_k + \frac{1}{n_k N} \sum_{j \in \mathcal{I}_k} \mu_{ij}(t-1) C_k(t-1) \right]^+ \quad (3)$$

with  $X_{ik}(0) = 0$ . In (3), we construct the virtual collision queue  $X_{ik}(t)$  with the collision rate threshold  $\rho_k$  being its service rate and the collisions in the previous time slot (normalized by the number of channels licensed to PU  $k$ ) being its arrival rate at each time slot  $t$ . Thus, if an algorithm ensures the stability of the collision queues, by queuing theory, the collision rate constraint (2) is satisfied. We will show via Theorem 2 and Corollary 1 in Section III-B that the collision queues are asymptotically stable under the many-channel regime for at least a constant fraction of the capacity region.

At the beginning of each time slot  $t$ , the collision-queue-regulated algorithm consists of two phases: *Exchange Phase* and *Scheduling Phase*, the duration of which we assume is

negligible compared with that of a unit time slot. The exchange phase is detailed as follows:

#### Exchange Phase:

The exchange phase takes place over the CCC. Specifically, the transmitter of each SU link  $i \in \mathcal{L}$  broadcasts the following three binary vectors to all its neighbors its (intended receiver and all nodes in  $\mathcal{L}$ ) over the CCC: its schedules at the previous time slot  $(\mu_{ij}(t-1))_{j=1}^N$ , a vector of contention variables  $(a_{ij}(t))_{j=1}^N$ , and a vector of transmission variables  $(p_{ij}(t))_{j=1}^N$ .

The *contention variables*  $(a_{ij}(t))$  are i.i.d. over SUs  $i$  and channels  $j$  with

$$a_{ij}(t) = \begin{cases} 1, & \text{w.p. } \frac{1}{L} \\ 0, & \text{w.p. } \frac{L-1}{L}. \end{cases}$$

The *transmission variables*  $(p_{ij}(t))$  are independent over channels  $j$  and SUs  $i$  with

$$p_{ij}(t) = \begin{cases} 1, & \text{w.p. } \frac{e^{y_{ik}(t)}}{e^{y_{ik}(t)+1}} \\ 0, & \text{w.p. } \frac{1}{e^{y_{ik}(t)+1}} \end{cases}$$

where  $j \in \mathcal{I}_k$ . The *collision-queue-regulated weight*  $y_{ik}(t)$  is defined as

$$y_{ik}(t) \triangleq q_i(t-1) S_k(t) - \gamma X_{ik}(t-1) (1 - S_k(t)) \quad (4)$$

and  $\gamma > 0$  is a constant parameter that serves as a weight to the collision queue  $X_{ik}(t-1)$ .

The contention variables and the transmission variables will be used in the scheduling phase to determine the schedules of SU links. Specifically, contention variables  $(a_{ij}(t))_{i \in \mathcal{L}}$  are used to simulate a contention of SUs over channel  $j$  at time slot  $t$ . Given  $p_{ij}(t) = 1$ , SU  $i$  that succeeds in the contention is scheduled to transmit over channel  $j$  if other SUs did not transmit over channel  $j$  in the previous time slot; if SU  $i$  fails in the contention, its schedule remains the same as its previous time slot.

As detailed in the scheduling phase, schedules are in part determined by transmission variables  $p_{ij}(t)$ :  $\mu_{ij}(t) = 0$  (i.e., the transmission of SU link  $i$  over channel  $j$  is not scheduled) if  $p_{ij}(t) = 0$ ;  $\mu_{ij}(t) = 1$  if  $p_{ij}(t) = 1$  and some other conditions are satisfied. According to the definition of  $p_{ij}(t)$ , its probability mass function is determined by  $y_{ik}(t)$  defined in (4). Specifically, the larger  $y_{ik}(t)$  is, the more likely  $p_{ij}(t) = 1$ ; hence, the more likely the transmission of SU link  $i$  over channel  $j$ . Thus, we can consider the two components  $(q_i(t-1) S_k(t) - \gamma X_{ik}(t-1) (1 - S_k(t)))$  of  $y_{ik}(t)$  in (4) as the reward and the cost, respectively, of scheduling SU link  $i$  over channel  $j$  (belonging to PU  $k$ ). Specifically, the larger the data queue backlog  $q_i(t-1)$  (representing the congestion level of SU  $i$ ), the more likely the transmission of SU link  $i$  over channel  $j$ . On the other hand, the larger the collision queue backlog  $X_{ik}(t-1)$  (representing the collision level of PU), the less likely the transmission of SU link  $i$  over channel  $j$ .

After the exchange phase, the transmitter and the receiver of each SU link  $i$  have the following information:  $((\mu_{lj}(t-1))_{j=1,\dots,N}^{l \in \mathcal{L}}, (a_{ij}(t))_{j=1,\dots,N}^{i \in \mathcal{L}}, (p_{ij}(t))_{j=1}^N)$ , which will be used to determine the transmission schedules for SU link  $i$ .

To assist the development of scheduling phase, we define the following three conditions, for any given SU link  $i$  and channel  $j$ .

*Condition (i):* The ‘‘contention’’ of SU link  $i$  for channel  $j$  is successful, i.e.,  $a_{ij}(t)\prod_{l \in \mathcal{L} \setminus \{i\}}(1 - a_{lj}(t)) = 1$ .

*Condition (ii):*  $\sum_{l \in \mathcal{L} \setminus \{i\}} \mu_{lj}(t-1) = 0$ , i.e., none of the neighbors were scheduled at the previous time slot.

*Condition (iii):* The transmission variable  $p_{ij}(t) = 1$ .

The scheduling phase is introduced as follows:

#### Scheduling Phase:

The transmitter and the receiver of each SU link  $i$  determine the schedules  $\mu_{ij}(t)$ ,  $j = 1, \dots, N$ , according to the following:

*Case 1:*  $\mu_{ij}(t) = 1$  if Conditions (i)(ii)(iii) hold.

*Case 2:* If Condition (i) does not hold and Condition (iii) holds, then  $\mu_{ij}(t) = \mu_{ij}(t-1)$ .

*Case 3:* Otherwise,  $\mu_{ij}(t) = 0$ .

According to the scheduling phase, we conclude that,  $\forall i \in \mathcal{L}$ ,  $\forall j \in \{1, 2, \dots, N\}$ , we have

$$\begin{aligned} & \mu_{ij}(t) \\ &= p_{ij}(t) \left\{ a_{ij}(t)\prod_{l \in \mathcal{L} \setminus \{i\}}(1 - a_{lj}(t)) \left( 1 - \sum_{l \in \mathcal{L} \setminus \{i\}} \mu_{lj}(t-1) \right) \right. \\ & \quad \left. + [1 - a_{ij}(t)\prod_{l \in \mathcal{L} \setminus \{i\}}(1 - a_{lj}(t))] \mu_{ij}(t-1) \right\} \quad (5) \end{aligned}$$

where the first and second terms in the  $\{\cdot\}$  in (5) correspond to Cases 1 and 2 in the scheduling phase, respectively.

Since both the transmitter and the receiver of SU link  $i \in \mathcal{L}$  have a copy of the schedule vector  $(\mu_{ij}(t))_{j=1}^N$  when the scheduling phase ends, they will tune to the set of channels  $\{j : \mu_{ij}(t) = 1\}$  for SU data transmission in the remaining time slot  $t$ .

We show in Proposition 1 that the collision-queue-regulated algorithm is feasible in that interfering links are never scheduled over a same channel in any time slot.

