

Maximizing Social Welfare in Operator-based Cognitive Radio Networks under Spectrum Uncertainty and Sensing Inaccuracy

Shuang Li, Zizhan Zheng, Eylem Ekici and Ness B. Shroff

Abstract—In Cognitive Radio Networks (CRNs), secondary users (SUs) are allowed to opportunistically access the unused/under-utilized channels of primary users (PUs). To utilize spectrum resources efficiently, an auction scheme is often applied where an operator serves as an auctioneer and accepts spectrum requests from SUs. Most existing works on spectrum auctions assume that the operator has perfect knowledge of PU activities. In practice, however, it is more likely that the operator only has statistical information of the PU traffic when it is trading a spectrum hole, and it is acquiring more accurate information in real time. In this paper, we distinguish PU channels that are under the control of the operator, where accurate channel states are revealed in real-time, and channels that the operator acquires from PUs out of its control, where a sense-before-use paradigm has to be followed. Considering both spectrum uncertainty and sensing inaccuracy, we study the social welfare maximization problem for serving SUs with various levels of delay tolerance. We first model the problem as a finite horizon Markov decision process when the operator knows all spectrum requests in advance, and propose an optimal dynamic programming based algorithm. We then investigate the case when spectrum requests are submitted online, and propose a greedy algorithm that is 1/2-competitive for homogeneous channels and is comparable to the offline algorithm for more general settings. We further extend the online algorithm to an online auction scheme, which ensures incentive compatibility for the SUs and also provides a way for trading off social welfare and revenue.

I. INTRODUCTION

With the ever-growing demand for wireless spectrum, Cognitive Radio Networks (CRNs) have been proposed to better utilize spectrum holes in wireless networks. In CRNs, secondary users (SUs) are allowed to opportunistically access the channels of primary users (PUs). To utilize spectrum resources efficiently, an auction framework is often applied where an operator serves as an auctioneer and accepts requests from SUs. These frameworks are implemented via a resource allocation and a payment scheme with the objective of maximizing either social welfare or revenue [2], [4]–[6], [16].

Most existing works on spectrum auctions, however, assume that the operator has perfect knowledge of PU activities in a given period of time. They ignore the uncertainty of channel states caused by the uncertain and frequent PU usage. For instance, in cellular networks, the subscribers access channels

at their own will and they do not make any reservation for the spectrum usage. Hence, these existing auction schemes are mainly applicable to spectrum resources that tend to be available for relatively long periods of time. For instance, the interval between two adjacent auctions is assumed to be 30 minutes or longer in [4]. However, to allow more efficient spectrum utilization and relieve spectrum congestion, spectrum holes at smaller time scales need to be explored. A straightforward extension of current approaches to this more dynamic environment would require auctions to be conducted frequently, which would incur high communication and management overhead. A more reasonable approach is to again consider a relatively long period of time, where the operator only has statistical information of the PU traffic when trading spectrum holes. More accurate information is acquired later in real-time. *Therefore, an auction scheme that takes spectrum uncertainty into account is needed.*

To further improve spectrum utilization, besides trading spectrum holes that are fully under the control of the operator, as commonly assumed in the spectrum auction literature, the operator may choose to acquire licensed channels out of its control to further improve social welfare or revenue. To avoid interference with PUs, a *sense-before-use* paradigm must be followed in this case. The operator must first identify spectrum holes in a channel, e.g., by coordinating SUs to sense the channel, before allocating the holes to SUs. While spectrum sensing has been extensively studied in the CRN literature [8], [11], [12], the joint problem of sensing and spectrum auction remains unexplored.

In this paper, we propose a spectrum allocation framework that takes both *spectrum uncertainty* and *sensing inaccuracy* into account. In particular, we consider two types spectrum resources: PU channels that are under the control of the operator, and the channels that the operator acquires from PUs out of its control. In practice, wireless service providers (WSP) act as operators, and they may cover areas that almost completely overlap. SUs registered with one of them may access spectrum from other WSPs as will be introduced in our model. In both types of channels, PU traffic on each channel is assumed to follow a known i.i.d. Bernoulli distribution. For the first type of channels, the real-time channel state can be learned accurately by the operator. For the second type of channels, a sense-before-use paradigm must be followed, where a collision with the PU traffic due to sensing inaccuracy incurs a penalty.

S. Li is with the Department of CSE; Z. Zheng and E. Ekici are with the Department of ECE; N. B. Shroff is with the Department of ECE and CSE at The Ohio State University. Emails: li.908@osu.edu, {zhengz, ekici, shroff}@ece.osu.edu. This work has been funded in part by the Army Research Office MURI award W911NF-08-1-0238 and National Science Foundation awards CNS-1065136, CNS-1012700, and CCF-0914912.

Using a fixed set of channels of each type, we study the joint spectrum sensing and allocation problem to serve spectrum requests with arbitrary valuations and arbitrary levels of delay tolerance. The objective of the operator is to maximize social welfare, i.e., the total valuations obtained from successfully served requests minus the cost due to collisions. We consider both the scenario where the operator knows all spectrum requests in advance, and the setting when spectrum requests are submitted online. While our online setting is similar to the online spectrum auction schemes considered in [3], [15], the key difference is that sensing inaccuracy is not considered in these existing works. Hence, the approaches in [3], [15] can only be applied to cases where accurate real time channel states are obtainable, which is not always the case.

Our contributions can be summarized as follows:

- We model the joint sensing and spectrum allocation problem as a finite horizon Markov decision process when all spectrum requests are revealed to the operator offline, i.e., ahead of time. We develop an optimal dynamic programming based algorithm, which serves as a baseline for the achievable social welfare.
- We propose a greedy algorithm for the case when spectrum requests are submitted online. We prove that the online algorithm is 1/2-competitive for homogeneous channels, and we show that it achieves performance comparable to the offline algorithm for the more general heterogeneous channel case by numerical results.
- We further extend the online algorithm by proposing an online auction scheme, which ensures incentive compatibility for SUs and also provides a way for trading off social welfare and revenue using a reservation price.

The paper is organized as follows: The system model and problem formulation are introduced in Section II. Our solutions to the problem with offline and online requests are presented in Sections III and IV, respectively. Our online auction scheme is then discussed in Section V. In Section VI, numerical results are presented to illustrate the performance of the greedy online algorithm in general cases, and the tradeoff between social welfare and revenue. We conclude the paper in Section VII.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a cognitive radio network with a single operator and multiple SUs registered with it (see Figure 1). The operator manages multiple orthogonal channels and controls the corresponding network composed of PUs. We focus on downlink transmission at the operator with power control. A time slotted system is considered with all PU and SU transmissions synchronized. All SUs are assumed to be in the interference range of each other and that of PUs, hence, each channel can be assigned to at most one SU at any time when it is not used by PUs. In this paper, we focus on the temporal reuse of spectrum, and we will consider spatial reuse [3], [9], [15] to further improve allocation efficiency in the future.

