

Minimum Maintenance Cost Routing in Cognitive Radio Networks

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Abstract

Cognitive Radio Networks (CRNs) are composed of frequency-agile radio devices that allow licensed (primary) and unlicensed (secondary) users to coexist, where secondary users opportunistically access channels without interfering with the operation of primary ones. From the perspective of secondary users, spectrum availability is a time varying network resource over which multi-hop end-to-end connections must be maintained. In this work, a theoretical outlook on the problem of routing secondary user flows in a CRN is provided. The investigation aims to characterize optimal sequences of routes over which a secondary flow is maintained. The optimality is defined according to a novel metric that considers the maintenance cost of a route as channels and/or links must be switched due to the primary user activity. Different from the traditional notion of route stability, the proposed approach considers subsequent path selections, as well. The problem is formulated as an integer programming optimization model and shown to be of polynomial time complexity in case of full knowledge of primary user activity. Properties of the problem are also formally introduced and leveraged to design a heuristic algorithm to solve the minimum maintenance cost routing problem when information on primary user activity is not complete. Numerical results are presented to assess the optimality gap of the heuristic routing algorithm.

1. Introduction

With the prolific use of wireless devices, wireless spectrum, especially the ISM band, becomes increasingly scarce. Recent studies by the Federal Communications Commission (FCC) highlighted that many spectrum bands allocated through static assignment policies are used only in bounded geographical areas or over limited periods of time, and that the average utilization of such bands varies between 15-85% [5].

Spectrum scarcity and the under-utilization of statically allocated spectrum motivate the investigation of allocation and regulatory policies to allow unlicensed users to utilize the licensed spectrum bands in a dynamic and non-interfering manner. Advancements in the field of software-defined radios allow the development of spectrum-agile devices that can be programmed to operate on a wide spectrum range and tuned to any frequency band in that range with negligible delay [18]. Resulting so-called *Cognitive Radios* (CRs) sense a wide spectrum range, dynamically identify currently unused spectrum blocks for data communications, and intelligently access the unoccupied spectrum called *Spectrum Opportunities (SOPs)* [2].

We consider in this paper a general scenario of *Cognitive Radio Networks (CRNs)* [1] based on the model of “commons”, where two kinds of users share a common spectrum portion with different rules: Primary (or licensed) Users

(PUs) have priority in spectrum utilization, and Secondary Users (SUs) must access the spectrum in a non-intrusive manner. PUs use traditional wireless communication systems with static spectrum allocation. SUs are equipped with CRs and exploit SOPs to sustain their communication activities without interfering with PU transmissions. Roughly speaking, SUs can access a spectrum band that is not being used by PUs as long as they immediately vacate that spectrum when PUs start using it.

Realization of CRNs requires research in many areas, including spectrum sensing techniques [14], [8], [7], spectrum management policies among SUs [11], non-disruptive spectrum transition procedures, and cognitive multiple access strategies [13], [28], [12], [23]. Although most of the research has focused on single-hop scenarios of CRNs, the analysis of end-to-end communication has very recently attracted the attention of the research community, being of profound importance for networking applications [9], [16], [10]. In their most general form, CRNs are wireless multi-hop networks where information may be forwarded through several relay points to its destination. Yet, CRNs possess properties that set them apart from traditional multi-hop wireless networks, like ad-hoc networks and wireless mesh networks. Different from ad-hoc networks, where topological changes are caused by node mobility, topological changes in CRNs occur primarily due to changes in PU activity, which may affect several links at a same time. Furthermore, the channel availability in CRNs is also significantly different than in traditional wireless multi-channel multi-hop networks. Nodes in CRNs potentially have partially overlapping or non-overlapping sets of available channels, and the available channel set at a SU is of time-varying nature and changes in correlated or uncorrelated manner with respect to sets of other nodes. Consequently, development of network layer solutions in CRNs is a non-trivial task that requires development of new and specific algorithms able to cope with the spectrum availability throughout the entire path, and the necessity of re-routing in case specific portions of the currently active path are “impaired” by the presence of an activating PU.

In this paper, we address the problem of routing in CRNs by offering a novel point of view on the definition of *route stability*. Traditionally, a stable route is defined as a

route that must be repaired with a small probability, which ignores the cost of route repair or *route maintenance*. The maintenance cost represents the effort needed or penalty paid to maintaining end-to-end connectivity. Such cost may account for the service interruption time while switching routes, the amount of resources used to find new available links and routes, and/or the signaling required to include new links into the new route.

To alleviate problems associated with the traditional definition of route stability, which may lead to selection of routes that require large repair costs, we aim to find a *sequence of paths* throughout the lifetime of a connection that *minimizes the maintenance cost*. To this end, we associate a cost to each new link activation and define a stable route as a route that is maintained during its lifetime with a small total cost. This definition intrinsically considers a trade-off between the reliability of a given route and the effort needed to repair that route during its entire lifetime.

The salient contributions of the work presented in this paper can be listed as follows:

- A new approach to route selection in multi-hop CRNs is presented that accounts for route maintenance costs.
- Fundamental properties of the problem and the optimal solutions are formally shown.
- A polynomial time algorithm is derived to compute the optimal solutions under full knowledge of PU activity.
- A heuristic algorithm is introduced to solve the problem under partial knowledge of PU activity.
- Numerical results are presented to assess the quality of minimum maintenance cost route selection.

2. Related Work

Despite its importance, only a limited number of research efforts related to routing in multi-hop CRNs appear in the literature. Previously published work in the field is primarily concerned with the design of novel routing protocols in cognitive environments, and with the definition of proper routing metrics. The general and most diffused approach resorts to some type of on-demand routing protocol, leveraging different metrics to assess the quality of a given path. In [6], the authors consider a metric given by a linear combination of route availability and interference along the route. In [24], source nodes discover candidate routes through standard route discovery procedures and collect information on link connectivity and quality. For each candidate route, the algorithm finds all feasible channel assignment combinations and selects the route with the best estimated throughput. The authors of [3] and [4] propose an on-demand routing protocol based on a metric capturing the end-to-end delivery delay, further integrating the proposed routing approach with spectrum assignment techniques in [27]. Reference [20] proposes a routing metric based on the power efficiency of the considered link/path.

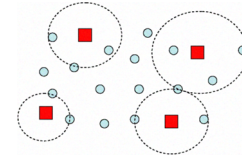


Figure 1: Reference CRN Scenario.

Similar on-demand based routing protocols are proposed in [29], and [19], where, on the other hand, the criterion steering routing decisions is secondary user throughput maximization. A slightly different approach is followed in [15], where the quality of the end-to-end path is weighted also with the probability for a secondary user transmission to cause interference on primary users.

Roughly speaking, the main purpose of the aforementioned work is finding “high quality” routes in a dynamic network setting. Although some of the proposed routing metrics consider future link availability statistics, the definition of the “best path” does not account directly the cost associated with switching to another path when necessary. In contrast, we provide a mathematical framework to design routes in CRNs with minimum route maintenance cost.