*Proposition 1:* The collision-queue-regulated algorithm provides a feasible schedule for each time slot  $t$ , i.e.,  $\forall i, j, t$ ,  $\sum_{l \in \mathcal{L} \setminus \{i\}} \mu_{lj}(t) = 0$ , if  $\mu_{ij}(t) = 1$ .

*Proof:* We prove Proposition 1 by mathematical induction over time slot  $t$ . We note that  $\mu_{ij}(0) = 0$ , for all  $i \in \mathcal{L}$  and  $j = 1, 2, \dots, N$  establishing the base case. Suppose the induction hypothesis holds for  $t-1$ , i.e.,  $\forall i, j$ ,  $\sum_{l \in \mathcal{L} \setminus \{i\}} \mu_{lj}(t-1) = 0$ , if  $\mu_{ij}(t-1) = 1$ . For any given  $i \in \mathcal{L}$  and channel  $j$  such that  $\mu_{ij}(t) = 1$ , we have the following two cases according to the scheduling phase.

- Case 1: Conditions (i)–(iii) hold. Since Condition (ii) holds,  $\sum_{l \in \mathcal{L} \setminus \{i\}} \mu_{lj}(t-1) = 0$ . Since Condition (i) holds (i.e., SU link  $i$  succeeds in the contention for channel  $j$ ),

for any other SU link  $l$ , i.e.,  $l \in \mathcal{L} \setminus \{i\}$ , according to the scheduling dynamics (5) for  $\mu_{lj}(t)$

$$\begin{aligned} \mu_{lj}(t) &\leq [1 - a_{lj}(t)\prod_{l' \in \mathcal{L} \setminus \{l\}}(1 - a_{l'j}(t))] \mu_{lj}(t-1) \\ &= \mu_{lj}(t-1). \end{aligned}$$

Thus, we have  $\sum_{l \in \mathcal{L} \setminus \{i\}} \mu_{lj}(t) \leq \sum_{l \in \mathcal{L} \setminus \{i\}} \mu_{lj}(t-1) = 0$ .  
• Case 2: Condition (i) does not hold, Condition (iii) holds, and  $\mu_{ij}(t-1) = \mu_{ij}(t) = 1$ . Consequently,  $\sum_{l \in \mathcal{L} \setminus \{i\}} \mu_{lj}(t-1) = 0$ . For any other SU link  $l$ , i.e.,  $l \in \mathcal{L} \setminus \{i\}$ , we have according to (5)

$$\begin{aligned} \mu_{lj}(t) &\leq a_{lj}(t)\prod_{l' \in \mathcal{L} \setminus \{l\}}(1 - a_{l'j}(t)) \\ &\quad \times \left( 1 - \sum_{l' \in \mathcal{L} \setminus \{l\}} \mu_{l'j}(t-1) \right) \\ &\quad + [1 - a_{lj}(t)\prod_{l' \in \mathcal{L} \setminus \{l\}}(1 - a_{l'j}(t))] \mu_{lj}(t-1) \\ &\leq \max \left\{ 1 - \sum_{l' \in \mathcal{L} \setminus \{l\}} \mu_{l'j}(t-1), \mu_{lj}(t-1) \right\} = 0. \end{aligned}$$

Thus, we have  $\sum_{l \in \mathcal{L} \setminus \{i\}} \mu_{lj}(t) = 0$ .

Since the given results hold for any given SU link  $i$  and channel  $j$  with  $\mu_{ij}(t) = 1$ , we have proved that the induction step holds, completing the proof. ■

### III. PERFORMANCE ANALYSIS IN A MANY-CHANNEL REGIME

In Section III-A, we show that under the collision-queue-regulated algorithm, the original system of the queue lengths  $(q_i(t))$  and the collision queues  $(X_{ik}(t))$  converge to an equivalent queue system as the number of channels  $N$  grows. Based on the analysis of the equivalent queue system, we show in Section III-B that the algorithm achieves at least  $1/e$  fraction of the capacity region asymptotically with respect to  $N$ .

#### A. Asymptotic Queuing Behavior of the Collision-Queue-Regulated Algorithm

We present the asymptotic queuing behavior of the collision-queue-regulated algorithm in Theorem 1.

*Theorem 1:* Given  $\mathcal{H}(t) \triangleq (\mathcal{H}(t), \mathcal{H}(t-1), \dots, \mathcal{H}(1))$ , there exists an equivalent queuing system  $(q(t), (x_i(t))_{i=1}^M)$  with equivalent schedule variables  $(u_k(t))_{k=1}^M$ , such that the following four arguments (I(t), II(t), III(t), and IV(t)) hold under the collision-queue-regulated algorithm for each time slot  $t$ .

I(t): The queue lengths  $q_i(t)$  and the collision queue lengths  $X_{ik}(t)$  converge to  $q(t)$  and  $x_{ik}(t)$ , respectively, as follows:

$$q_i(t) \xrightarrow{P} q(t) \quad X_{ik}(t) \xrightarrow{P} x_{ik}(t), \quad \forall i \in \mathcal{L}; \quad \forall k \quad (6)$$

where  $\xrightarrow{P}$  denotes the convergence in probability [20] as  $N \rightarrow \infty$ .

II(t): The schedules  $\mu_{ij}(t)$  converge to the equivalent schedule variable  $u_k(t)$  with  $j \in \mathcal{I}_k$ , i.e.,

$$\mu_{ij}(t) \xrightarrow{L}_N u_k(t) \quad \forall i; \quad \forall k; \quad \forall j \in \mathcal{I}_k \quad (7)$$

where  $\xrightarrow{L}_N$  denotes the convergence in distribution [20] as  $N \rightarrow \infty$ .

III(t): The schedules  $\mu_{ij}(t)$  follow a law of large numbers (LLN), i.e.,

$$\frac{1}{n_k N} \sum_{j \in \mathcal{I}_k} \mu_{ij}(t) \xrightarrow{P}_N \mathbb{E}\{u_k(t) | \mathcal{H}'(t)\} \quad \forall i \in \mathcal{L}; \quad \forall k. \quad (8)$$

IV(t): The schedules  $\mu_{ij}(t)$  are asymptotically mutually independent. Specifically, for any given SU links  $i_1, i_2 \in \mathcal{L}$ , and any two distinct channels  $j_1 \neq j_2 \in \{1, 2, \dots, N\}$ , the scheduling decisions are independent, i.e.,  $\forall w_1, w_2 \in \{0, 1\}$  we have

$$\begin{aligned} \lim_{N \rightarrow \infty} \Pr \{ \mu_{i_1 j_1}(t) = w_1, \mu_{i_2 j_2}(t) = w_2 | \mathcal{H}'(t) \} \\ = \Pr \{ u_{k_1}(t) = w_1 | \mathcal{H}'(t) \} \Pr \{ u_{k_2}(t) = w_2 | \mathcal{H}'(t) \} \end{aligned} \quad (9)$$

where  $k_1$  and  $k_2$  satisfy  $j_1 \in \mathcal{I}_{k_1}$  and  $j_2 \in \mathcal{I}_{k_2}$ .