The spectrum pool consists of two types of channels, those managed by the operator and those that are not. The operator is

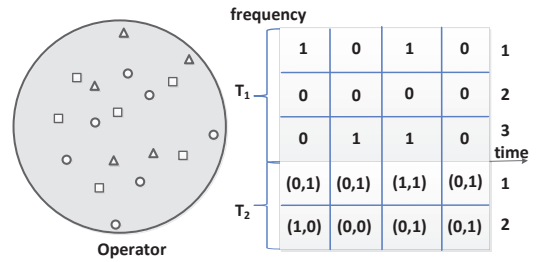


Fig. 1. System model of the CRN. In the left figure, small circles are SUs, squares represent PUs registered at the operator, and triangles are PUs out of the operator's control. The big circle is the coverage area of the operator. The right figure shows the availability of channels in T_1 and T_2 . For T_1 channels: 0 means idle state and 1 means busy state. For T_2 channels, the first element represents the actual state (0: idle; 1: busy) and the second element represents the sensed state (0: sensed idle; 1: sensed busy).

aware of the downlink activity of its own PUs at the beginning of each time slot. The set of the spectrum bands¹ managed by the operator is denoted by T_1 . However, the activities of PUs not managed by the operator are unknown. Bands accessed by these PUs are denoted by T_2 . To access bands in T_2 , SUs cooperatively sense them and report their sensing results to the operator. The operator then makes a fusion decision on the activities of bands in T_2 and selects a subset of channels sensed idle to serve the SUs. We only consider the set of PUs located in the coverage area of the operator so that all SUs in the system have the cognitive capability and can sense spectrum in T_2 . We assume that the sensing cost is low and even negligible. In practice, wireless service providers (WSP) act as operators, and they may cover areas that almost overlap. SUs registered with one of them may access spectrum from other WSPs as introduced in our model.

We assume that the spectrum bands in T_1 and T_2 have the same capacity, which is normalized to 1. PU activities on these channels follow an i.i.d. Bernoulli distribution in each time slot. For instance, in Figure 1, there are three channels in T_1 and two channels in T_2 . In time slot 1, channels 2 and 3 in T_1 are idle and channel 1 in T_2 is idle. However, channel 1 in T_2 is sensed busy and it will not be allocated. Also, channel 2 in T_2 is incorrectly sensed to be idle and scheduling a request on this channel will lead to a collision. We let $\pi_1(i)$ denote the probability that channel i in T_1 is idle and $\pi_2(j)$ the probability that channel j in T_2 is idle. We also assume that the prior distribution of the PU activity is accurately acquired over time. We assume that state changes occur at the beginning of a time slot. Let $C \triangleq |T_1| + |T_2|$ denote the total number of channels, which remains constant over time.

The availabilities of channels in T_1 and T_2 at t are denoted by binary vectors $\vec{I}_1(t) = (I_1^1(t), \dots, I_1^k(t), \dots)$ and $\vec{I}_2(t) = (I_2^1(t), \dots, I_2^j(t), \dots)$, respectively, where 0 represents idle and 1 represents busy states. Moreover, $\vec{I}_2^s(t)$ denotes the sensed availabilities of channels in T_2 at t . Let $P_f(k)$, $k \in T_2$, denote the **probability of false alarm** for channel k , i.e., the probability that SUs cooperatively sense channel k to be busy given that it is actually idle. Let $P_m(k)$ represent the **probability of misdetection** for channel k , i.e.,

¹We use channel and spectrum band interchangeably.

TABLE I
NOTATION LIST

Symbol	Meaning
\mathcal{N}	Set of spectrum requests submitted to the operator
T_1	Set of channels managed by the operator
T_2	Set of channels not managed by the operator
$\pi_1(i)$	Probability that channel i in T_1 is idle
$\pi_2(j)$	Probability that channel j in T_2 is idle
C	The total number of channels ($C = T_1 + T_2 $)
$\vec{I}_1(t)$	Availabilities of channels in T_1 at t
$\vec{I}_2(t)$	Availabilities of channels in T_2 at t
$\vec{I}_2^s(t)$	Sensed availabilities of channels in T_2 at t
$P_f(k)$	Probability of false alarm for channel $k \in T_2$
$P_m(k)$	Probability of misdetection for channel $k \in T_2$
$P_I(k)$	Probability that channel k is sensed idle
$P_0(k)$	Probability of channel k being idle given that it is sensed idle
a_i	Earliest service time for request i
d_i	Deadline of request i
w_i	Valuation of the request i
H	The time period where spectrum allocation has to be made
r	Maximum number of outstanding requests in the system at any time
Q	Penalty price per collision

the probability that SUs cooperatively sense channel k to be idle given that it is actually busy. Our problem formulation and solutions are independent of the cooperative sensing scheme used. We further define $P_I(k)$ as the probability that channel k is sensed idle and $P_0(k)$ as the conditional probability of channel k being idle given that it is sensed idle. Note that $P_I(k) = \pi_2(k)(1 - P_f(k)) + (1 - \pi_2(k))P_m(k)$ and $P_0(k) = \frac{\pi_2(k)(1 - P_f(k))}{P_I(k)}$. We assume that $P_f(k)$ and $P_m(k)$ are constant for any channel $k \in T_2$, which occurs e.g. when SUs are static in the system. Some of our technical results apply to the special case when all channels in T_2 are *homogenous*, that is, when the channels have the same $\pi_2(i)$, $P_m(i)$ and $P_f(i)$. Thus, they also have the same $P_0(i)$ and $P_I(i)$.

We assume each spectrum request is for a single time-frequency chunk, i.e., a single time slot of any channel in T_1 or T_2 . Each request i submitted at time t is of the form (a_i, d_i, w_i) , where $a_i \geq t$ is the required service starting time, d_i is the deadline, at the beginning of which request i leaves the system, and w_i is the valuation of request i , which will be added to the social welfare if request i is served by d_i . We denote the set of requests by $\mathcal{N} = \{1, \dots, N\}$. $H = \max_{i \in \mathcal{N}} d_i - \min_{i \in \mathcal{N}} a_i$ denotes the time period spectrum allocation needs to be made, and $\min_{i \in \mathcal{N}} a_i$ is normalized to 1. The maximum number of outstanding requests in the system at any time is denoted as r . Table I summarizes the notations used in the paper.