Finally, since we make wide use of mathematical programming formulations throughout the paper, it is worth mentioning here also the related literature leveraging optimization approaches in the field of CRNs. Miao and Tsang propose in [17] a Mixed Integer Linear Programming formulation for the problem of achieving throughput optimal routing and scheduling for secondary transmissions. Shi and Hou propose in [22] a non-linear mathematical programming formulation for the same problem, which is solved through a distributed algorithm. A centralized heuristic based on a graph-layering approach is introduced in [26]. To the best of our knowledge, ours is the first effort in the literature to analyze the interactions of successive route selections for a given flow and to propose an algorithm to minimize the route maintenance cost in a multi-hop CRN.

3. Problem Definition and Notation

We focus on the reference network scenario represented in Figure 1, where Secondary Users, SUs (*circles*), coexist in a given geographical area with Primary Users, PUs (*squares*). PUs have licensed access to a given spectrum portion, whereas SUs can opportunistically access the spectrum whenever vacated by PUs. From the SUs’ perspective, the spectrum availability is a random process which depends on the activity of PUs. In the figure, the dashed circles around PUs represent blocked areas for SUs whenever the corresponding PU is actively using its licensed spectrum portion. In this scenario, SUs are interested in establishing and maintaining multi-hop paths towards destination SUs. Since PUs become active according to specific activity patterns, re-routing and route repair are needed to maintain multi-hop connectivity among SUs. Route repair may involve link

switching and channel switching operations. In the former case, one or more links along the route must be replaced by other ones not interfered by PUs, whereas in the latter case, the same link can be maintained, but the transmission must be carried out over another spectrum portion (or channel). In either case, re-routing and route repair involve a route maintenance cost which accounts for the signaling overhead to coordinate with other SUs, the corresponding consumed power, etc. To this extent, in the following, we address the problem of designing routes among SUs which require the minimum maintenance cost.

The aforementioned network scenario can be abstracted as follows: Given a two-dimensional Euclidean space \mathcal{S} , a set of SU nodes \mathcal{N} is deployed in \mathcal{S} . Nodes in \mathcal{N} are equipped with a single transceiver and communicate through wireless links. Along each link, transmissions can be carried out on a given channel m that belongs to the set of available channels \mathcal{M}^1 . A link from node i to node j using channel m is denoted by $l(i, j)^m$. The set of available links is denoted by \mathcal{L} . Sets \mathcal{N} , \mathcal{L} , and \mathcal{M} define a scenario.

We introduce an *epoch* set $\mathcal{E} \triangleq \{1..E\}$. An *epoch* is defined as the time period during which there is no change in PU activities. We have an epoch transition when at least one PU switches from an idle state to a transmission state, or vice versa. Progress in time is given by the ordered sequence of epochs, from 1 to E .

Finally, we focus on route change costs. In general, since the set of all routes from node s to node d , $\mathcal{P}_{sd}(e)$, varies in subsequent epochs, we may be forced to switch from route $P_i \in \mathcal{P}_{sd}(e)$ at epoch e , denoted as $P_i(e)$, to a new route $P_j \in \mathcal{P}_{sd}(e+1)$, denoted as $P_j(e+1)$. We associate a cost to this change given by $C[P_i(e), P_j(e+1)]$. We say a *link change* occurs, when a new link must be established in epoch $e+1$, that is, $\exists l(a, b)^m : (l(a, b)^m \in P_j) \wedge (l(a, b)^{m'} \notin P_i, \forall m' \in \mathcal{M})$. A *channel change* occurs, instead, when a link already used in path P_i at epoch e must change only its operating channel in path P_j at epoch $e+1$, that is, $\exists l(a, b)^m : (l(a, b)^m \in P_j) \wedge (l(a, b)^{m'} \in P_i, m \neq m')$. We assign a cost c_L to each link that experiences a link change. Similarly, we assign a cost c_C to each link that experiences a channel change. Note that a link can be subject to either a link change or a channel change; they cannot occur together. Finally, if a link does not experience any change from epoch e to epoch $e+1$, that is, $l(a, b)^m : (l(a, b)^m \in P_j) \wedge (l(a, b)^m \in P_i)$, then its change cost is 0.

The transition cost associated to the switch from route P_i at epoch e to route P_j at epoch $e+1$, $C[P_i(e), P_j(e+1)]$, is given by the total cost of changes involving routes' links, that is

$$C[P_i(e), P_j(e+1)] = \sum_{l(a, b)^m \in P_j} c[l(a, b)^m].$$

1. Note that the definition of a channel is completely arbitrary. It can be a carrier, a set of sub-carriers, a slot in a frame, etc.

Note that the cost $C[P_i(e), P_j(e+1)]$ is independent of the epoch at which it is computed. It depends only on the pair of selected routes. Thus, we can write $C[P_i(e), P_j(e+1)] = C[P_i, P_j]$.

The minimum maintenance cost **Route Selection Problem (RSP)** in CRNs can be stated as follows:

(RSP) Given two nodes s and d in \mathcal{N} , find a route $\hat{P}(e)$ from s to d for each epoch $e \in \mathcal{E}$, $\hat{P}(e) \in \mathcal{P}_{sd}(e)$, such that the sum of the initial route $\hat{P}(1)$'s setup cost and all the required transition costs is minimized,

$$\min_{\{\hat{P}(e)\}_{e \in \mathcal{E}}} \left\{ C[\emptyset, \hat{P}(1)] + \sum_{e=1}^{|\mathcal{E}|-1} C[\hat{P}(e), \hat{P}(e+1)] \right\}$$

$$\text{s.t. } \hat{P}(e) \in \mathcal{P}_{sd}(e), \forall e \in \mathcal{E},$$

where $\{\hat{P}(e)\}_{e \in \mathcal{E}}$ is the sequence of selected routes.

4. Problem Formulation

We consider a set \mathcal{K} of traffic demands where S_k denotes the source node, D_k the destination node, and V_k the traffic load for a given flow k , $\forall k \in \mathcal{K}$. We define two sets of variables. Route selection variables $x_{(ij)_c}^k[e]$ are defined as:

$$x_{(ij)_c}^k[e] = \begin{cases} 1 & \text{demand } k \text{ path goes through link } (i, j) \\ & \text{over channel } c \text{ at epoch } e \\ 0 & \text{otherwise} \end{cases}$$

and link cost variables $p_{(ij)_c}^k[e]$ that express the change cost when link (i, j) with channel c is selected for demand k at epoch e .

The model for the (RSP) is the following:

$$\min \sum_{e \in \mathcal{E}: e \neq 1} \sum_{k \in \mathcal{K}} \sum_{c \in \mathcal{M}} \sum_{(i, j) \in \mathcal{L}} x_{(ij)_c}^k[e] p_{(ij)_c}^k[e] + \sum_{k \in \mathcal{K}} \sum_{c \in \mathcal{M}} \sum_{(i, j) \in \mathcal{L}} c_L x_{(ij)_c}^k[1] \quad \text{s.t.} \quad (1)$$

$$\sum_{c \in \mathcal{M}} \sum_{(j, i) \in \mathcal{L}} x_{(ji)_c}^k[e] - \sum_{c \in \mathcal{M}} \sum_{(i, j) \in \mathcal{L}} x_{(ij)_c}^k[e] = \begin{cases} -1 & i = S_k \\ 1 & i = D_k \\ 0 & \text{o/w} \end{cases} \quad \forall i \in \mathcal{N}, k \in \mathcal{K}, e \in \mathcal{E} \quad (2)$$