The equivalent queuing system  $(q(t), (x_k(t))_{k=1}^M)$  and the equivalent schedule variables  $(u_k(t))_{k=1}^M$  evolve as follows:

$$q(t) = \left[ q(t-1) - \sum_{k=1}^M (n_k K (1 - C_k(t-1))) \times \mathbb{E}\{u_k(t-1) | \mathcal{H}'(t-1)\} + \lambda \right]^+ \quad (10)$$

$$x_k(t) = [x_k(t-1) - \rho_k + C_k(t-1) \times \mathbb{E}\{u_k(t-1) | \mathcal{H}'(t-1)\}]^+ \quad \forall k \quad (11)$$

$$u_k(t) = U_k(t)u_k(t-1) + V_k(t)(1 - u_k(t-1)) \quad (12)$$

where  $U_k(t)$  and  $V_k(t)$  are independent over time and PUs and defined as follows:

$$U_k(t) = \begin{cases} 1, & \text{w.p. } (1 - (L-1)\beta) \frac{e^{y_k(t)}}{e^{y_k(t)+1}} \\ 0, & \text{otherwise} \end{cases}$$

$$V_k(t) = \begin{cases} 1, & \text{w.p. } \beta \frac{e^{y_k(t)}}{e^{y_k(t)+1}} \\ 0, & \text{otherwise} \end{cases}$$

with

$$\beta \triangleq \frac{(L-1)^{L-1}}{L^L}$$

$$y_k(t) \triangleq q(t-1)S_k(t) - \gamma x_k(t-1)(1 - S_k(t)).$$

The initial conditions of the equivalent queue system are set as

$$q(0) = 0, \quad x_k(0) = 0, \quad \text{and} \quad u_k(0) = 0, \quad \forall k. \quad (13)$$

*Proof:* The proof for Theorem 1 is provided in Appendix A. ■

*Remark 1:* According to (6) in Theorem 1, given  $\mathcal{H}'(t)$ , the data queues  $q_i(t)$  and the collision queues  $X_{ik}(t)$  converge (in probability) asymptotically to *deterministic* equivalent queues

$q(t)$  and  $x_k(t)$ , respectively. By the dynamics of  $u_k(t)$  in (12), we find the dynamics of  $\mathbb{E}\{u_k(t) | \mathcal{H}'(t)\}$  as follows:

$$\begin{aligned} \mathbb{E}\{u_k(t) | \mathcal{H}'(t)\} &= \beta \frac{e^{y_k(t)}}{e^{y_k(t)+1}} \\ &+ (1 - L\beta) \frac{e^{y_k(t)}}{e^{y_k(t)+1}} \mathbb{E}\{u_k(t-1) | \mathcal{H}'(t-1)\} \end{aligned} \quad (14)$$

where we note that  $u_k(t-1)$  is independent of  $\mathcal{H}(t)$  given  $\mathcal{H}'(t-1)$ .

In Section III-B, we will study the stability of the equivalent queuing system  $(q(t), (x_k(t))_{k=1}^M)$ , which becomes the asymptotic network stability (i.e., the stability for the data queues  $q_i(t)$  and the collision queues  $X_{ik}(t)$ ) under the collision-queue-regulated algorithm.

### B. Performance Analysis

We have shown through Theorem 1 that the equivalent system  $(q(t), (x_k(t))_{k=1}^M)$  can represent the asymptotic queuing behavior of the data queues  $q_i(t)$  and the collision queues  $X_{ik}(t)$ . Here, we will show that, under the collision-queue-regulated algorithm,  $(q(t), (x_k(t))_{k=1}^M)$  are stable for *at least*  $1/e$  fraction of the capacity region.

Specifically, we define the capacity region  $\Lambda$  as

$$\Lambda = \{ \lambda \geq 0 : \text{the arrivals } A_i(t) = \lambda, \quad \forall i \in \mathcal{L} \text{ are stabilizable by some scheduling algorithm} \}.$$

For any given  $0 < \alpha < 1$ , we let  $\alpha\Lambda$  denote an  $\alpha$  fraction of the capacity region such that

$$\alpha\Lambda \triangleq \left\{ \lambda \geq 0 : \exists \lambda' \in \Lambda \text{ s.t. } \frac{\lambda}{\alpha} < \lambda' \right\}.$$

Before we present the asymptotic stability in Theorem 2, we introduce the following two lemmas to assist the proof of Theorem 2.

*Lemma 1:* For any given  $0 < \delta < L\beta$ , there exists  $B_2(\delta) < 0$  such that, for any time slot  $t$ , whenever  $y_k(t) \geq B_2$  for any given PU  $k$ , we have

$$\Pr \{ u_k(t) = 1 \} \geq \beta - \frac{\delta}{L}.$$

*Proof:* Let  $B_2 \triangleq \log((L\beta/\delta) - 1)$ . By taking the expectation of both sides of (14) over  $\mathcal{H}'(t)$  conditioned on  $y_k(t) \geq B_2$ , we have

$$\begin{aligned} &\mathbb{E}\{u_k(t) | y_k(t) \geq B_2\} \\ &= \beta \mathbb{E}\left\{ \frac{e^{y_k(t)}}{e^{y_k(t)+1}} | y_k(t) \geq B_2 \right\} \\ &+ (1 - \beta L) \mathbb{E}\left\{ \frac{e^{y_k(t)}}{e^{y_k(t)+1}} u_k(t-1) | y_k(t) \geq B_2 \right\} \\ &\geq \beta \frac{e^{B_2}}{e^{B_2+1}} = \beta - \frac{\delta}{L}. \end{aligned} \quad \blacksquare$$

We show that, for any  $\lambda' \in \Lambda$ , there exist (auxiliary) random variables  $(\mu_k^{\text{STAT}}(t))_{k=1}^M$  for each time slot  $t$  satisfying the properties described in Lemma 2.

*Lemma 2:* For any  $\lambda' \in \Lambda$ , there exist random variables  $(\mu_k^{\text{STAT}}(t))$  with  $\mu_k^{\text{STAT}}(t) \in \{0, 1\} \forall k$ , which are dependent only on  $\mathcal{H}(t)$  for each time slot  $t$ , such that the following holds:

$$\sum_{k=1}^M \mathbb{E} \{n_k K \mu_k^{\text{STAT}}(t) S_k(t)\} = \lambda' \quad (15)$$

$$\mathbb{E} \{ \mu_k^{\text{STAT}}(t) (1 - S_k(t)) \} \leq \rho_k \quad (16)$$

$$\mathbb{E} \{ \mu_k^{\text{STAT}}(t) | \mathcal{H}(t) \} \leq \frac{1}{L}, \quad \forall \mathcal{H}(t). \quad (17)$$

*Proof:* Proof of Lemma 2 is provided in Appendix B. ■

Utilizing Lemmas 1 and 2, we show in Theorem 2 that the equivalent system is stable for at least a constant fraction of the capacity region  $\Lambda$  under the collision-queue-regulated algorithm.

*Theorem 2:*  $\forall \lambda \in \alpha \Lambda$ , with  $\alpha = ((L - 1)/L)^{L-1}$ ,  $q(t)$  and  $x(t)$  are stable under the collision-queue-regulated algorithm, i.e.,

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} \left\{ q(t) + \sum_{k=1}^M x_k(t) \right\} \leq \frac{B_3}{\epsilon_2} \quad (18)$$

where positive constants  $B_3$  and  $\epsilon_2$  will be defined in the proof.

*Proof:* The proof of Theorem 2 is provided in Appendix C, where we have employed Lemmas 1 and 2. ■

The following corollary directly follows Theorem 2, where we note that  $((L - 1)/L)^{L-1} > 1/e, \forall L > 0$ .

*Corollary 1:*  $\forall \lambda \in (1/e)\Lambda$ ,  $q(t)$  and  $(x_k(t))$  are stable under the collision-queue-regulated algorithm.

Since the queue lengths  $q_i(t)$  and the collision queues  $X_{ik}(t)$  converge to  $q(t)$  and  $(x_k(t))$ , respectively, i.e., asymptotically with respect to  $N$ , the collision-queue-regulated algorithm achieves at least  $\alpha = 1/e$  fraction of the capacity region in the many-channel regime by Theorem 2.