We are interested in maximizing the social welfare of the operator and the SUs in the system, which is defined as the total valuations from the requests successfully served (by their deadlines) minus the collision cost to channels in T_2 . Let Q denote the penalty incurred per collision. Let $x_{il}(t)$ ($i \in \mathcal{N}$, $l \in T_1 \cup T_2$, $t = 1, \dots, H$) denote the allocation indicator: $x_{il}(t) = 1$ if request i is allocated to channel l at t ; $x_{il}(t) =$

0 otherwise. Let y_i denote the service indicator: $y_i = 1$ if request i is served by d_i ; $y_i = 0$ otherwise. The social welfare maximization problem is then formulated as follows, where $Z(\cdot)$ denotes the number of 0 elements in a vector:

Problem (A):

$$\max_{\mathbf{x}, \vec{y}} E_{\mathbf{I}_1, \mathbf{I}_2, \mathbf{I}_2^s} \left[\sum_{i \in \mathcal{N}} y_i w_i - Q \sum_{i \in \mathcal{N}} \sum_{l \in T_2} \sum_{t=1}^H x_{il}(t) I_2^l(t) \right]$$

$$\text{s.t. } \sum_{t=a_i}^{d_i} \left(\sum_{l \in T_1} x_{il}(t) + \sum_{k \in T_2} x_{ik}(t)(1 - I_2^k(t)) \right) \geq y_i, \quad (1)$$

for all $i \in \mathcal{N}$

$$\sum_{i \in \mathcal{N}} \sum_{l \in T_1} x_{il}(t) \leq Z(\vec{I}_1(t)), \text{ for all } t = 1, \dots, H \quad (2)$$

$$\sum_{i \in \mathcal{N}} \sum_{k \in T_2} x_{ik}(t) \leq Z(\vec{I}_2^s(t)), \text{ for all } t = 1, \dots, H, \quad (3)$$

where $\mathbf{x} = (x_{il}(t))_{i,l,t}$, $\vec{y} = (y_i)_{i \in \mathcal{N}}$, $\mathbf{I}_1 = (\vec{I}_1(t))_{t=1, \dots, H}$, $\mathbf{I}_2 = (\vec{I}_2(t))_{t=1, \dots, H}$, $\mathbf{I}_2^s = (\vec{I}_2^s(t))_{t=1, \dots, H}$. The cost $Q \sum_{i \in \mathcal{N}} \sum_{l \in T_2} \sum_{t=1}^H x_{il}(t) I_2^l(t)$ takes into account the current availabilities of channels in T_2 . Inequality (1) reflects the relationship between the allocation indicator $x_{il}(t)$ and the service indicator y_i . Inequality (2) guarantees that a channel in T_1 will not be allocated unless it is observed idle. Likewise, Inequality (3) guarantees that a channel in T_2 will not be allocated unless it is sensed idle.

The challenges of Problem (A) are threefold: 1) The requests are uncertain since they may be submitted at different time slots; 2) Spectrum availabilities of T_1 and T_2 in the future are not known at the current time slot; 3) Sensing is not accurate for channels in T_2 . In the following, we propose an offline optimal solution in Section III and an online solution in Section IV. We define the offline algorithm as an algorithm that decides the channel allocation for outstanding requests in each time slot with only the observed availabilities of channels in T_1 and sensed availabilities in T_2 of the current slot. All requests, including future arrivals, are assumed to be known. For instance, SUs submit their requests at the beginning of H . The operator then knows the full arrival information. In each time slot, the operator has to make channel allocation decisions based on the observed availabilities of its own channels and the sensed availabilities of channels managed by other operators. The only difference between online and offline algorithms is that online algorithm does not assume the full arrival information to be known ahead of time. *Both algorithms are designed under the challenges of spectrum uncertainty and sensing inaccuracy.*

III. OPTIMAL OFFLINE ALGORITHM

In this section, we study Problem (A) under the assumption that the operator has full knowledge of the spectrum requests in advance. By our assumptions on channel statistics, the problem can be modeled as a finite horizon Markov Decision Process (MDP) [14]. In this section, we propose an optimal dynamic programming based solution to the problem. We start

with the simple case where $T_1 = \emptyset$ and all the channels for serving SUs are in T_2 , which models the case where all the channels owned by the operator are overloaded by PU traffic. Then, we proceed with the general case where both T_1 and T_2 channels are available in the system. In each time slot, based on the knowledge of the spectrum requests and the current channel state, the operator makes a joint decision including 1) which subset of requests to schedule; 2) which subset of channels to allocate; 3) which request to assign to which channel. In our solution, we consider all possible scenarios for each time slot and find the schedule that maximizes the expected social welfare. We show that our algorithm has a complexity of $O(2^r 3^C (\max\{C, r\})^{\min\{C, r\}} HCr)$. When (a_i, d_i) of requests do not have a dense overlap, i.e., $r = O(\log N)$ where N is the total number of requests in $[1, H]$, our algorithms are of polynomial complexity. We also provide important structural properties which further substantially reduce the complexity and help design a simple online greedy algorithm.

We first define $F(D, t)$ as the maximum expected social welfare from the beginning of slot t till the end of slot H given that the set of outstanding requests at time t is D . The expectation takes into account all possible channel realizations and sensing results. We define $F(D, H+1) = 0, \forall D$. Our goal is to calculate $F(\{i : i \in \mathcal{N}, a_i = 1\}, 1)$ (Algorithm III.1). We calculate it backward from $t = H$ till $t = 1$ is reached since requests requiring service in future time slots have an impact on the current optimal scheduling decision. Note that at any time t , it is sufficient to consider D in $F(D, t)$'s for being any subset of the requests that satisfy $a_i \leq t < d_i$.

A. With no available channels in T_1

When no channel is in T_1 , the spectrum bands managed by the operator, SUs can only be served by channels in T_2 . SUs may request spectrum in arbitrary time slots. The success of serving request i contributes w_i to the social welfare while the assignment of a request to a busy channel causes collisions, incurring a penalty of Q .