$$\sum_{c \in \mathcal{M}} x_{(ij)_c}^k[e] \leq 1 \quad \forall (i, j) \in \mathcal{L}, e \in \mathcal{E}, k \in \mathcal{K} \quad (3)$$

$$\sum_{k \in \mathcal{K}} x_{(ij)_c}^k[e] V_k \leq B_{(ij)_c}[e] \quad \forall (i, j) \in \mathcal{L}, c \in \mathcal{M}, e \in \mathcal{E} \quad (4)$$

$$p_{(ij)_c}^k[e] = \begin{cases} 0 & x_{(ij)_c}^k[e-1] = 1 \\ c_C & x_{(ij)_c}^k[e-1] = 0 \wedge \sum_{c' \in \mathcal{M}} x_{(ij)_c'}^k[e-1] = 1 \\ c_L & \sum_{c' \in \mathcal{M}} x_{(ij)_c'}^k[e-1] = 0 \end{cases} \quad \forall (i, j) \in \mathcal{L}, k \in \mathcal{K}, c \in \mathcal{M}, e \in \mathcal{E} : e \neq 1 \quad (5)$$

$$\sum_{c \in \mathcal{M}} \sum_{k \in \mathcal{K}} \left(\sum_{(i, j) \in \mathcal{L}} \frac{x_{(ij)_c}^k[e] V_k}{B_{(ij)_c}} + \sum_{(j, i) \in \mathcal{L}} \frac{x_{(ji)_c}^k[e] V_k}{B_{(ij)_c}} \right) \leq 1 \quad \forall i \in \mathcal{N}, e \in \mathcal{E} \quad (6)$$

$$\sum_{k \in \mathcal{K}} \sum_{(lm)_c \in \mathcal{I}S_{(ij)}} x_{(lm)_c}^k[e] V_k \leq B_{(ij)_c} \quad \forall (i, j) \in \mathcal{L}, c \in \mathcal{M}, e \in \mathcal{E} \quad (7)$$

$$x_{(ij)_c}^k[e] \in \{0, 1\}, p_{(ij)_c}^k[e] \in \mathcal{R}^+ \quad \forall (i, j) \in \mathcal{L}, k \in \mathcal{K}, c \in \mathcal{M}, e \in \mathcal{E} \quad (8)$$

The objective function (1) minimizes the sum of change costs of the links included in the routes selected for every epoch and the setup cost of the initial route. Constraints (2) are flow balance equations that define routing paths. Constraints (3) force the use of a single channel along each link for any demand in a given epoch, as we assume that nodes only have one transceiver each. Constraints (4) are capacity constraints for each link on each channel. Note that the parameter $B_{(ij)c}[e]$ expresses the capacity of the link $l(i, j)^c$ during epoch e and it is equal to 0 if a PU activation at epoch e prevents its use. Constraints (5) define the change cost for each link in each epoch. If link $l(i, j)^c$ is used in epoch e and is:

- 1) used at epoch $e - 1$ for the same demand, then its change cost at epoch e is 0.
- 2) used at epoch $e - 1$ for the same demand with a different channel, then it implies a channel change cost.
- 3) not used at epoch $e - 1$, then it implies a link change cost.

Constraints (6) limit the per-node total traffic on a given channel c to the capacity of that channel, while constraints (7) limit the total traffic of links belonging to the interference set of each link $l(i, j)$, denoted by $IS_{(ij)}$. Finally, constraints (8) define variable domains.

The model above is non-linear due to the Objective Function and Constraints (5). We linearize the model to obtain an equivalent formulation which allows to get optimal solutions with a state-of-the-art integer linear programming solver. The linearization consists of replacing the Objective Function (1) with the following:

$$\min \sum_{e \in \mathcal{E}: e \neq 1} \sum_{k \in \mathcal{K}} \sum_{c \in \mathcal{M}} \sum_{(i, j) \in \mathcal{L}} p_{(ij)c}^k[e] + \sum_{k \in \mathcal{K}} \sum_{c \in \mathcal{M}} \sum_{(i, j) \in \mathcal{L}} c_L x_{(ij)c}^k[1]$$

and Constraints (5) with the following two sets of constraints:

$$p_{(ij)c}^k[e] \geq c_L - c_L \left(\sum_{c' \in \mathcal{M}} (x_{(ij)c'}^k[e-1]) + 1 - x_{(ij)c}^k[e] \right) \\ \forall (i, j) \in \mathcal{L}, k \in \mathcal{K}, c \in \mathcal{M}, e \in \mathcal{E} : e \neq 1$$

$$p_{(ij)c}^k[e] \geq c_C - c_C (x_{(ij)c}^k[e-1] + 1 - x_{(ij)c}^k[e]) \\ \forall (i, j) \in \mathcal{L}, k \in \mathcal{K}, c \in \mathcal{M}, e \in \mathcal{E} : e \neq 1$$

The proposed formulation is rather complete, accounting for all the relevant aspects related to CRNs routing. Nevertheless, in specific but realistic CRN scenarios, some of the assumptions (constraints) given in the formulation may be relaxed. As an example, in some cases connectivity among nodes is the main issue, whereas capacity limitations are much more loose. Think of network scenario where SUs perform transmissions over short opportunistic, high-capacity links. Roughly speaking, in this scenario flows carry small amounts of traffic with respect to link capacities. Constraints (6), (7) and demand index “ k ” can be dropped in the previous formulation, thus focusing on routing a single traffic demand throughout epochs. This problem where

$\mathcal{K} = 1$ and $v_k = 1$ is denoted as **(RSP-1)**. The global routing can be found by the superimposition of the route of each single demand at every epoch determined solving (RSP-1).

In the remainder of the paper we focus on this first simplified scenario in order to analyze its properties and behaviors. The useful insights found with this work have strongly inspired the ongoing work on the complete scenario, which we have not included in this paper for reasons of space.

5. Problem Complexity

In this section, we show that (RSP-1) is solvable in polynomial time. We take here a constructive approach: We first show that (RSP-1) in the case of a Single Channel, (SC-RSP-1), ($|\mathcal{M}| = 1$) belongs to P , and propose a polynomial time solution approach. Then, we show that the general (RSP-1) can be reduced to an equivalent single channel formulation through a polynomial reduction.

5.1. Single Channel RSP-1 (SC-RSP-1)

(SC-RSP-1) can be written as:

$$\min \sum_{e \in \mathcal{E}: e \neq 1} \sum_{(i, j) \in \mathcal{L}} C_L p_{(ij)}[e] + \sum_{(i, j) \in \mathcal{L}} c_L x_{(ij)}[1] \quad \text{s.t.} \quad (9)$$

$$\sum_{(j, i) \in \mathcal{L}} x_{(ji)}[e] - \sum_{(i, j) \in \mathcal{L}} x_{(ij)}[e] = \begin{cases} -1 & i = S \\ 1 & i = D \\ 0 & \text{otherwise} \end{cases} \\ \forall i \in \mathcal{N}, e \in \mathcal{E} \quad (10)$$

$$x_{(ij)}[e] \leq B_{(ij)}[e] \quad \forall (i, j) \in \mathcal{L}, e \in \mathcal{E} \quad (11)$$

$$p_{(ij)}[e] \geq x_{(ij)}[e] - x_{(ij)}[e-1] \\ \forall (i, j) \in \mathcal{L}, e \in \mathcal{E} : e \neq 1 \quad (12)$$

$$x_{(ij)}[e] \in \{0, 1\}, p_{(ij)}[e] \in \mathbb{R}^+ \quad \forall (i, j) \in \mathcal{L}, e \in \mathcal{E} \quad (13)$$

Variable and constraint meanings are the same as in the previous MILP formulation (1)-(8). Indices k and c are dropped due to the use of a single channel and a single demand, as well as Constraints (3), (6), and (7). We build the proof of polynomial time characteristic of (SC-RSP-1) (9)-(13) on the following theorems.