We refer to this fraction  $\alpha$  as the *efficiency factor* of the capacity region. For comparison, the efficiency factor of the purely localized distributed scheme (PLDS) algorithm [16] proposed for a general multiradio multinonfading-channel wireless network is  $1/3e$ .<sup>1</sup> Note that, although both algorithms are proposed for a multichannel scenario, the setting for the collision-queue-regulated algorithm is more stringent than that for PLDS, in that nonfading channels are assumed in [16], whereas we consider channels modulated by PU activities in this paper. However, the provable efficiency factor of the collision-queue-regulated algorithm is larger than that of PLDS.

#### IV. NUMERICAL RESULTS

Here, via simulation, we compare the throughput performance of the proposed algorithm with a back-pressure-based

<sup>1</sup>In [16], the key to the PLDS is an *access hash function*, i.e., a binary function parameterized by an SU pair, channel, and time slot, which probabilistically determines the schedules of SUs via random access. In a fully connected wireless network, the efficiency factor of PLDS is  $1/3e$ .

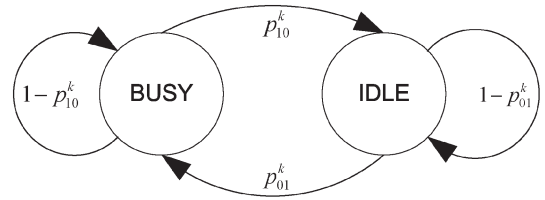


Fig. 1. Transition diagram of PU activity.

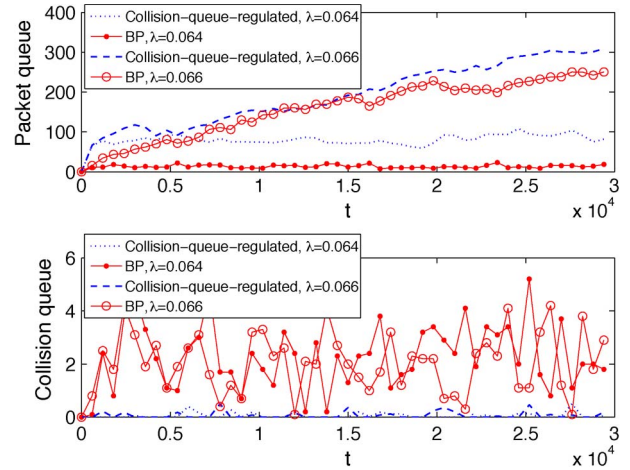


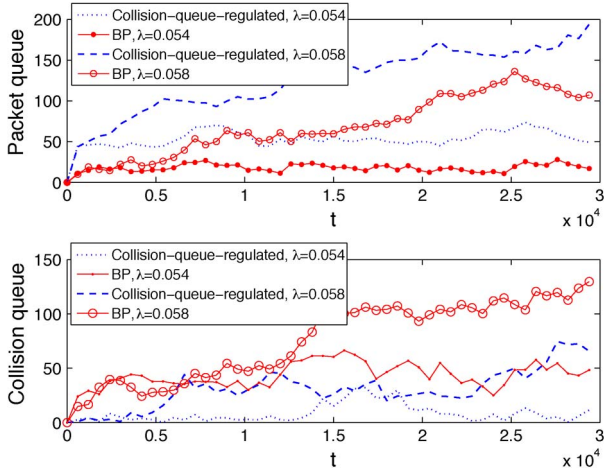
Fig. 2. Queue dynamics  $\rho = 0.1$ .

*centralized throughput-optimal* algorithm, which is denoted as the BP algorithm. The BP algorithm is based on the throughput-optimal back-pressure algorithm [21],<sup>2</sup> where we substitute the (generic) weight in [21] with the collision-queue-regulated weight  $y_{ik}(t)$  defined in (4) for each  $i \in \mathcal{L}$  and  $k = 1, 2, \dots, M$ . It can be shown, similar to the analysis [3], [4] that the BP algorithm is optimal given the collision rate constraints.

We consider a fully connected CRN with ten SU communication links. We set the parameter in (4) as  $\gamma = 1$  and consider two PUs with  $N = 100$  and  $n_1 = n_2 = 0.5$ . The channel states of PUs evolve according to the transition diagram in Fig. 1, where the “busy” and the “idle” states are represented as  $C_k(t) = 1$  and  $C_k(t) = 0$ , respectively. Note that, in Fig. 1,  $p_{01}^k$  and  $p_{10}^k$  represent channel  $k$ ’s transition probability from the idle state to the busy state and that from the busy state to the idle state, respectively, with  $k = 1, 2$ . In the numerical evaluation, we let  $p_{01}^1 = 0.3, p_{10}^1 = 0.7, p_{01}^2 = 0.4,$  and  $p_{10}^2 = 0.6$ .

We illustrate the stability of queues through Fig. 2 under the collision rate constraint  $\rho = 0.1$ , where queue backlogs are summed over all links/channels. In our numerical studies, we have observed that both algorithms stabilize data and collisions queues for  $\lambda = 0.064$ . Again, both algorithms fail to stabilize the system for  $\lambda = 0.066$ , where the data queues keep growing, indicating network instability. Since the BP algorithm is throughput optimal, we can expect that the maximum stabilizable

<sup>2</sup>The back-pressure algorithm in [21] achieves throughput optimality in a general wireless network by solving a maximal weight matching each time slot, where the weight is equal to the data queue backlog. However, the back-pressure algorithm needs centralized control and its complexity increases with the network size.

Fig. 3. Queue dynamics  $\rho = 0.03$ .

$\lambda$  is in between 0.064 and 0.066. Hence, the collision-queue-regulated algorithm achieves at least  $0.064/0.066 = 97\%$  of the throughput optimality under this simulation setting. Note that 0.97 is significantly higher than the efficiency factor  $\alpha = 1/e = 0.37$  in Theorem 2.

We now post a more stringent collision rate constraint  $\rho = 0.03$  in Fig. 3. Under both algorithms, at  $\lambda = 0.054$ , the data queues and collision queues are stable; at  $\lambda = 0.058$ , the data queues and collision queues are both increasing over the time slots  $t$ , indicating network instability and collision rate violation. That is, both algorithms can stabilize  $\lambda = 0.054$  but cannot stabilize  $\lambda = 0.058$ . We can expect that the collision-queue-regulated algorithm achieves at least  $0.054/0.058 = 93\%$  of the throughput optimality under this simulation setting.

## V. CONCLUSION AND FUTURE WORKS

In this paper, we have proposed a distributed collision-queue-regulated scheduling algorithm for CRNs. We have proven theoretically that the proposed algorithm can achieve at least  $1/e$  of the capacity region asymptotically in the many-channel regime via a novel equivalent queue system analysis. We have also illustrated through numerical evaluation that the throughput performance of the proposed algorithm is close to optimal. We have assumed a fully connected network in this paper. The proposed algorithm can be readily extended to general interference-graph-based network topologies (e.g., see [11]–[15]). Thus, our future work involves performance analysis for a more general scenario: a general interference graph model with heterogeneous arrival processes.

### APPENDIX A PROOF OF THEOREM 1

We prove Theorem 1 by mathematical induction over time slot  $t$ . Given the initial conditions (13), the base case holds for time slot  $t = 0$ . Suppose the induction hypothesis (I( $t-1$ ), II( $t-1$ ), III( $t-1$ ), and IV( $t-1$ )) holds, we prove I( $t$ ), II( $t$ ), III( $t$ ), and IV( $t$ ) hold in Appendices A-1 to A-4, respectively.