We define $X(D, S, t)$ as the maximum expected social welfare from t ($t = 1, \dots, H$) to the end of the period, given that the set of outstanding requests is D and channels in S are sensed idle ($S \subseteq T_2$). The expectation is taken over all possible realizations of \mathbf{I}_2 . Then,

$$X(D, S, t) = \max_{\mathbf{x}(t)} \left[\sum_{S_1 \subseteq S} \prod_{l \in S_1} P_0(l) \prod_{m \in S \setminus S_1} (1 - P_0(m)) \right. \\ \left. (W(D, S, S_1, \mathbf{x}(t), t) + F(D', t+1)) \right], \quad (4)$$

where $W(D, S, S_1, \mathbf{x}(t), t)$

$$= \sum_{n \in D} \left[w_n \sum_{k \in S_1} x_{nk}(t) - Q \sum_{k \in S \setminus S_1} x_{nk}(t) \right]$$

is defined as the social welfare achieved in time slot t , for a given D , the set of outstanding requests; $S \subseteq T_2$, the set of channels sensed idle; $S_1 \subseteq S$, the set of channels that are sensed idle and actually idle; and $\mathbf{x}(t)$, the channel allocation at t . Recall that $x_{nk}(t)$ is the allocation indicator used to determine whether the SU is served by this allocation. We

form D' based on D as follows: If request m is allocated to channels in S_1 , then remove m from D , which means it is served and the request no longer exists. If request n satisfies $a_n = t + 1$, then add n to D , which indicates it is a new request. Among the remaining requests, those that expire at the beginning of $t + 1$ are removed from D .

Based on $X(D, S, t)$, we calculate $F(D, t)$ as follows. The expectation in $F(D, t)$ in the form of the product of $P_I(l)$ and $(1 - P_I(m))$ takes into account all realizations of \mathbf{I}_2 .

$$F(D, t) = \sum_{S \subseteq T_2} \prod_{l \in S} P_I(l) \prod_{m \in T_2 \setminus S} (1 - P_I(m)) X(D, S, t) \quad (5)$$

In Algorithm III.1, our objective $F(\{i : i \in \mathcal{N}, a_i = 1\}, 1)$ is calculated by dynamic programming. It first calculates the maximum social welfare and the corresponding schedule for each time slot, and then specifies the real time operations. Lines 1-5 calculate $F(D, t)$ backward from H to 1 given the initial condition defined earlier $F(D, H+1) = 0$ for all D . Line 4 calculates the optimal scheduling policy for time t given D , the request set; S , the set of channels sensed idle; and S_1 , the set of channels sensed idle and actually idle, according to Equation (4). The value of $F(D, t)$ is updated in Line 5 according to Equation (5). The complexity of the Equation (5) is $O(3^{|T_2|} (\max\{|T_2|, r\})^{\min\{|T_2|, r\}} |T_2| r)$: The number of possible channels realizations is $3^{|T_2|}$ since different social welfare values will be generated in the cases where the channel is sensed idle but actually busy, it is sensed idle and actually idle, and all other cases. It takes at most $(\max\{|T_2|, r\})^{\min\{|T_2|, r\}}$ combinations to find the optimal \mathbf{x} in Equation (4). The complexity for the calculation of $W(D, S, S_1, \mathbf{x}(t), t)$ is $O(|T_2| r)$. On the other hand, given t , the number of possible argument combinations in $F(D, t)$ is $O(2^r H)$ by assumption. The total time complexity is $O(2^r 3^C (\max\{C, r\})^{\min\{C, r\}} HCr)$. Note that C is assumed to be a constant in our model. When there are only homogeneous T_2 channels, allocation to different channels in T_2 makes no difference. Then, we can replace $(\max\{C, r\})^{\min\{C, r\}}$ with 2^r , resulting in a complexity of $O(2^{2r} 3^C HCr)$.

B. With at least one channel in T_1

With channels in T_1 , requests can be served by channels in both T_1 and T_2 . Since the channel availabilities of T_1 are known at the beginning of each time slot, they can serve SU requests without any cost. Thus, once observed idle, channels in T_1 could be assigned to requests so as to maximize the sum of valuations. Our focus is still the allocation of channels in T_2 if they are sensed idle. The differences from the case without T_1 channels are as follows: 1) All realizations of \mathbf{I}_1 needs to be taken into account; 2) In a schedule $\mathbf{x}(t)$, assignment of any request to a channel in T_1 causes no cost. The detailed algorithm can be found in our online technical report [10].

Correspondingly, the algorithm for the general case is similar to Algorithm III.1 except that in Line 4, the allocations of T_1 channels is included and in Line 5, $F(D, t)$ needs to take into account all realizations of \mathbf{I}_1 . Following a similar argument as in the case where $|T_1| = 0$, the total time complexity is still $O(2^r 3^C (\max\{C, r\})^{\min\{C, r\}} HCr)$.

Algorithm III.1 Dynamic Programming based Optimal Algorithm for Social Welfare Maximization

Offline computation

- 1: **for** $t = H$ to 1 **do**
- 2: **for all** $D \subseteq \{i : a_i \leq t < d_i\}$ **do**
- 3: **for all** $S \subseteq T_2$ **do**
- 4: $X(D, S, t) \leftarrow \max_{\mathbf{x}(t)} \left[\sum_{S_1 \subseteq S} \prod_{l \in S_1} P_0(l) \prod_{m \in S \setminus S_1} (1 - P_0(m)) \left(\sum_{n \in D} \left[w_n \sum_{k \in S_1} x_{nk}(t) - Q \sum_{k \in S \setminus S_1} x_{nk}(t) \right] + F(D', t + 1) \right) \right]$
- 5: $F(D, t) \leftarrow \sum_{S \subseteq T_2} \prod_{l \in S} P_l(l) \prod_{m \in T_2 \setminus S} (1 - P_l(m)) X(D, S, t)$

Real-time scheduling

- 1: At each time slot t with a set of requests D that are currently in the system and a set of channels S that are sensed idle, allocate channels to the requests based on the schedule $\mathbf{x}(t)$ that maximizes $X(D, S, t)$.
-

C. Discussion

In this section, we prove some structural properties of the optimal solution, which helps to further reduce the time complexity of the algorithm and also provides insight to the design of the online algorithm discussed in Section IV. Note that at any time t , for an active request i and a channel $k \in T_2$ that is sensed idle, $P_0(k)w_i - Q(1 - P_0(k))$ is the expected immediate social welfare contributed by request i if i is assigned to k in the current slot. Proposition III.1 shows that a non-negative immediate social welfare is necessary for request i to be served by channel k in the optimal solution, which turns out to be a sufficient condition in certain scenario as stated in Proposition III.2, as well.

Proposition III.1. *At any time t , if a request i is scheduled on channel $k \in T_2$ in Algorithm III.1, then $P_0(k)w_i \geq Q(1 - P_0(k))$.*

Proof: See our online technical report [10]. ■

Proposition III.2 shows that the condition $P_0(k)w_i > Q(1 - P_0(k))$ is also sufficient for a request to be scheduled for homogenous channels. To simplify notation, we drop the index for channel related parameters for the homogeneous case.