Theorem 5.1. *The matrix of coefficients of (SC-RSP-1) is totally unimodular.*

PROOF SKETCH Denoting the sets of variables $x_{(ij)}[e]$ and $p_{(ij)}[e]$ as $\mathbf{v} = [\mathbf{x}, \mathbf{p}]^T$, the set of constraints (10)-(13) can be written as $\mathbf{A}\mathbf{v} \leq \mathbf{b}$, where \mathbf{b} is called vector of known coefficients and \mathbf{A} matrix of coefficients. The proof is based on two observations: First, the rows of \mathbf{A} are composed only of elements in $\{-1, 0, +1\}$; Second, matrix \mathbf{A} satisfies the following: Each collection Q of rows can be partitioned into two subsets (Q_1, Q_2) such that the sum of the rows in one subset minus the sum of the rows in the other subset is a vector with entries only in the set $\{-1, 0, +1\}$. This is a necessary and sufficient condition for \mathbf{A} to be totally unimodular [25]. \square

Knowing that its matrix of coefficients is totally unimodular, we can leverage the following theorem to assess the complexity of (SC-RSP-1).

Theorem 5.2. *Let \mathbf{A} be an integer matrix. The matrix \mathbf{A} is totally unimodular if and only if the polyhedron $P(\mathbf{b}) = \{\mathbf{x} \in \mathbb{R}_+^n \mid \mathbf{A}\mathbf{x} \leq \mathbf{b}\}$ is integral, $\forall \mathbf{b} \in \mathbb{Z}^m$ for which $P(\mathbf{b}) \neq \emptyset$ [25].*

This second theorem states that vertices of the solution space polyhedron of problems with totally unimodular coefficient matrices are integer, even if the problem variables \mathbf{x} are fractional. This allows to solve the continuous relaxation of the MILP guaranteeing an integer optimum solution. Moreover, since the vertices of the solution space polyhedron are integer regardless of the variables' integrality, optimum solutions of the integer formulation and the linear relaxation are guaranteed to be equal. Consequently, we can obtain the optimal solution of (SC-RSP-1) by solving a Linear Program (LP). Since LP solution methods are known to take polynomial time [21], we can state that (SC-RSP-1) $\in P$.

5.2. Multiple Channel RSP-1

In this section, we describe a polynomial time reduction algorithm to reduce any formulation of the general (RSP-1) to an equivalent formulation of (SC-RSP-1). This proves that the general (RSP-1) is also polynomial time since its solution time complexity is given by the sum of the complexities of (SC-RSP-1) and of the reduction procedure, both polynomial with respect to instance sizes.

The driving idea of the reduction procedure is to create an equivalent SU connectivity graph epoch by epoch such that channel switching operations in the original graph are equivalent to link switching operations in the equivalent one. To this extent, for each link $l(i, j)$ in the original graph, we introduce the following elements in the equivalent graph:

- 1) Node i itself and an additional node i_j .
- 2) Nodes i_j^c , one for each available channel c on $l(i, j)$.
- 3) For each channel c , a link from node i to node i_j^c and one from node i_j^c to node i_j are added. In addition, a link from node i_j to node j is added to the graph.

The link switching costs, $C_{L(lm)}$, in the equivalent graph are defined as follows:

- $\frac{C_C}{2}$, for links $l(l, m)$ from node i to nodes i_j^c .
- $\frac{C_C}{2}$, for links $l(l, m)$ from nodes i_j^c to nodes i_j .
- $C_L - C_C$, for the link $l(l, m)$ from node i_j to node j .

The new multiple channel link representation is shown in Fig. 2. The objective function (9) is replaced by:

$$\min \sum_{e \in \mathcal{E}: e \neq 1} \sum_{(i,j) \in \mathcal{C}} C_{L(ij)} p_{(ij)}[e] + \sum_{(i,j) \in \mathcal{C}} C_{L(ij)} x_{(ij)}[1]. \quad (14)$$

Note that Objective Function (14) does not modify the matrix of coefficients, which is still unimodular.

The modeling of channel c unavailability between nodes i and j during epoch e is obtained by imposing $B_{i, i_j^c}[e] =$

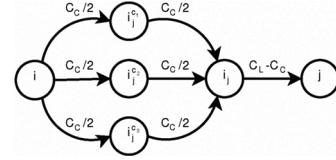


Figure 2: Equivalent Structure of a Multi-Channel Link.

$B_{i_j^c, i_j}[e] = 0$. Note that the unavailability of multiple channel link $l(i, j)$ is equivalent to the unavailability of every available channel between nodes i and j . Imposing $B_{i, i_j^c}[e] = B_{i_j^c, i_j}[e] = 0, \forall c \in \mathcal{M}$, prevents flows from entering link $l(i_j, j)$, thus the route cannot go directly from node i to node j .

As for the maintenance cost from epochs $e - 1$ to e , if channel c becomes unavailable between nodes i and j and the new selected route uses channel d , we have $x_{(i, i_j^d)}[e] = x_{(i, i_j^d)}[e - 1] = 1$ and $x_{(i, i_j^c)}[e] = x_{(i, i_j^c)}[e - 1] = 0$, while $x_{(i_j, j)}[e] = x_{(i_j, j)}[e - 1] = 1$. Therefore, the objective function (14) adds $\frac{C_C}{2} + \frac{C_C}{2} = C_C$ for a channel change. In case the link must be switched completely, the newly established link is characterized by $x_{(i, i_j^c)}[e] = x_{(i, i_j^c)}[e] = x_{(i_j, j)}[e] = 1$, thus the additional cost in the objective function (14) is $\frac{C_C}{2} + \frac{C_C}{2} + C_L - C_C = C_L$, which is consistent with the original problem formulation.

Finally, since integer solutions provide a unique non-split path from source to destination in the modified single channel scenario, per-link per-demand single channel selection constraints (3) are implicitly satisfied. With the procedure described above, (RSP-1) can be solved with the following polynomial time algorithm.

Algorithm 1 PolyAlg (RSP-1)

- 1: Reduce(RSP-1) \rightarrow SC-RSP-1 $_{eq}$
 - 2: LP-SOLVE(SC-RSP-1 $_{eq}$)
-

The algorithm is composed of two steps: In Step 1, the instance of RSP-1 is reduced to an equivalent single channel formulation leveraging the reduction procedure described above. Note that the reduction steps are polynomial in number, i.e., $O(|\mathcal{L}||\mathcal{M}|)$. The second and final step solves the equivalent single channel formulation leveraging LP solvers, which are known to be of polynomial complexity [21].