#### 1. Proof of I( $t$ )

Given any SU link  $i \in \mathcal{L}$ , I( $t-1$ ), and III( $t-1$ ), we have

$$\begin{aligned} q_i(t-1) &= \frac{K}{N} \sum_{k=1}^M \sum_{j \in \mathcal{I}_k} ((1 - C_k(t-1)) \mu_{ij}(t-1)) \\ &+ A_i(t-1) \xrightarrow{P} q(t-1) \\ &- \sum_{k=1}^M (n_k K (1 - C_k(t-1)) \mathbb{E} \{u_k(t-1) | \mathcal{H}'(t-1)\}) + \lambda. \end{aligned}$$

By queue dynamics (1), (10), and the continuity of  $[\cdot]^+$ , we conclude that  $q_i(t) \xrightarrow{P} q(t)$ ,  $\forall i \in \mathcal{L}$ .

Similarly, we have

$$X_{ik}(t-1) - \rho_k + \frac{1}{n_k N} \sum_{j \in \mathcal{I}_k} \mu_{ij}(t-1) C_k(t-1)$$

$$\xrightarrow{P} x_k(t-1) - \rho_k + C_k(t-1) \mathbb{E} \{u_k(t-1) | \mathcal{H}'(t-1)\}.$$

By queue dynamics (3), (11), and the continuity of  $[\cdot]^+$ , we conclude that  $X_{ik}(t) \xrightarrow{P} x_k(t)$ , which completes the proof of I( $t$ ).

#### 2. Proof of II( $t$ )

Given any  $i \in \mathcal{L}$ , PU  $k$ , and  $j \in \mathcal{I}_k$ , by taking conditional expectation on both sides of (5), we obtain the following:

$$\begin{aligned} &\mathbb{E} \{ \mu_{ij}(t) | q_i(t-1), X_{ik}(t-1), (\mu_{lj}(t-1))_{l \in \mathcal{L}}, \mathcal{H}'(t) \} \\ &= \beta \frac{e^{y_{ik}(t)}}{e^{y_{ik}(t)} + 1} + \frac{e^{y_{ik}(t)}}{e^{y_{ik}(t)} + 1} \\ &\times \left[ (1 - \beta) \mu_{ij}(t-1) - \beta \sum_{l \neq i} \mu_{lj}(t-1) \right] \end{aligned}$$

where we recall that  $y_{ik}(t)$  is defined in (4), and  $S_k(t)$  in  $y_{ik}(t)$  is defined as  $S_k(t) \triangleq \mathbb{E} \{1 - C_k(t) | \mathcal{H}(t)\}$ . By taking the expectation over  $(q_i(t-1), X_{ik}(t-1), (\mu_{lj}(t-1))_{l \in \mathcal{L}})$  on both sides of the earlier equation, we conclude that

$$\begin{aligned} \mathbb{E} \{ \mu_{ij}(t) | \mathcal{H}'(t) \} &= \mathbb{E} \left\{ \beta \frac{e^{y_{ik}(t)}}{e^{y_{ik}(t)} + 1} + \frac{e^{y_{ik}(t)}}{e^{y_{ik}(t)} + 1} \right. \\ &\times \left. \left[ (1 - \beta) \mu_{ij}(t-1) - \beta \sum_{l \neq i} \mu_{lj}(t-1) \right] | \mathcal{H}'(t-1) \right\} \quad (19) \end{aligned}$$

where  $S_k(t)$  in  $y_{ik}(t)$  is deterministic, and we note that  $q_i(t-1)$ ,  $X_{ik}(t-1)$ , and  $(\mu_{lj}(t-1))_{l \in \mathcal{L}}$  are independent of  $\mathcal{H}(t)$  given  $\mathcal{H}'(t-1)$ .

From I( $t-1$ ) and III( $t-1$ ), we have the following convergence result given  $\mathcal{H}'(t-1)$ :

$$(q_i(t-1), X_{ik}(t-1), \mu_{lj}(t-1))$$

$$\xrightarrow{L} (q(t-1), x_k(t-1), u_k(t-1)) \quad \forall l \in \mathcal{L}$$

to which, by applying the continuous mapping theorem and the bounded convergence theorem [20], we obtain

$$\lim_{N \rightarrow \infty} \mathbb{E} \left\{ \frac{e^{y_{ik}(t)}}{e^{y_{ik}(t)} + 1} |\mathcal{H}'(t-1) \right\} = \frac{e^{y_k(t)}}{e^{y_k(t)} + 1} \quad (20)$$

$$\begin{aligned} & \lim_{N \rightarrow \infty} \mathbb{E} \left\{ \frac{e^{y_{ik}(t)}}{e^{y_{ik}(t)} + 1} \mu_{lj}(t-1) |\mathcal{H}'(t-1) \right\} \\ &= \frac{e^{y_k(t)}}{e^{y_k(t)} + 1} \mathbb{E} \{u_k(t-1) |\mathcal{H}'(t-1)\}, \quad \forall l \in \mathcal{N}. \quad (21) \end{aligned}$$

By taking the limit of  $N$  over both sides of (19) and applying (20) and (21), we conclude that

$$\begin{aligned} & \lim_{N \rightarrow \infty} \Pr \{ \mu_{ij}(t) = 1 | \mathcal{H}'(t) \} \\ &= \lim_{N \rightarrow \infty} \mathbb{E} \{ \mu_{ij}(t) | \mathcal{H}'(t) \} \\ &= \beta \frac{e^{y_k(t)}}{e^{y_k(t)} + 1} + (1-L)\beta \frac{e^{y_k(t)}}{e^{y_k(t)} + 1} \mathbb{E} \{u_k(t-1) | \mathcal{H}'(t-1)\} \\ &= \mathbb{E} \{u_k(t) | \mathcal{H}'(t)\} = \Pr \{u_k(t) = 1 | \mathcal{H}'(t)\} \end{aligned}$$

where the third equality follows from (14). Hence, we have completed the proof of  $\Pi(t)$ .

### 3. Proof of $IV(t)$

We prove  $IV(t)$  by enumerating all four cases of  $w_1$  and  $w_2$  in (9). Specifically, proving (9) is equivalent to proving that,  $\forall i_1, i_2 \in \mathcal{L}, \forall j_1 \neq j_2$  with  $j_1 \in \mathcal{I}_{k_1}$  and  $j_2 \in \mathcal{I}_{k_2}$ , and  $\forall g_1, g_2 \in \{g^{(0)}, g^{(1)}\}$

$$\begin{aligned} & \lim_{N \rightarrow \infty} \mathbb{E} \{g_1(\mu_{i_1 j_1}(t)) g_2(\mu_{i_2 j_2}(t))\} \\ &= \mathbb{E} \{g_1(u_{k_1}(t))\} \mathbb{E} \{g_2(u_{k_2}(t))\} \quad (22) \end{aligned}$$

where  $g^{(0)}, g^{(1)} : \{0, 1\} \rightarrow \{0, 1\}$  are defined as  $g^{(1)}(x) = x$  and  $g^{(0)}(x) = 1 - x, \forall x \in \{0, 1\}$ . The four cases are listed as follows.

- Case 1:  $w_1 = w_2 = 1$  in (9) corresponds to  $g_1 = g_2 = g^{(1)}$  in (22).
- Case 2:  $w_1 = 1, w_2 = 0$  in (9) corresponds to  $g_1 = g^{(1)}, g_2 = g^{(0)}$  in (22).
- Case 3:  $w_1 = 0, w_2 = 1$  in (9) corresponds to  $g_1 = g^{(0)}, g_2 = g^{(1)}$  in (22).
- Case 4:  $w_1 = w_2 = 0$  in (9) corresponds to  $g_1 = g_2 = g^{(0)}$  in (22).