Proposition III.2. *In a system with no channels in T_1 and homogeneous channels in T_2 , if there exists at least one request i that satisfies $P_0 w_i > Q(1 - P_0)$ in a slot t and there is at least one channel sensed idle, then in the optimal solution at least one of the requests satisfying this condition will be scheduled, for all t .*

Proof: We provide a proof sketch due to page limits. The detailed proof is in our online technical report [10]. It is equivalent to show that for any schedule at time t , adding a request i with $P_0 w_i > Q(1 - P_0)$ to it will achieve at least the same social welfare as before. We compare the expected social welfare of these two schedules, and observe that it is sufficient to prove that $P_0 F(\{i\}, t) \leq w_i P_0 - Q(1 - P_0)$ for all t . We show this result by induction. ■

Based on these propositions, we can reduce the candidate set of requests for scheduling in each time slot. For instance, no requests should be scheduled if $P_0(k)w_i \leq Q(1 - P_0(k))$ for all existing requests i and all k sensed idle. Also, in a system with no channels in T_1 and homogeneous channels in T_2 , the candidate set is composed of all requests that satisfy

$P_0(k)w_i > Q(1 - P_0(k))$. We utilize these propositions in the design of our online algorithm.

IV. ONLINE ALGORITHM

In this section, we introduce a greedy online algorithm (Algorithm IV.1) that does not need future arrival information. For systems where requests are not submitted ahead of the required service starting time a_i , the online algorithm makes decisions based on the information available in the current slot. An online algorithm for a maximization problem is α -competitive ($\alpha \leq 1$) if it achieves at least a fraction c of the objective value of an optimal offline algorithm for any finite input sequence [1], where c is called a *competitive ratio*. We show that the greedy online algorithm is 1/2-competitive for homogeneous T_2 channels in Proposition IV.1, and achieves performance comparable to the optimal offline algorithm for the more general heterogenous channel case by numerical results (see Figure 2(b)).

In Algorithm IV.1, the main idea is to (greedily) offer requests with higher valuation channels with better quality. We define $c_k \triangleq Q(1 - P_0(k))/P_0(k)$, which is the expected cost of serving one request on channel k (will be shown in Lemma IV.1). Note that $c_k = 0$ for $k \in T_1$. Lines 2 and 3 sort channels sensed idle by c_k and current requests by w_j , respectively. Since accessing channels in T_1 causes no cost if observed idle, they are allocated first to requests with highest valuations (Lines 5-8). In Lines 11-13, the remaining requests are allocated to channels in T_2 sensed idle from highest valuation to lowest if they satisfy $w_n > \theta(k)$ where $\theta(k)$ serves as a threshold for using channel k . We set $\theta(k) = c_k$ in this section, which is motivated by Propositions III.1 and III.2. Setting different thresholds provides a way for trading off the social welfare and the revenue of the operator, which will be discussed in detail in Section V.

The time complexity of Algorithm IV.1 is $O(C \log C + r \log r)$ since the complexity of sorting in Lines 2 and 3 dominates that of allocation in Lines 4-13. We then show that the greedy online algorithm is 1/2-competitive when $|T_1| = 0$ and channels in T_2 are homogeneous in Proposition IV.1. To establish this result, we first show that c_k is the expected cost per a request served by channel k in Lemma IV.1.

Algorithm IV.1 Greedy Online Algorithm

In each time slot t :

- 1: **if** $D = \emptyset$ **then** exit
 - 2: Sort channels in S (sensed idle in T_2) by c_k in ascending order
 - 3: Sort requests in D (outstanding ones) by w_j in descending order
 - 4: $i \leftarrow 1$
 - 5: **for all** l in Γ (channels in T_1 observed idle) **do**
 - 6: $x_{il}(t) \leftarrow 1$; $D \leftarrow D \setminus \{i\}$
 - 7: **if** $D = \emptyset$ **then** exit
 - 8: $i \leftarrow i + 1$
 - 9: **if** $D = \emptyset$ **then** exit
 - 10: $n \leftarrow |\Gamma| + 1$
 - 11: **for all** k in S (channels sensed idle in T_2) **do**
 - 12: **if** $w_n \leq \theta(k)$ or $D = \emptyset$ **then** break
 - 13: $x_{nk} \leftarrow 1$; $D \leftarrow D \setminus \{n\}$; $n \leftarrow n + 1$
-

Lemma IV.1. *For the greedy online policy, c_k is the expected cost of serving a request on channel k when $H \rightarrow \infty$.*

Proof: Let A'_k denote the the number of time slots after the last request is served by channel k in Algorithm IV.1. We have $A'_k/H \rightarrow 0$ if $H \rightarrow \infty$. Consider the time interval right after a request is served by channel k and before the next request is served by channel k . Remove all time slots in the interval when there are no requests in the system or channel k is sensed but not allocated. Given that a channel is sensed idle, the probability that collision happens is $1 - P_0(k)$. Thus the number of slots where collisions happen follows a geometric distribution and the expected cost per a request service on channel k is $Q(1 - P_0(k))/P_0(k)$. ■

Based on Lemma IV.1, we show the competitive ratio of Algorithm IV.1 for homogenous T_2 channels.

Proposition IV.1. *If $|T_1| = 0$ and channels in T_2 are homogeneous, Algorithm IV.1 is 1/2-competitive when $H \rightarrow \infty$.*

Proof: Let the random variable γ denote the set of requests that are eventually served by the algorithm. Let $P_0 = P_0(k)$ for any channel $k \in T_2$. Since the channels in T_2 are homogeneous, we have $c = Q(1 - P_0)/P_0$, which is the expected cost for serving a single request in Algorithm IV.1 when $H \rightarrow \infty$ by Lemma IV.1. Then the expected social welfare achieved by Algorithm IV.1 can be written as follows:

$$\begin{aligned} & \sum_{k=1}^N \left[\left(\sum_{|\gamma|=k} \Pr(\gamma) \sum_{i \in \gamma} w_i \right) - kc \Pr(|\gamma|=k) \right] \\ &= \sum_{k=1}^N \left[\sum_{|\gamma|=k} \Pr(\gamma) \sum_{i \in \gamma} (w_i - c) \right] \end{aligned} \quad (6)$$

Let γ' denote the set of requests that are eventually served by the optimal offline algorithm, then c serves as an lower bound for the expected cost of serving a request by ignoring the time slots after the last request is served. Hence, Equation (6) with γ replaced by γ' serves as an upper bound for the expected social welfare achieved by the optimal offline algorithm.