6. Minimum Maintenance Cost Routing

To show the effect of parameter values on solution characteristics, we solve the RSP-1 in the reference cognitive network scenario depicted in Figure 3. SUs are deployed on a grid, and their communication range is equal to the grid step. The transmitting SU and the receiving one are located on the left and right edges of the grid, respectively. A variable number of PUs are active in the scenario and induce *interference areas*, defined as the geographical portion of the network where SU transmissions are inhibited whenever a PU is active. Since we want to stress the routing design algorithm, we further assume that *interference areas* of PUs form vertical barriers between source and destination.

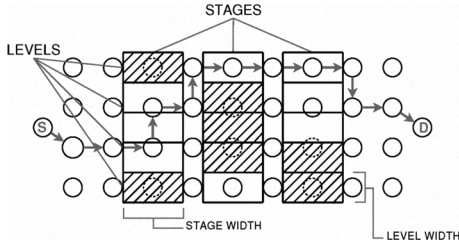


Figure 3: CRN Scenario Under Evaluation.

The barriers can be described as sets of rectangular interference areas organized in rows (*levels*) and columns (*stages*). In case multiple channels are available, we have a superimposition of elementary PUs (one for each channel) within the same rectangular interference area.

Unless specified differently, the reference scenario considers 104 SUs (on a 8x13 grid), a two-stages barrier covering 3 columns of the grid per-stage, and 5 levels with a total height equal to the grid height; optimal routes are evaluated over 15 epochs with a PU activity probability of 0.3 on each channel, in each epoch. The channel switching cost is 1, and the link switching cost is 10. All results are averaged on 10 realizations of the PU activity process.

As first step of analysis, the route maintenance cost is observed for different PU activity probability and interference area shapes and dimensions. Fig. 4(a) shows minimum routing maintenance cost as the PU activity probability increases. As expected, as PU activity increases, links among SUs are more likely to be unavailable, thus, the frequency of switching link and/or channel increases. Interestingly, when multiple channels are available, route maintenance cost decreases. Indeed, if transmissions can be carried out over many channels, the probability of finding at least one available channel between two nodes in an epoch is higher. As a result, the more expensive link changes occur less frequently, and routes can be repaired just by changing the operating channel of their links. We can further observe that the gain of having more channels is higher when PUs are more active. Finally, note how the addition of just one channel to the single-channel case can substantially reduce the route maintenance costs.

As for the shape of the interference areas, Fig. 4(b) shows the effect of the barrier width on route maintenance. The stage width is expressed in number of grid columns affected by PU activity. Intuitively, maintenance costs increase with the stage width since a larger number of SU links are affected by the interference areas. The effect of PU barrier locations is shown in Fig. 4(c). Results are obtained by splitting a single wide barrier into several geographically disjoint stages (3, 4, 6).² Results show that the more dispersed the PU activity is, the higher is the route maintenance cost. Crossing a wide interference area with an almost synchronous behavior induces a smaller maintenance cost

2. To provide a fair comparison, the splitting is carried out in such a way that the number of links affected by the PU activity is kept constant.

than crossing several smaller but uncorrelated interference areas.

In Fig. 4(d) the effect of the initial route on route maintenance cost is shown. Here, the number of hops of the initial route is shown as number of stages varies. In the single-channel case, since the maintenance cost among epochs has a big impact on the overall cost, the initial route tends to be longer, preferring links that require less changes in the future. In case of multiple channels, the initial setup is the dominant cost, so the first epoch route is shorter. The availability of multiple channels allows to perform channel changes instead of link changes. This is readily confirmed by in Fig. 4(e), which reports the number of hops of the initial route when varying the ratio between link change and channel change costs. For the case of 4 channels, the curve also reports the cost share of the route at the first epoch with respect to the overall route maintenance cost. The impact of the route setup cost in the first epoch grows as the link setup cost increases. Route maintenance, exploiting channel changes, becomes increasingly negligible. In addition, note that, while the initial route length is actually constant with 2 and 3 available channels, with more available channels it shortens when the maintenance cost ratio is higher.

Fig. 4(f) shows the minimum route maintenance costs as the number of epochs increases. In case of a single available channel, the route maintenance cost increases by 120% going from 5 to 25 epochs. On the other hand, cost increases are less relevant when multiple channels are available: 83% and 32% increase in case of 2 and 3 channels, respectively. As more channels are available, maintenance costs among epochs are less influent, and the main component of the overall routing cost is due to the initial route setup.

7. A Routing Heuristic

Although the algorithm presented in Sec. 5 provides an efficient way to solve (RSP-1) even for large-size scenarios, it is based on a centralized approach which assumes full knowledge on the PU activity. In this section, we design an algorithm operating with a more limited knowledge of the PU activity, dropping the full knowledge assumption.

Again, we follow a constructive approach: we start off by highlighting some properties of the minimum maintenance cost routing (Sec. 7.1), which we leverage in the design of a novel routing metric (Sec. 7.2) to be used in an operational algorithm (Sec. 7.3). The optimality gap of the proposed algorithm is also assessed through numerical evaluations.

7.1. Properties and Observations

The fundamental property of minimum maintenance cost routing is given in the form of theorem. It states that if

- 1) Two PU activity processes are equivalent from an epoch e_0 to the last one, and
- 2) Corresponding optimal route selection sequences include the same route at the same epoch e_0 ,

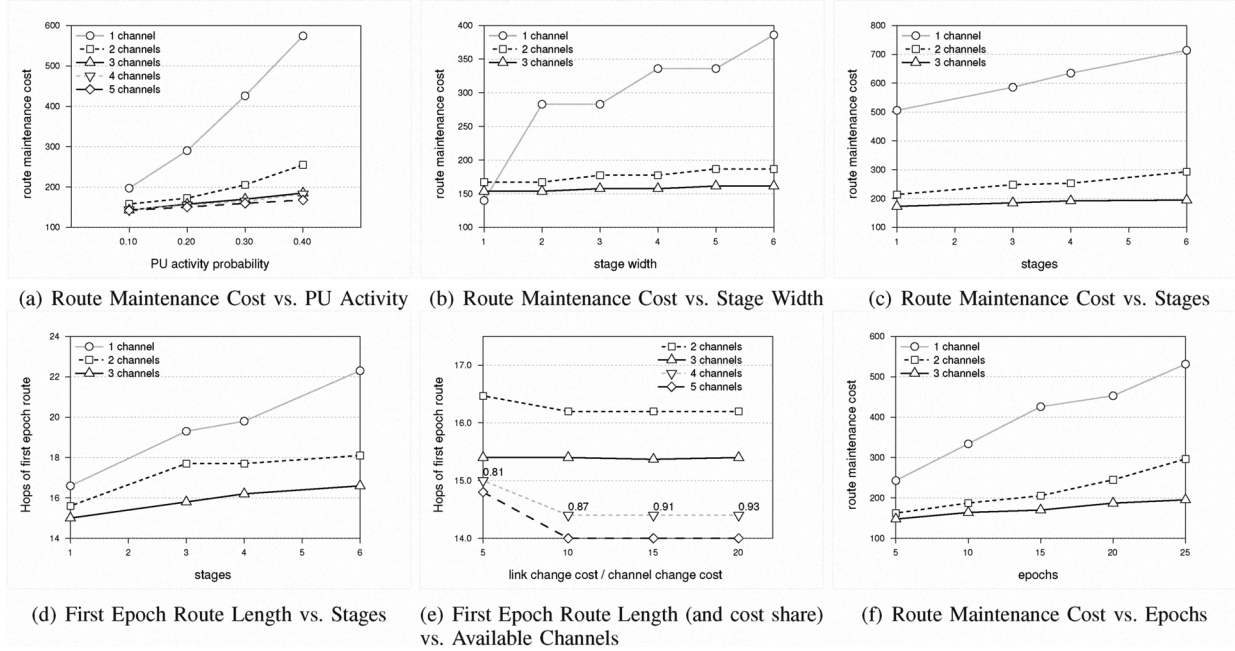


Figure 4: Quality of Minimum Maintenance Cost Routing.