We note that  $a_{ij}(t)$  are independent over  $i, j$  and that  $p_{ij}(t)$  is independent over  $j$  by definition. From (5), for any given  $i_1, i_2 \in \mathcal{L}$  and  $j_1 \neq j_2, \mu_{i_1 j_1}(t)$  and  $\mu_{i_2 j_2}(t)$  are independent, given that

$$\begin{aligned} \mathcal{X}(t) \triangleq & \left( q_{i_1}(t-1), q_{i_2}(t-1), X_{i_1 k_1}(t-1), \right. \\ & \left. X_{i_2 k_2}(t-1), (\mu_{ij}(t-1))_{i \in \mathcal{L}}^{j=j_1, j_2}, \mathcal{H}'(t) \right) \end{aligned}$$

where we note that  $S(t)$  is deterministic given  $\mathcal{H}(t)$ . Thus, we have,  $\forall i_1, i_2 \in \mathcal{L}, \forall k_1, k_2$ , and  $\forall j_1 \in \mathcal{I}_{k_1}, j_2 \in \mathcal{I}_{k_2}$

with  $j_1 \neq j_2$

$$\begin{aligned} & \mathbb{E} \{g_1(\mu_{i_1 j_1}(t)) g_2(\mu_{i_2 j_2}(t)) | \mathcal{X}(t)\} \\ &= \mathbb{E} \{g_1(\mu_{i_1 j_1}(t)) | \mathcal{X}(t)\} \mathbb{E} \{g_2(\mu_{i_2 j_2}(t)) | \mathcal{X}(t)\} \\ &= \left[ F_{g_{1,0}}(q_{i_1}(t-1), X_{i_1 k_1}(t-1), S_{k_1}(t)) + \sum_{l_1 \neq i_1} F_{g_{1,2}} \right. \\ & \quad \times (q_{i_1}(t-1), X_{i_1 k_1}(t-1), S_{k_1}(t)) \mu_{l_1 j_1}(t-1) \\ & \quad \left. + F_{g_{1,1}}(q_{i_1}(t-1), X_{i_1 k_1}(t-1), S_{k_1}(t)) \mu_{i_1 j_1}(t-1) \right] \\ & \quad \times \left[ F_{g_{2,0}}(q_{i_2}(t-1), X_{i_2 k_2}(t-1), S_{k_2}(t)) + \sum_{l_2 \neq i_2} F_{g_{2,2}} \right. \\ & \quad \times (q_{i_2}(t-1), X_{i_2 k_2}(t-1), S_{k_2}(t)) \mu_{l_2 j_2}(t-1) + F_{g_{2,1}} \\ & \quad \left. \times (q_{i_2}(t-1), X_{i_2 k_2}(t-1), S_{k_2}(t)) \mu_{i_2 j_2}(t-1) \right] \quad (23) \end{aligned}$$

where we have used the following definitions:

$$\begin{aligned} F_{g^{(1)},0}(z_1, z_2, S) & \triangleq \beta \frac{e^{z_1 S - \gamma z_2 (1-S)}}{e^{z_1 S - \gamma z_2 (1-S)} + 1} \\ F_{g^{(1)},1}(z_1, z_2, S) & \triangleq (1-\beta) \frac{e^{z_1 S - \gamma z_2 (1-S)}}{e^{z_1 S - \gamma z_2 (1-S)} + 1} \\ F_{g^{(1)},2}(z_1, z_2, S) & \triangleq -\beta \frac{e^{z_1 S - \gamma z_2 (1-S)}}{e^{z_1 S - \gamma z_2 (1-S)} + 1} \\ F_{g^{(0)},0}(z_1, z_2, S) & \triangleq 1 - F_{g^{(1)},0}(z_1, z_2, S) \\ F_{g^{(0)},1}(z_1, z_2, S) & \triangleq -F_{g^{(1)},1}(z_1, z_2, S) \\ F_{g^{(0)},2}(z_1, z_2, S) & \triangleq -F_{g^{(1)},2}(z_1, z_2, S). \end{aligned}$$

By taking the limit of  $N$  and the expectation on both sides of (23), we obtain (24) after rearranging terms and employing the induction hypothesis ( $\Pi(t-1)$ ,  $\Pi(t-1)$ , and  $IV(t-1)$ ). Note the last equality in the following:

$$\begin{aligned} & \lim_{N \rightarrow \infty} \mathbb{E} \{g_1(\mu_{i_1 j_1}(t)) g_2(\mu_{i_2 j_2}(t)) | \mathcal{H}'(t)\} \\ &= F_{g_{1,0}}(q(t-1), x_{k_1}(t-1), S_{k_1}(t)) \\ & \quad \times F_{g_{2,0}}(q(t-1), x_{k_2}(t-1), S_{k_2}(t)) \\ & \quad + \mathbb{E} \{u_{k_2}(t-1) | \mathcal{H}'(t-1)\} \\ & \quad \times [(L-1)F_{g_{1,0}}(q(t-1), x_{k_1}(t-1), S_{k_1}(t)) \\ & \quad \times F_{g_{2,2}}(q(t-1), x_{k_2}(t-1), S_{k_2}(t)) \\ & \quad + F_{g_{1,0}}(q(t-1), x_{k_1}(t-1), S_{k_1}(t)) \\ & \quad \times F_{g_{2,1}}(q(t-1), x_{k_2}(t-1), S_{k_2}(t))] \\ & \quad + \mathbb{E} \{u_{k_1}(t-1) | \mathcal{H}'(t-1)\} \\ & \quad \times [(L-1)F_{g_{2,0}}(q(t-1), x_{k_2}(t-1), S_{k_2}(t)) \\ & \quad \times F_{g_{1,2}}(q(t-1), x_{k_1}(t-1), S_{k_1}(t)) \end{aligned}$$



$$\begin{aligned}
& + F_{g_{1,1}}(q(t-1), x_{k_1}(t-1), S_{k_1}(t)) \\
& \quad \times F_{g_{2,0}}(q(t-1), x_{k_2}(t-1), S_{k_2}(t)) \\
& + \mathbb{E}\{u_{k_1}(t-1)|\mathcal{H}'(t-1)\} \mathbb{E}\{u_{k_2}(t-1)|\mathcal{H}'(t-1)\} \\
& \times [(L-1)^2 F_{g_{1,2}}(q(t-1), x_{k_1}(t-1), S_{k_1}(t)) \\
& \quad \times F_{g_{2,2}}(q(t-1), x_{k_2}(t-1), S_{k_2}(t)) \\
& \quad + (L-1)F_{g_{2,1}}(q(t-1), x_{k_2}(t-1), S_{k_2}(t)) \\
& \quad \times F_{g_{1,1}}(q(t-1), x_{k_1}(t-1), S_{k_1}(t)) \\
& \quad + (L-1)F_{g_{1,1}}(q(t-1), x_{k_1}(t-1), S_{k_1}(t)) \\
& \quad \times F_{g_{2,2}}(q(t-1), x_{k_2}(t-1), S_{k_2}(t)) \\
& \quad + F_{g_{1,1}}(q(t-1), x_{k_1}(t-1), S_{k_1}(t)) \\
& \quad \times F_{g_{2,1}}(q(t-1), x_{k_2}(t-1), S_{k_2}(t))] \\
& = [F_{g_{1,0}}(q(t-1), x_{k_1}(t-1), S_{k_1}(t)) \\
& \quad + \mathbb{E}\{u_{k_1}(t-1)|\mathcal{H}'(t-1)\} \\
& \quad \times (F_{g_{1,1}}(q(t-1), x_{k_1}(t-1), S_{k_1}(t)) + (L-1)) \\
& \quad \times F_{g_{1,2}}(q(t-1), x_{k_1}(t-1), S_{k_1}(t))] \\
& \times [F_{g_{2,0}}(q(t-1), x_{k_2}(t-1), S_{k_2}(t)) \\
& \quad + \mathbb{E}\{u_{k_2}(t-1)|\mathcal{H}'(t-1)\} \\
& \quad \times (F_{g_{2,1}}(q(t-1), x_{k_2}(t-1), S_{k_2}(t)) + (L-1)) \\
& \quad \times F_{g_{2,2}}(q(t-1), x_{k_2}(t-1), S_{k_2}(t))] \\
& = \mathbb{E}\{g_1(u_{k_1}(t)|\mathcal{H}'(t)) \mathbb{E}\{g_2(u_{k_2}(t)|\mathcal{H}'(t))\} \quad (24)
\end{aligned}$$

follows (14). Since  $g_1, g_2$  are arbitrarily chosen from  $\{g^{(0)}, g^{(1)}\}$ , we have (9) holds  $\forall w_1, w_2 \in \{0, 1\}$ , completing the proof of IV(t).