Note that the greedy algorithm always chooses the active request with highest valuation. For any sample path, consider the set of requests served by the optimal offline algorithm and those by the greedy algorithm with $w'_i = w_i - c$ as the valuation. We follow the same argument as in [7]: We consider

any request i that is scheduled offline but not online. Since request i is not scheduled online, it is present at time t and the greedy algorithm schedules another request j in that slot, the valuation of request j should be as least as large as that of request i . For any request i that is allocated offline and also online, it makes the same contribution to the social welfare. Then the offline solution achieves a social welfare at most twice that in the online solution since $\frac{w'_i}{w'_i + w'_j} \geq \frac{1}{2}$. Therefore, Algorithm IV.1 is 1/2-competitive when $H \rightarrow \infty$. ■

Note that the factor 2 in Proposition IV.1 does not depend on request arrival patterns or channel related parameters. Algorithm IV.1 can always achieve at least $\frac{1}{2}$ of the social welfare of the optimal offline algorithm (Algorithm III.1) when the system is only composed of homogeneous T_2 channels.

V. ACHIEVING INCENTIVE COMPATIBILITY

When the available spectrum resource cannot satisfy all the requests, which is often the case, a selfish SU may choose to cheat on its valuation or arrival and deadline times to obtain some priority of being served. Such strategic behavior leads to a less efficient system. In this section, an online auction scheme is presented, which utilizes the online greedy algorithm (Algorithm IV.1) together with a payment scheme to suppress the cheating behavior. We first develop a natural payment scheme that uses c_k , the expected cost for serving a request by channel k , as the reservation price for using channel k , and show that the mechanism achieves incentive compatibility (formally defined below). For heterogeneous channels, however, such a variable reservation price has the weakness that the payment charged to an SU not only depend on the valuations of SUs but also the particular channel that serves the SU. Hence, two SUs with the same valuation served simultaneously may be charged different prices simply because they are served by different channels. This leads to arbitrary and unfair treatment of SUs. To avoid this issue, we also introduce a fixed reservation pricing scheme that is independent of channel assignment, which also provides a more straightforward way of trading off social welfare and revenue. The revenue of the operator is composed of two parts: payments collected from the SUs by serving their requests and the penalty paid for causing collisions. For an actual business model to be viable, it is important that the revenue of the operator is taken into account.

A. Online Auction with Variable Reservation Price

In this section, an online auction scheme using a variable reservation price is presented (see Auction 1) to suppress the cheating behavior of SUs. At any time slot t , the operator accepts bids of the form $(\hat{a}_i, \hat{d}_i, \hat{w}_i)$, where $\hat{a}_i = t$ and \hat{d}_i denote the reported required service starting time and the deadline, respectively, and \hat{w}_i denotes the reported valuation. All these values could be different from the true values of request i . We assume there is no early-arrival misreport and late-departure misreport in the system, that is, $\hat{a}_i \geq a_i$ and $\hat{d}_i \leq d_i$ in any bid. In practice, both of them can easily be detected since the request is no longer in the system when either misreport occurs.

Let p_i denote the payment that the operator charges a SU for having its request i served. The *net utility* for request i is defined as: $u_i = w_i - p_i$ if request i is served and $u_i = 0$ if not. A mechanism is said to be *dominant-strategy incentive compatible* (DSIC) if for any given sample path of channel state realizations and sensing realizations and a set of requests, each request maximizes its utility when it truthfully reveals the private information independent of the bids from other requests (adapted from Definition 16.5 in [13]).

In Auction 1, channels are assigned to requests according to Algorithm IV.1 by setting $\theta(k) = c_k$, where a request is assigned to channel k only if its valuation is higher than c_k . Hence c_k serves as a reservation price. For every request successfully served by its deadline using channel k , the charged price is the maximum of the reservation price c_k and a critical value. A *critical value* is defined as the maximum reported valuation under which it will not be served, assuming the other bids are fixed. Note that by the definition of payment, the net utility of a SU is always non-negative, and the revenue of the operator never exceeds the social welfare. Furthermore, since a payment is lower bounded by the corresponding reservation price, a non-negative expected revenue of the operator is obtained for large enough H by Lemma IV.1.

Auction 1: Requests $(\hat{a}_i, \hat{d}_i, \hat{w}_i)$ are reported to the operator at time $t = \hat{a}_i$.

(i) At the beginning of each t , allocate requests according to Algorithm IV.1 by setting $\theta(k) = c_k$.

(ii) A request successfully served by channel k pays $\max(c_k, \text{critical value})$, collected at its reported deadline.

According to Theorem 16.13 in [13], to show that Auction 1 is DSIC, it is sufficient to show that the mechanism is monotonic in terms of both valuation and timing. That is, for a given sample path of channel realizations and sensing realizations and a set of requests, if request i submitting a bid $(\hat{a}_i, \hat{d}_i, \hat{w}_i)$ wins, then it continues to win if it instead submits a bid $(\hat{a}'_i, \hat{d}'_i, \hat{w}'_i)$ with $\hat{w}'_i > \hat{w}_i$, $\hat{a}'_i \leq \hat{a}_i$, and $\hat{d}'_i \geq \hat{d}_i$, assuming other bids are fixed. This condition can be easily verified. So, Auction 1 is DSIC. Furthermore, a binary search algorithm can be applied to find the critical value for requests successfully served, which is presented in our online technical report [10].

Remark: In a traditional VCG like auction [13], the payment charged to a winning bidder is only determined by the valuations of other bidders competing for the same resource. In Auction 1, due to the heterogeneity of spectrum resource, however, it also depends on the particular channel that serves the request. For instance, consider two heterogeneous T_2 channels with $c_1 \ll c_2$, both of which are sensed idle and really idle at time slot t . Assume that two requests of same valuation $w > c_2$ arrive at t and expire at the beginning of $t + 1$. Since $w > c_2$, the greedy algorithm will serve request 1 by channel 1 and request 2 by channel 2. For request 1, the critical value is c_1 since given that request 2 reports a valuation of w , request 1 will be served iff it reports a valuation higher than c_1 . Hence, the payment charged to request 1 is also c_1 . Similarly, the payment charged to request 2 is c_2 . The two

requests are charged different prices even if they have the same valuation and face the same competitive environment. Ideally, we would like to hide such resource heterogeneity from SUs.

B. Online Auction with a Fixed Reservation Price

To overcome the weakness of the variable reservation pricing scheme for the heterogenous channel case, we consider a fixed reservation price in this section. Let $\theta(k) = q$ in Auction 1, where q is computed using channel related parameters, and is fixed for a given set of channels. Note that the mechanism is still monotonic, hence, DSIC is still ensured in this case. Setting q to different values provide a way for trading off social welfare and revenue. At a very low reservation price, the payment collected cannot recover the expected cost and hence the average revenue becomes negative. A low reservation price may also harm social welfare by our necessary condition for serving requests (Proposition III.1). On the other hand, when the reservation price is too high, fewer requests will be accepted, which harms both social welfare and revenue. Furthermore, the optimal social welfare and revenue are usually achieved at very different reservation prices (see Figures 3(a) and 3(b)). In this section, we would like to find a reservation price that matches the average expected cost for serving a request, such that nearly optimal social welfare can be achieved while ensuring the *expected* revenue of the operator to be non-negative.