then sums of transition costs given by the two sequences from e_0 to the end are *equal* and *minimum* although path selections, except at epoch e_0 , are not necessarily the same. We define two PU activity processes equivalent from epoch f to epoch l when their induced sets $\mathcal{P}_{sd}(e)$ are equivalent for $e \in [f, l]$. Leveraging the following theorem, we show in Corollary 7.2 that if the route selection sequence reaches a particular route in epoch e_0 , then routes at future epochs do not depend on choices made before e_0 . A sketch of the proof of Thm. 7.1 is given in Appendix A.

Theorem 7.1. *Given a scenario, two PU activity processes Υ , and Υ' , and indicating with $\hat{P}_\Upsilon(e)$ the route selected at epoch e under PU activity process Υ , if the following conditions hold:*

- $\Upsilon \equiv \Upsilon'$ from epoch e_0 to epoch $|E|$,
- $\{\hat{P}_\Upsilon(e)\}_{e \in \mathcal{E}}$ and $\{\hat{P}_{\Upsilon'}(e)\}_{e \in \mathcal{E}}$ are min-cost route selection sequences under corresponding PU activity processes, where $\hat{P}_\Upsilon(e)$ and $\hat{P}_{\Upsilon'}(e)$ are not necessarily the same except at epoch e_0 , and
- $\hat{P}_\Upsilon(e_0) = \hat{P}_{\Upsilon'}(e_0)$,

then the sums of transition costs of $\{\hat{P}_\Upsilon(e)\}_{e \in \{e_0..|E|\}}$ and $\{\hat{P}_{\Upsilon'}(e)\}_{e \in \{e_0..|E|\}}$ are equal and minimum.

Corollary 7.2. (Memoryless Property). *If the route selection reaches a particular route in epoch e_0 , routes at future epochs do not depend on choices made before e_0 .*

PROOF: Suppose two PU activity processes Υ and Υ' are equivalent from epoch e_0 to epoch $|E|$, but not necessarily equivalent for epochs $e < e_0$. Furthermore, let the optimum routes selected under the two PU activity processes at epoch

e_0 be equal, i.e., $(\hat{P}_\Upsilon(e_0) = \hat{P}_{\Upsilon'}(e_0))$. Let us proceed by contradiction and assume that the future route selection depends on selections made at epochs $e < e_0$. This means that routes selected after epoch e_0 in the two optimum sequences $\{\hat{P}_\Upsilon(e)\}_{e \in \mathcal{E}}$ and $\{\hat{P}_{\Upsilon'}(e)\}_{e \in \mathcal{E}}$ can be different, in particular, the sums of transition costs of $\{\hat{P}_\Upsilon(e)\}_{e \in \{e_0..|E|\}}$ and $\{\hat{P}_{\Upsilon'}(e)\}_{e \in \{e_0..|E|\}}$ can be different. This contradicts Theorem 7.1, thus, future route selections cannot depend on the past. \square

Corollary (7.2) introduces a nice property of the minimum maintenance cost routing which is proved to be memoryless in time under specific assumptions.

We highlight here further properties of minimum maintenance cost routing as observed from numerical results. Such properties, given here in form of observations, are leveraged to develop a heuristic routing algorithm in the following sections.

- **Observation 1** - Using minimum hop routing in individual epochs results in extremely high route maintenance costs even though the average path length of optimal routes converges to minimum hop path lengths with increased number of channels per link.

Figure 5 reports the average number of hops obtained when solving RSP-1 compared against the average number of hops obtained solving shortest path problem in each epoch for different numbers of available channels. The shortest path curve is labeled with the percentage increase in route maintenance cost with respect to RSP-1. As clearly demonstrated by the figure, routes created by RSP-1 feature larger number of hops, but on the other hand, the cost for

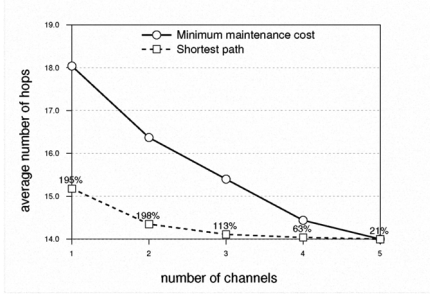


Figure 5: Ave. Number of Hops vs. Number of Channels

maintaining the shortest path is considerably and expectedly higher than the one to maintain RSP-generated routes (up to 200% increase).

• *Observation II* - A good indicator to assess the quality of a link is the average uninterrupted link lifetime under the minimum cost route selection problem.

Previous work in the field [6] considers the average link availability to assess the stability of a given link. Here, we are more interested in the expected *time-to-switch*. Time-to-switch is the time between the selection of a link until a forced switch due to a failure of that link. If a link goes up and down frequently, its use incurs a high maintenance cost even if its average availability is high.

7.2. Routing Metric Design

From the previous section, any consistent routing metric for setting minimum maintenance cost path cannot be based on the hop count only (*Observation I*), but must also account for the continuous lifetime of the links (*Observation 2*). Moreover, the memoryless property (Corollary 7.2) allows us to neglect the past history in the definition of the metric thus focusing on the current epoch and the future ones only.

Leveraging these properties and observations, we propose hereafter a new routing metric for RSP, which is able to capture the “quality” of a given link l between secondary users as far as route maintenance is concerned. Ideally, the link “quality” depends on two factors: the cost of switching from the current link to another link l , C_l^{sw} , (*switching cost*), and the expected cost to repair link l in the future, C_l^{Rep} , (*repair cost*). The former represents the “short-term” investment to maintain the route, the latter the expected “long-term” one.

Each link l can be weighted by the following metric:

$$w_l = \frac{C_l^{sw} + \alpha C_l^{Rep}}{E[TTSl]} \quad (15)$$

Parameter α allows gauging of different cost contributions, and $E[TTSl]$ represents the average time to switch for the link l . Following *Observation II*, the longer the continuous lifetime of a link is, the lower maintenance cost incurs. Therefore, the denominator is used to give lower weights to links available for longer time periods.

The *switching cost* for link l operating on channel c from epoch e to epoch $e + 1$ can be defined as:

$$C_l^{sw} = dist \cdot ch.cost, \quad (16)$$

where the value of $dist$ is computed considering the minimum number of hops between link l and the current route at epoch e , and the value of $ch.cost$ depends on whether link l appears in the current route with the same channel c ($ch.cost = 0$), with a different channel ($ch.cost = C_C$), or not ($ch.cost = C_L$).