#### D. Proof of III(t)

For notational simplicity, all expectations that appear in the following proof are conditioned on  $\mathcal{H}'(t)$ . For any  $i \in \mathcal{L}$ , taking the variance of  $(1/(n_k N)) \sum_{j \in \mathcal{I}_k} \mu_{ij}(t)$  leads to

$$\begin{aligned}
& \text{Var} \left\{ \frac{1}{n_k N} \sum_{j \in \mathcal{I}_k} \mu_{ij}(t) \right\} \\
& = (n_k N)^{-2} \left[ \sum_{j_1, j_2 \in \mathcal{I}_k: j_1 \neq j_2} \text{Cov} \{ \mu_{ij_1}(t), \mu_{ij_2}(t) \} \right. \\
& \quad \left. + \sum_{j \in \mathcal{I}_k} \text{Var} \{ \mu_{ij}(t) \} \right] \\
& \stackrel{(a)}{=} \frac{n_k N - 1}{n_k N} \text{Cov} \{ \mu_{ij'_1}(t), \mu_{ij'_2}(t) \} + (n_k N)^{-1} \text{Var} \{ \mu_{ij'_1}(t) \} \\
& \stackrel{(b)}{\rightarrow} 0, \text{ as } N \rightarrow \infty \quad (25)
\end{aligned}$$

where  $j'_1$  and  $j'_2 \in \mathcal{I}_k$  are any two given channels with  $j'_1 \neq j'_2$ . The equality (a) in (25) follows from the exchangeability of

$(\mu_{ij}(t))_{j \in \mathcal{I}_k}$ , and (b) in (25) follows the property of asymptotic mutual independence by IV(t), which has been proven in Appendix A-3.

By employing Chebyshev's inequality to the variance  $\text{Var}\{(1/n_k N) \sum_{j \in \mathcal{I}_k} \mu_{ij}(t)\}$ , we obtain, for any arbitrarily small  $\eta > 0$ , the following:

$$\lim_{N \rightarrow \infty} \Pr \left\{ \left| \frac{1}{n_k N} \sum_{j \in \mathcal{I}_k} \mu_{ij}(t) - \mathbb{E} \left\{ \frac{1}{n_k N} \sum_{j \in \mathcal{I}_k} \mu_{ij}(t) \right\} \right| \geq \eta \right\} = 0.$$

Hence, we conclude

$$\begin{aligned}
& \frac{1}{n_k N} \sum_{j \in \mathcal{I}_k} \mu_{ij}(t) - \mathbb{E} \{ \mu_{ij'_1}(t) \} \\
& = \frac{1}{n_k N} \sum_{j \in \mathcal{I}_k} \mu_{ij}(t) - \mathbb{E} \left\{ \frac{1}{n_k N} \sum_{j \in \mathcal{I}_k} \mu_{ij}(t) \right\} \xrightarrow{P} 0 \quad (26)
\end{aligned}$$

where the first equality follows from the exchangeability of  $(\mu_{ij}(t))_{j \in \mathcal{I}_k}$ . By II(t) (which has been proved in Appendix A-2), III(t) follows (26).

## APPENDIX B PROOF OF LEMMA 2

We introduce the following lemma to assist the proof of Lemma 2.

*Lemma 3:* For any stabilizable arrival rate  $\lambda' \in \Lambda$ , there exists a randomized stationary algorithm STAT with schedules  $(\mu_{ij}^{\text{STAT}}(t))$  dependent only on  $\mathcal{H}(t)$ , such that for each time slot  $t$

$$\begin{aligned}
& \frac{K}{N} \mathbb{E} \left\{ \sum_{k=1}^M \sum_{j \in \mathcal{I}_k} \mu_{ij}^{\text{STAT}}(t) (1 - C_k(t)) \right\} = \lambda' \\
& \frac{1}{n_k N} \mathbb{E} \left\{ \sum_{j \in \mathcal{I}_k} \mu_{ij}^{\text{STAT}}(t) C_k(t) \right\} \leq \rho_k \quad \forall i \in \mathcal{L}; \quad \forall k. \quad (27)
\end{aligned}$$

Similar formulation and proof have been provided in [5] and [6]; therefore, the proof for Lemma 3 is omitted for brevity.

Consider any given  $\lambda' \in \Lambda$ . We define random variables  $\mu_k^{\text{STAT}}(t)$  as follows:

$$\begin{aligned}
& \mu_k^{\text{STAT}}(t) | (\mathcal{H}(t) = h) \\
& = \begin{cases} 1, & \text{w.p. } \frac{1}{L} \sum_{i \in \mathcal{L}} \frac{1}{n_k N} \mathbb{E} \left\{ \sum_{j \in \mathcal{I}_k} \mu_{ij}^{\text{STAT}}(t) | (\mathcal{H}(t) = h) \right\} \\ 0, & \text{otherwise} \end{cases} \quad (28)
\end{aligned}$$

$\forall k$  and for all possible  $\mathcal{H}(t) = h$ . Since  $(\mu_{ij}^{\text{STAT}}(t))$  are feasible schedules, given any  $\mathcal{H}(t) = h$

$$\sum_{i \in \mathcal{L}} \mu_{ij}^{\text{STAT}}(t) \leq 1, \quad \forall j$$

and hence

$$\begin{aligned} \mathbb{E} \{ \mu_k^{\text{STAT}}(t) | \mathcal{H}(t) = h \} \\ = \frac{1}{n_k N} \sum_{j \in \mathcal{I}_k} \frac{1}{L} \sum_{i \in \mathcal{L}} \mathbb{E} \{ \mu_{ij}^{\text{STAT}}(t) | \mathcal{H}(t) = h \} \leq \frac{1}{L}. \end{aligned}$$

According to definition (28), we also have the following:

$$\begin{aligned} \sum_{k=1}^M \mathbb{E} \{ n_k K \mu_k^{\text{STAT}}(t) S_k(t) | \mathcal{H}(t) = h \} \\ = \frac{K}{NL} \sum_{i \in \mathcal{L}} \mathbb{E} \left\{ \sum_{k=1}^M \sum_{j \in \mathcal{I}_k} \mu_{ij}^{\text{STAT}}(t) (1 - C_k(t)) | \mathcal{H}(t) = h \right\} \\ \mathbb{E} \{ \mu_k^{\text{STAT}}(t) (1 - S_k(t)) | \mathcal{H}(t) = h \} \\ = \frac{1}{L} \sum_{i \in \mathcal{L}} \frac{1}{n_k N} \mathbb{E} \left\{ \sum_{j \in \mathcal{I}_k} \mu_{ij}^{\text{STAT}}(t) C_k(t) | \mathcal{H}(t) = h \right\} \end{aligned}$$

from which, by employing Lemma 3, we conclude

$$\begin{aligned} \sum_{k=1}^M \mathbb{E} \{ n_k K \mu_k^{\text{STAT}}(t) S_k(t) \} = \lambda' \\ \mathbb{E} \{ \mu_k^{\text{STAT}}(t) (1 - S_k(t)) \} \leq \rho_k \end{aligned}$$

proving Lemma 2.