Consider any given set of requests. Let n'_j denote the expected fraction of requests served by channel j , with the expectation taken over all possible channel and sensing realizations. Then by Lemma IV.1, for large enough H , the average expected cost per request can be represented as $q' \triangleq \sum_{j \in T_1 \cup T_2} c_j n'_j$. Note that, for homogenous T_2 channels, $q' = c_j$ for any j . For heterogenous channels, however, finding the accurate value of q' is hard, if not impossible, without accurate knowledge of the request set. We therefore consider an upper bound of q' that can be computed using channel related parameters only. To this end, let v_j denote the probability that the channel j is sensed idle and it is really idle. Then $v_j = \pi_1(j)$ if $j \in T_1$, and $v_j = P_I(j)P_0(j)$ if $j \in T_2$. We then define $m_j \triangleq \frac{v_j}{\sum_{i \in T_1 \cup T_2} v_i}$, and use it to estimate n'_j . Note that, $m_j = n'_j$ when the system is always overloaded, that is, if whenever a channel is (sensed) available, it will be used to serve some active request in the system. Now define $q_1 \triangleq \sum_{j \in T_1 \cup T_2} c_j m_j$. Since q_1 only depends on channel parameters, it can be easily computed. Note that $q_1 = q'$ for homogenous channels. We will show that $q_1 \geq q'$ and therefore a non-negative expected revenue is obtained using q_1 as the reservation price in Proposition V.1. Furthermore, using q_1 also achieves good performance in terms of social welfare as shown in numerical results (see Figures 3(a) and 3(b)).

In the following, we assume that channels have been sorted by a non-decreasing order of c_j . We start with Lemma V.1 that provides a sufficient condition for $q_1 \geq q'$.

Lemma V.1. *If $\frac{m_j}{m_{j+1}} \leq \frac{n'_j}{n'_{j+1}}$ for all j , then $q_1 \geq q'$.*

Proof: We calculate $q_1 - q' = \sum_{j \in T_1 \cup T_2} c_j(m_j - n'_j)$. In the following, we will show that there exists i such that for all $j \leq i$ we have $m_j \leq n'_j$ and for all $k > i$ we have $m_k > n'_k$. Since $\frac{m_j}{m_{j+1}} \leq \frac{n'_j}{n'_{j+1}}$ for all j , it is easy to see that: if $m_j \geq n'_j$, then $m_k \geq n'_k$ by multiplying $\frac{m_{j+1}}{m_j} \dots \frac{m_k}{m_{k-1}}$ and $\frac{n'_{j+1}}{n'_j} \dots \frac{n'_k}{n'_{k-1}}$, respectively, on both sides. Then we can find such i . We divide $q_1 - q'$ into two parts:

$$q_1 - q' = \sum_{j \leq i} c_j(m_j - n'_j) + \sum_{k > i} c_k(m_k - n'_k) \quad (7)$$

If $i = |T_1| + |T_2|$, $q_1 - q' = \sum_{j \in T_1 \cup T_2} c_j(m_j - n'_j) \geq (\max_{j \in T_1 \cup T_2} c_j) \left(\sum_{j \in T_1 \cup T_2} m_j - \sum_{j \in T_1 \cup T_2} n'_j \right) = (\max_{j \in T_1 \cup T_2} c_j)(1 - 1) = 0$. If $i = 0$, then all terms in $q_1 - q'$ are positive. Next we consider the case where neither sums in Equation (7) has no terms. Since all terms in the first term in the sum are non-positive and all terms in the second term in the sum are positive, we obtain

$$\begin{aligned} q_1 - q' &\geq (\max_{j \leq i} c_j) \sum_{j \leq i} (m_j - n'_j) + (\min_{k > i} c_k) \sum_{k > i} (m_k - n'_k) \\ &\stackrel{(a)}{\geq} (\max_{j \leq i} c_j) \left(\sum_{j \leq i} (m_j - n'_j) + \sum_{k > i} (m_k - n'_k) \right) \\ &= (\max_{j \leq i} c_j) \left(\sum_{j \in T_1 \cup T_2} m_j - \sum_{j \in T_1 \cup T_2} n'_j \right) = 0, \end{aligned}$$

where (a) is by the assumption that $c_1 \leq \dots \leq c_{|T_1|+|T_2|}$. Hence $q_1 \geq q'$ holds. \blacksquare

Based on Lemma V.1, we claim that a reservation price of q_1 results in a non-negative revenue for the operator.

Proposition V.1. *The operator obtains a non-negative expected revenue with reservation price q_1 when H is large enough.*

Proof: It suffices to show that $q_1 \geq q'$. Consider a given set of requests and any sample path of channel states. Without loss of generality, consider the first two channels in the sorted list. Let n_1 and n_2 denote the number of requests served by channels 1 and 2, respectively. Let s_i denote the number of time slots that are sensed and allocated in the interval between $(i-1)$ th and i -th requests served by channel 1, and define b_i similarly for channel 2. Let A denote the total number of time slots between 0 and H that are not sensed or sensed but not allocated for channel 1, and A' the number of time slots after the last request is served by channel 1. Define B and B' similarly for channel 2. Note that by the ordering of channels, when there is only one request in the system, and both channels are available, channel 1 will be used. It follows that $A \leq B$. We then have $H = \sum s_i + A + A' = \sum b_i + B + B'$. Therefore, $H = E(\sum s_i + A + A') = E(n_1)/m_1 + E(A) + E(A')$ (by geometric distribution) and $H = E(n_2)/m_2 + E(B) + E(B')$. Note that $E(A')/H \rightarrow 0$ and $E(B')/H \rightarrow 0$ when $H \rightarrow \infty$. Therefore $\frac{n_1}{n_2} = \frac{E(n_1)}{E(n_2)} = \frac{[H - E(A)]m_1}{[H - E(B)]m_2} \geq \frac{m_1}{m_2}$ since

$H - E(A) \geq H - E(B)$. It then follows that $q_1 \geq q'$ by Lemma V.1. Hence, a reservation price of q_1 leads to a non-negative revenue at the operator. \blacksquare

VI. NUMERICAL RESULT

In this section, we evaluate the performance of the greedy online algorithm (Algorithm IV.1) and the tradeoff between social welfare and revenue for different reservation prices. We first show the performance of the greedy online algorithm compared with the optimal offline algorithm under different channel settings and request related parameters, respectively. We then apply Auction 1 with varying reservation prices and show the performances of social welfare and revenue.