The *repair cost* is defined as:

$$C_l^{Rep} = \sum_{n=1}^{\infty} \left(n \cdot \frac{ch.cost}{\langle E[TTL] \rangle_n} \cdot P_n^l \cdot \prod_{j=0}^{n-1} (1 - P_j^l) \right). \quad (17)$$

The term $\langle E[TTL] \rangle_n$ denotes the expected time-to-switch averaged over the n -hop neighborhood. The sum gives the expected cost to pay to repair link l when it becomes unavailable. Different from the centralized solution approach which assumes full knowledge of the PU activity, the proposed metrics only leverage a statistical knowledge of the PU behavior. To this extent, P_n^l denotes the probability to find an available link n -hops away from l which is available as a backup when l goes down:

$$P_n^l = \langle Pr\{\text{link } i \text{ is UP} | \text{link } l \text{ is DOWN}\} \rangle_{i \in N_n^l}$$

where N_n^l denotes the set of links at exactly n -hops from link l . Note that this equation provides an approximated value of the exact probability since an exact prediction on where the new route will go through after link l 's failure is not possible.

We call $X_u(t)$ the ergodic random binary process describing the activity of a PU u : $X_u(t) = 1$ if PU u is active at time t , X_u is a time sample of $X_u(t)$. Given a link k : \mathcal{U}_k is the set of PUs that prevent k from transmitting when they become active. Since $Pr\{\text{link } i \text{ is UP} | \text{link } l \text{ is DOWN}\} = \frac{Pr\{i \text{ is UP} \cap l \text{ is DOWN}\}}{Pr\{l \text{ is DOWN}\}}$, we compute the following probabilities for link in N_n^l .

$$\begin{aligned} Pr\{l \text{ is DOWN}\} &= 1 - Pr\{X_i = 0, \forall i \in \mathcal{U}_l\} = 1 - E[\prod_{i \in \mathcal{U}_l} (1 - X_i)] \\ &= \sum_{n=1}^{|\mathcal{U}_l|} (-1)^{n+1} \sum_{c \in \mathcal{C}^{|\mathcal{U}_l|}} E[\sum_{i \in c} X_i] \\ Pr\{i \text{ is UP} \cap l \text{ is DOWN}\} &= Pr\{\cap_{j \in \mathcal{U}_i} X_j = 0 \cap \cup_{k \in \mathcal{U}_l} X_k = 1\} = \\ &= E[\prod_{j \in \mathcal{U}_i} (1 - X_j) (1 - \prod_{k \in \mathcal{U}_l} X_k)] = \\ &= \sum_{n=1}^{|\mathcal{U}_i|} ((-1)^n \sum_{c \in \mathcal{C}^{|\mathcal{U}_i|}} (E[\prod_{j \in c} X_j] - E[\prod_{j \in c} X_j \prod_{k \in \mathcal{U}_l} X_k])) \end{aligned}$$

where \mathcal{C}_s^g is the set of combinations of s PUs in groups of g elements. If the correlation among PUs with overlapping interference areas is known, then the value of $E[\prod_i X_i]$ can be computed exactly. When no correlation information is available, the best possible approximation is splitting the expected value of the X_i product into the product of their expected values $E[X_i]$, i.e., the product of average activities of each single PU ($E[X_i] = Pr\{X_i = 1\}$). This is

Table 1: Optimality Gap of Algorithm 2.

PU act.	Optimality Gap				
	1 ch.	2 ch.	3 ch.	4 ch.	5 ch.
0.1	0.22	0.10	0.04	0.02	0.01
0.2	0.34	0.28	0.14	0.06	0.05
0.3	0.32	0.35	0.14	0.09	0.05
0.4	0.36	0.36	0.19	0.17	0.07

equivalent to considering PU activity processes independent of each other.

Finally, the average *time-to-switch* is determined considering the mean length of the uninterrupted availability periods of link l . Since PU activity processes are usually ergodic, the estimate of $E[\text{TTS}]$ value can be computed averaging over past observations.

7.3. Routing Algorithm Structure

We use the routing metric provided in the previous section in a heuristic to solve the minimum maintenance cost routing problem when exact information on PU activity is not available. The main idea of the algorithm is to assign weights to each link in every epoch based on the metric given in Equation (15) and iteratively find the shortest source-destination route from epoch to epoch. The algorithm is composed of the following steps. In Step 1, the algorithm

Algorithm 2 Routing Algorithm

```

1: Initialize
2: for  $e \in \mathcal{E}$  do
3:   AssignWeights( $e, e - 1$ )
4:   GetShortestPath( $e$ )
5: end for

```

computes the shortest path between the intended source and destination, assuming all PUs are inactive (initialization phase). After that, for each epoch, the algorithm assigns weights to SU links according to metric in Equation (15), and then applies the Bellman-Ford algorithm to find the shortest path at epoch e .

Figures 6(a), 6(b), and 6(c) report the route maintenance cost obtained running our routing heuristic when varying PU activity, the number of epochs, and the number of stages, respectively. As observed in these figures, the maintenance cost obtained through the heuristics has a similar trend and behavior as the minimum maintenance cost obtained in Section 6 (See Figures 4(a), 4(f), and 4(c)). Table 1 reports the optimality gap between results obtained using the heuristic algorithm (without using exact future PU activity knowledge) and optimal results (using the centralized optimization algorithm) as the PU activity increases. The optimality gap is computed as $(\frac{C_{ALGO} - C_{OPT}}{C_{OPT}})$, where C_{ALGO} is the cost obtained using Algorithm 2, and C_{OPT} is the optimal cost obtained using Algorithm 1. The error trend is related to the number of changes faced by selected routes, and the gap increases when the probability of a change in the selected route is higher due to a higher PU activity probability.

Finally, it is worth assessing the quality of the routing metric proposed in Eq. (15) with respect to classical metrics

of route stability based on the average link availability [6]. To this extent, Figure 7 compares the solution of Algorithm 2 when these two metrics are used to weigh the links among the SUs epoch by epoch. As shown in the Figure (and anticipated in Observation 2), routing according to the average link availability induces considerably higher route maintenance cost (up to 4 times).

8. Conclusions

In this work, a theoretical outlook on the problem of routing secondary user flows in a CRN is provided. Defining optimality based on the cost of maintaining a connection as a sequence of paths, optimal centralized as well as distributed algorithms are proposed to address the problem when full PU activity is available and not available, respectively. To the best of our knowledge, this is the first attempt to analyze routing multi-hop CRNs considering route maintenance cost. Properties of the problem are also formally introduced. We are already investigating the capacity-constrained version of this problem, which we can prove to be NP-Hard. In our future work, we plan to study the relation of PU activity prediction accuracy and its effects on the heuristic solution accuracy.

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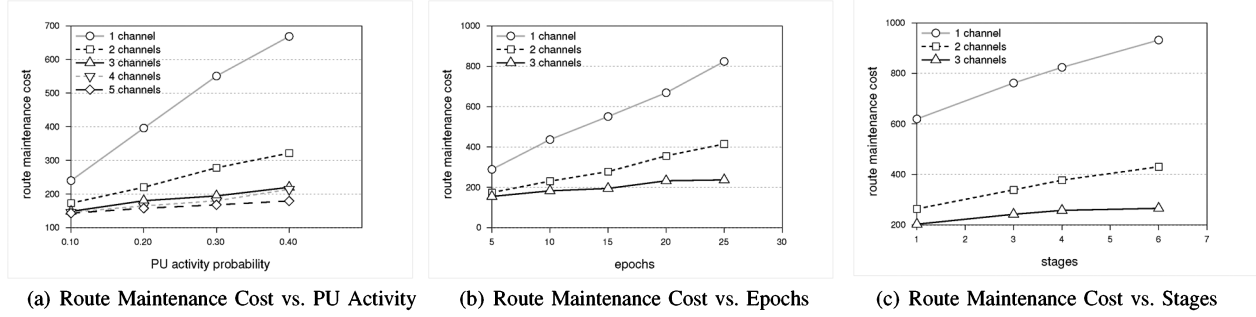


Figure 6: Route Maintenance Cost when Adopting Algorithm 2.