#### APPENDIX C PROOF OF THEOREM 2

For notational simplicity, we define  $\Delta(t) \triangleq \mathbb{E} \{ (1/2)[q(t)^2 - q(t-1)^2] + (\gamma K/2) \sum_{k=1}^M n_k [x_k(t)^2 - x_k(t-1)^2] \}$ . By squaring both sides of the queue dynamics (10) and (11), we have

$$\begin{aligned} \Delta(t) &\leq B_1 + \lambda \mathbb{E} \{ q(t-1) \} - \gamma K \sum_{k=1}^M n_k \rho_k \mathbb{E} \{ x_k(t-1) \} \\ &\quad - \mathbb{E} \left\{ \sum_{k=1}^M u_k(t-1) [n_k K (1 - C_k(t-1)) q(t-1) \right. \\ &\quad \quad \left. - \gamma n_k K C_k(t-1) x_k(t-1)] \right\} \\ &\leq B_1 + K \max\{K, \gamma\} \\ &\quad + \lambda \mathbb{E} \{ q(t-1) \} - \gamma K \sum_{k=1}^M n_k \rho_k \mathbb{E} \{ x_k(t-1) \} \\ &\quad - \mathbb{E} \left\{ K \sum_{k=1}^M n_k u_k(t-1) [(1 - C_k(t-1)) q(t-2) \right. \\ &\quad \quad \left. - \gamma C_k(t-1) x_k(t-2)] \right\} \\ &\stackrel{(c)}{=} B_1 + K \max\{K, \gamma\} + \lambda \mathbb{E} \{ q(t-1) \} \\ &\quad - \gamma K \sum_{k=1}^M n_k \rho_k \mathbb{E} \{ x(t-1) \} \\ &\quad - \mathbb{E} \left\{ K \sum_{k=1}^M n_k u_k(t-1) y_k(t-1) \right\} \end{aligned} \quad (29)$$

where  $B_1 \triangleq (1/2)\lambda^2 + (1/2)K^2 + (1/2)\gamma K + (1/2)\gamma K \sum_k \rho_k^2 n_k$ . Note that (c) follows from the following equality:

$$\begin{aligned} \mathbb{E} \{ u_k(t-1) [(1 - C_k(t-1)) q(t-2) \\ - \gamma C_k(t-1) x_k(t-2)] | \mathcal{H}'(t-1) \} \\ = \mathbb{E} \{ u_k(t-1) | \mathcal{H}'(t-1) \} y_k(t-1) \end{aligned}$$

where we utilized the fact that  $q(t-2)$ ,  $x_k(t-2)$ , and  $u_k(t-1)$  are independent of  $C_k(t-1)$  given  $\mathcal{H}'(t-1)$  by their dynamics (10)–(12).

Since  $\lambda \in (L-1/L)^{L-1} \Lambda$ , there exists  $\epsilon_1 > 0$  such that  $\lambda' \triangleq (\lambda/\beta L) + \epsilon_1 \in \Lambda$  by definition. We define  $\delta$  in Lemma 1 as follows:

$$0 < \delta \triangleq \frac{\beta L \epsilon_1}{2(\epsilon_1 + \frac{\lambda}{\beta L})} < 1.$$

By Lemma 1, we have

$$\begin{aligned} \mathbb{E} \{ u_k(t-1) y_k(t-1) | y_k(t-1) \geq B_2 \} \\ \geq \left( \beta - \frac{\delta}{L} \right) \mathbb{E} \{ y_k(t-1) | y_k(t-1) \geq B_2 \}. \end{aligned}$$

Employing the above inequality to (29), we obtain

$$\begin{aligned} \Delta(t) &\leq B_1 + K \max\{K, \gamma\} + \lambda \mathbb{E} \{ q(t-1) \} \\ &\quad - \gamma K \sum_{k=1}^M n_k \rho_k \mathbb{E} \{ x(t-1) \} - K \sum_{k=1}^M n_k \\ &\quad \times [\Pr \{ y_k(t-1) \geq B_2 \} \\ &\quad \times \mathbb{E} \{ u_k(t-1) y_k(t-1) | y_k(t-1) \geq B_2 \} \\ &\quad + \Pr \{ y_k(t-1) < B_2 \} \\ &\quad \times \mathbb{E} \{ u_k(t-1) y_k(t-1) | y_k(t-1) < B_2 \}] \\ &\leq B_1 + K \max\{K, \gamma\} + \lambda \mathbb{E} \{ q(t-1) \} \\ &\quad - \gamma K \sum_{k=1}^M n_k \rho_k \mathbb{E} \{ x_k(t-1) \} + K B_2 \left( \beta - \frac{\delta}{L} \right) \\ &\quad - K \left( \beta - \frac{\delta}{L} \right) \sum_{k=1}^M n_k \mathbb{E} \{ y_k(t-1) \}. \end{aligned} \quad (30)$$

Since  $\lambda' = (\lambda/\beta L) + \epsilon_1 \in \Lambda$ , by Lemma 2, there exists  $(\mu_k^{\text{STAT}}(t))$  such that (15)–(17) hold for each time slot  $t$  for this  $\lambda'$ . According to (17), we have

$$-\mathbb{E} \left\{ \sum_{k=1}^M n_k y_k(t-1) \right\} \leq -\mathbb{E} \left\{ L \sum_{k=1}^M n_k \mu_k^{\text{STAT}}(t-1) y_k(t-1) \right\}. \quad (31)$$

By applying (31) to (30) and employing (15) and (16), we obtain (32), shown at the top of the next page, where  $B_3 \triangleq B_1 + K \max\{K, \gamma\} + K B_2 (\beta - (\delta/L)) + K(\beta L - \delta) \sum_{k=1}^M n_k \max\{\lambda, \gamma \rho_k\}$ , and

$$\epsilon_2 \triangleq \min \left\{ \frac{1}{2} \epsilon_1 \beta L, \min_k \{ \gamma K \rho_k (1 - \beta L + \delta) n_k \} \right\} < 0$$

$$\begin{aligned}
\Delta(t) &\leq B_3 + \lambda \mathbb{E} \{q(t-1)\} - \gamma K \sum_{k=1}^M n_k \rho_k \mathbb{E} \{x_k(t-1)\} - K(\beta L - \delta) \mathbb{E} \\
&\quad \times \left\{ \sum_{k=1}^M n_k \mu_k^{\text{STAT}}(t-1) [S_k(t-1)q(t-1) - \gamma(1 - S_k(t-1))x_k(t-1)] \right\} \\
&= B_3 - \mathbb{E} \left\{ q(t-1) \left[ (\beta L - \delta) \sum_{k=1}^M n_k K \mu_k^{\text{STAT}}(t-1) S_k(t-1) - \lambda \right] \right\} \\
&\quad - \gamma K \sum_{k=1}^M n_k \mathbb{E} \{x_k(t-1) [\rho_k - (\beta L - \delta)(1 - S_k(t-1)) \mu_k^{\text{STAT}}(t-1)]\} \\
&\leq B_3 - \epsilon_2 \mathbb{E} \left\{ q(t-1) + \sum_{k=1}^M x_k(t-1) \right\} \tag{32}
\end{aligned}$$

From (32), by taking the time-average over  $t = 0, 1, \dots, T-1$  and taking the limsup of  $T$ , we can prove (18), completing the proof of Theorem 2.

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