We let the arrivals a_i of requests follow a Poisson distribution and the duration $d_i - a_i$ of the requests follow an exponential distribution. The valuations follow a uniform distribution in $[1, 15]$. We choose $Q = 10$, the penalty per collision, comparable to the valuations in all our simulations. We fix the number of requests as 20, and the inter-arrival mean as 3 slots, and vary the mean of request duration to adjust the density of requests. Given the means of inter-arrival and request durations, we generate 50 groups of requests and compare the average for the metrics we consider. We generate the channel availabilities in each time slot based on our assumption that channel states follow an i.i.d Bernoulli distribution and 100 samples of channel realizations are taken for our simulations. The channel parameters we use will be introduced in Section VI-A.

A. Performance of Greedy Online Algorithm

In Figure 2(a), we compare the performance of Algorithm IV.1 with that of Algorithm III.1 when there are three homogeneous T_2 channels in the system with $\pi_2 = 0.6324$, $P_m = 0.2218$, $P_f = 0.6595$ and various number of T_1 channels. The y -axis denotes the achieved performance ratio, i.e., the ratio between the social welfare of the online algorithm and that of the optimal offline algorithm. When $|T_1| = 1$, we set $\pi_1 = 0.5058$; When $|T_1| = 2$, we set $\pi_1(1) = 0.8147$ and $\pi_1(2) = 0.1270$. We observe that the performance of Algorithm IV.1 degrades as $|T_1|$ increases, independent of the request duration mean. With a high number of T_1 channels, a wrong decision made by the greedy online algorithm to schedule a request affects the performance more. Also, the greedy online algorithm serves requests of a larger density better than requests of a smaller density. When the system is overloaded with requests, even the optimal offline algorithm can not satisfy all requests. Thus, those with larger valuations tend to be chosen, as in the greedy online algorithm. All ratios plotted are strictly above $\frac{1}{2}$, even for those with $|T_1| \neq 0$.

In Figure 2(b), we evaluate the performance of Algorithm IV.1 with heterogeneous T_2 channels. We use the same T_1 channel parameters as in the homogeneous case. The parameters related to T_2 channels are as follows: $\pi_2(1) = 0.9134$, $\pi_2(2) = 0.6324$, $\pi_2(3) = 0.0975$, $P_m(1) = 0.1419$, $P_f(1) = 0.7922$, $P_m(2) = 0.2218$, $P_f(2) = 0.6595$, $P_m(3) = 0.6557$, $P_f(3) = 0.2157$. We observe similar results

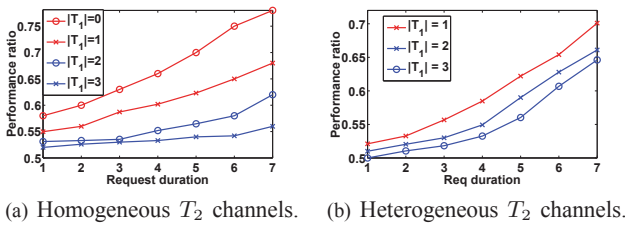


Fig. 2. Performance of online algorithm versus offline algorithm over various request duration means with homogeneous and heterogeneous T_2 channels ($|T_2| = 3$), respectively.

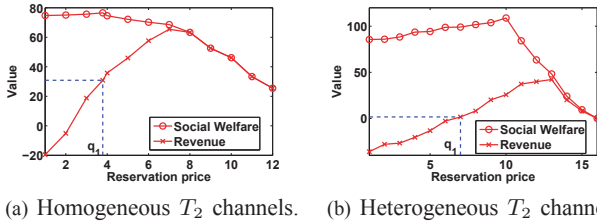


Fig. 3. Tradeoff between social welfare and revenue over reservation price with homogeneous and heterogeneous T_2 channels ($|T_2| = 3$), respectively. $|T_1| = 0$ in (a) and $|T_1| = 1$ in (b).

as in Figure 2(a): Algorithm IV.1 performs better with fewer T_1 channels and denser requests. Again all ratios are above $\frac{1}{2}$.

B. Tradeoff between Social Welfare and Revenue

We now study the tradeoff between social welfare and revenue generated by Auction 1 with a fixed reservation price. In Figure 3, the request duration mean is fixed at 3 time slots, and we vary the values of reservation price q . The channel related parameters are the same as in Section VI-A. We first show the tradeoff in a system with homogeneous T_2 channels and no T_1 channels in Figure 3(a). We observe that both social welfare and revenue first increase and then decrease as the reservation price increases. Note that the highest social welfare and revenue are obtained at very different reservation prices with $q = 3.8$ for the former and $q = 7$ for the latter. At a low reservation price (< 3), the payment collected cannot recover the expected cost and hence the average revenue becomes negative. We note that when the reservation price is $q_1 = 3.8$ (defined in Section V-B), a non-negative revenue is obtained together with a high social welfare. Furthermore, at a very high reservation price (≥ 8), the social welfare and the revenue converge, where the payment actually becomes the same as the valuation for requests served.

In Figure 3(b), we show the tradeoff in a system with T_1 and heterogeneous T_2 channels. The trend of social welfare and revenue is similar to that in Figure 3(a). The highest social welfare and revenue are obtained at $q = 10$ and $q = 13$, respectively. Note that with the reservation price $q_1 = 6.9$, the revenue obtained is right above 0, which is consistent with Proposition V.1 and also shows that q_1 is nearly a tight upper bound of the expected cost for this case.

VII. CONCLUSION

In this paper, we study the joint sensing and spectrum allocation problem for serving secondary users in cognitive

radio networks with the objective of maximizing the social welfare. Our problem formulation takes into account both spectrum uncertainty and sensing inaccuracy, which enables dynamic spectrum access at small time scales. Using only channel statistics and real time channel states, we develop an optimal solution for serving a given set of spectrum requests with various time elasticity. We then propose an online algorithm that achieves comparable performance as the offline algorithm. We further extend the online algorithm to an online auction scheme, which ensures incentive compatibility for the SUs and also provides a way for trading off social welfare and revenue. There are several open problems to be solved. First, in practice, a more flexible form of spectrum requests will be desirable. For instance, a request may ask for multiple chunks that may or may not be preemptive. Extending the current solutions to this more general setting will be part of our future work. Second, we plan to extend the problem formulation by including the notion of spatial spectrum reuse in addition to the time dimension considered in the paper. Third, we plan to relax the assumption on the i.i.d Bernoulli channels by considering correlated channels, which involves solving an *exploration vs. exploitation* problem in the context of an auction.

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