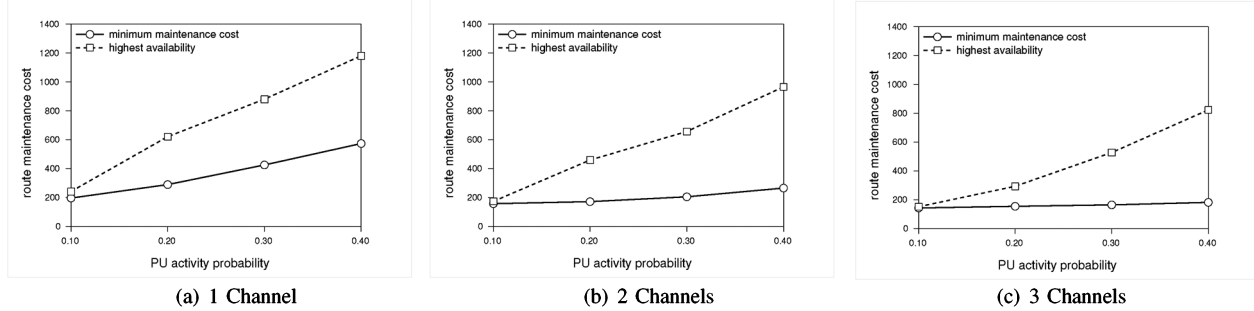


Figure 7: Route Maintenance Cost vs. PU Activity when varying the routing metric.

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Appendix

We define a graph $\mathbb{G}(\mathcal{V}, \mathcal{A})$ with a metric $\mathcal{C} : \mathcal{A} \rightarrow \mathbb{R}^+$ as follows.

- $\mathcal{V} \triangleq \{P_i(e), \forall e \in \mathcal{E}\} \cup \{P'_0, P''_0\}$. $P_i(e)$ represents route P_i at epoch e . P'_0 and P''_0 are two virtual vertices representing, respectively, the initial state in which no route is selected and the end of the time during which nodes s and d must stay connected.
- $\mathcal{A} \triangleq \{(P_i(e), P_j(e+1)) : \forall P_i(e) \in \mathcal{V} \setminus \{P'_0, P''_0\}\} \cup \{(P'_0,$

- $P_i(1)) : \forall P_i \in \mathcal{P}_{sd}(1)\} \cup \{(P_i(|\mathcal{E}|), P''_0) : \forall P_i \in \mathcal{P}_{sd}(|\mathcal{E}|)\}$. Arcs in the first set express route changes between epochs e and $e+1$. Arcs in the second set express the choice of the initial route. Finally, arcs in the last set join vertices $P_i(|\mathcal{E}|)$ to the end vertex P''_0 .
- $\mathcal{C} : \mathcal{A} \rightarrow \mathbb{R}^+$ is defined as: $\mathcal{C}[(P_i(e), P_j(e+1))] = C[P_i, P_j]$, $\mathcal{C}[(P'_0, P_i(1))] = C[\emptyset, P_i]$, and $\mathcal{C}[(P_i(|\mathcal{E}|), P''_0)] = 0$

Note that cost $C[\emptyset, P_i]$ is the cost of the selected initial route $P_i = \hat{P}(1)$. Arcs $(P_i(|\mathcal{E}|), P''_0)$ have length 0 as no route must be arranged after epoch $|\mathcal{E}|$, links of the last route from s to d can simply be destroyed. In addition, $\mathcal{C}[(P_i(e), P_i(e+1))] = C[P_i, P_i] = 0$ as route P_i does not change between epochs e and $e+1$, that is, no new links must be activated. A sequence of selected routes during epochs, $\{\hat{P}(e)\}_{e \in \mathcal{E}}$, can be mapped into a path \mathfrak{P}^* in \mathbb{G} , and vice versa. The mapping function \mathfrak{F} is defined as: $P_i = \hat{P}(1) \Leftrightarrow (P'_0, P_i(1)) \in \mathfrak{P}^*$, $P_i = \hat{P}(|\mathcal{E}|) \Leftrightarrow (P_i(|\mathcal{E}|), P''_0) \in \mathfrak{P}^*$, and $P_i = \hat{P}(e) \wedge P_j = \hat{P}(e+1) \Leftrightarrow (P_i(e), P_j(e+1)) \in \mathfrak{P}^*$. Thus, (RSP) reduces to find a shortest path from vertex P'_0 to vertex P''_0 in graph \mathbb{G} . Suppose \mathfrak{P} is a shortest path from vertex P'_0 to vertex P''_0 in \mathbb{G} . The length of such a path is equal to the cost of the corresponding route selection sequence. Since \mathfrak{P} is a shortest path, its length is minimum. This allow us to say that the sum of the initial route $\hat{P}(1)$'s setup cost and all transition costs between two epochs given by \mathfrak{P} is minimum. A node deployment and the communication range (scenario) define graph \mathbb{G} 's vertices and weights, while arcs in \mathbb{G} depend on the PU activity process. Therefore, PU activity processes Υ and Υ' induce graphs \mathbb{G}_Υ and $\mathbb{G}_{\Upsilon'}$, respectively. Since Υ and Υ' are equivalent from epochs e_0 to $|\mathcal{E}|$, the sub-graph of \mathbb{G}_Υ involving only vertices in $\{P_i(e) : e \geq e_0\} \cup P''_0$ is equal to the same sub-graph of $\mathbb{G}_{\Upsilon'}$. This common sub-graph is denoted by $\mathbb{G}^{[e_0, |\mathcal{E}|]}$. Given two sub-paths, $\mathfrak{X}_\Upsilon \in \mathbb{G}_\Upsilon$ and $\mathfrak{X}_{\Upsilon'} \in \mathbb{G}_{\Upsilon'}$, of two shortest paths going through $\mathbb{G}^{[e_0, |\mathcal{E}|]}$, for hypotheses, they cross, actually start, in $P_x(e_0)$, where $P_x = \hat{P}_\Upsilon(e_0) = \hat{P}_{\Upsilon'}(e_0)$, and end in P''_0 . Since they are sub-paths of shortest paths on a common sub-graph, the length of \mathfrak{X}_Υ from $P_x(e_0)$ to P''_0 must be equal to the length of $\mathfrak{X}_{\Upsilon'}$ from $P_x(e_0)$ to P''_0 and be minimum. Therefore, adding the costs on arcs of each sup-path, we have

$$\sum_{i=e_0}^{|\mathcal{E}|-1} C[\hat{P}_\Upsilon(e), \hat{P}_\Upsilon(e+1)] = \sum_{i=e_0}^{|\mathcal{E}|-1} C[\hat{P}_{\Upsilon'}(e), \hat{P}_{\Upsilon'}(e+1)]$$

and is minimum. \square