Distributed Multiple Access in Multichannel Cognitive Radio Networks via Potential Games

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***Abstract*—Cognitive Radio (CR) approaches constitute one of the most promising ways to solve the inefficiency of the static spectrum allocation procedures. Distributed and self-enforcing methods for multiple access/allocation have also become increasingly attractive for their scalability and robustness. In the light of this reality, we present in this work a game-based distributed algorithm for the multiple access of secondary users (SUs) in a multichannel CR environment. The main idea behind our method is to provide means for SUs to independently select a set of secondary channels to use for transmission, while achieving close-to-optimal equilibrium performance in a Signal-to-Interference-plus-Noise-Ratio (SINR) regime. Additionally, we analytically derive explicit performance bounds on the optimality and the convergence speed of the proposed algorithm. We corroborate the efficacy of our multiple access algorithm as well as the main conclusions on the optimality/convergence speed tradeoff through numerical results.**

***Index Terms*—Opportunistic Spectrum Access, Cognitive Radio, Distributed Multiple Access, Congestion Game**

I. Introduction

Opportunistic Spectrum Acess (OSA) approaches and, particularly, Cognitive Radio (CR) systems have been proposed to alleviate the inefficiency of the traditional static spectrum allocation procedures [1], [2]. In these contexts numerous research paths have been explored in recent years, as the new spectrum sharing perspectives span most aspects of wireless networks and pose important network challenges in all of them [3], [4].

Distributed CR solutions are more suitable for practical implementations, as they are more scalable, agile and robust to face the highly diverse and heterogeneous modern wireless settings [5]. For instance, there exists a myriad of works that consider distributed perspectives to the problem of multiple access in CR networks [3], [6]. Some examples of recent methods for distributed access in CR environments can be found in [7]–[9]. See also [10]–[13] for OSA-based distributed extensions to standard MAC protocols. The primary shortcoming of these approaches is that they rely on the assumption that all the secondary users (SUs) comply with the operation rules of the system at all times. This is not the case in practical next generation networks, since the increasing multiplicity of services obligates the users to adapt individually and, in many cases, pursuing conflicting goals. Thus, traditional methods with fully cooperative nodes and static operation are no longer sufficient [14]. The design of distributed, robust and self enforcing techniques for spectrum sharing in next generation wireless networks is still an open research problem.

In this work we develop a game-based distributed algorithm for the multiple access of SUs in an CR environment with multiple secondary channels. For this, we develop a method that provides a way for SUs to independently select a set of secondary channels to transmit over, while achieving close-to-optimal equilibrium points in a Signal-to-Interference-plus-Noise-Ratio (SINR) regime.

Game theoretical techniques have recently been used to design distributed algorithms that facilitate the dynamic operation of CR networks. These tools offer highly scalable spectrum sharing/access methods for modeling and constructing self-enforcing and distributed alternatives for SUs [15]. Non-cooperative game-based solutions for cognitive radio networks primarily focus on proving the existence of the Nash Equilibria (NE), analyzing their structure in case of existence, and/or studying their efficiency in terms of a particular performance metric [14]–[16]. To construct algorithms with provable convergence to equilibrium points, many approaches consider network models that can be mapped to specially constructed games. Among this type of games, *potential games* use a real-valued function that represents the entire player set to optimize some performance metric, while providing direct connection with the NE of the game [17]. Potential games have been used to solve CR-related problems in literature. The majority of works can be categorized in 3 classes, based mainly on user strategies: Waveform signature selection/adjustment, power level selection, and/or transmission channel selection.

Within the first class, [18] proposes a power/waveform adjustment analysis for the SUs to maintain a predefined level of SINR in a single channel setting. To minimize the aggregate perceived interference, a waveform-signature-based potential game is proposed in [19] for an Ad-Hoc secondary network where the convergence analysis is only addressed numerically. In contrast, the convergence aspect is covered theoretically in [20], but very little is mentioned on the rate of convergence to the desired solutions.

In the power selection class, [21] considers a secondary Ad-Hoc network and two potential games to avoid the change of the connectivity graph and to optimize the minimal set of non-interfering channels. However, the convergence speed of their methods was not discussed. The interference minimization path is taken in [22], where two schemes for spectrum sharing are proposed: one in which SUs cooperate with primary users (PUs) by relaying their communications, and one in which SUs do not cooperate with the PUs. No specific methods to reach the equilibria are provided in [22].

Lastly, a few approaches have considered the transmission channel selection by SUs. For instance, [23] proposes two game theoretic models for the interfer-

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ence minimization from channel selection strategies. Although some numerical analysis of convergence to equilibrium points is presented, no analytical arguments are provided in [23]. A similar issue occurs in [24] with the convergence analysis of the two proposed algorithms, as no theoretical arguments are provided for the convergence rates.

The existing work leaves the following challenges unaddressed concurrently: **(First)** Multiple-channel access methods for the secondary transmissions. **(Second)** Continuous-time operation for achieving the equilibrium points, so that the need for synchronization is alleviated. **(Third)** Analytical study of the convergence speed of the proposed algorithms to reach the equilibrium points, and its tradeoff with optimality. **(Fourth)** Explicit analytical bounds on the gaps between the approximate algorithmic solutions and the optimal values. We consider these factors to be crucial for the understanding and implementation of the CR paradigms in the near future, and therefore address them directly and concurrently in this paper.

The rest of the paper is organized as follows. Section II describes the network model and the problem formulation. Section III addresses the algorithmic solution to reach the equilibria of the proposed game. A numerical analysis is presented in Section IV. Section V concludes the paper.

II. Network Model and Problem Formulation

A. Network model

We consider a continuous-time network scenario with one primary channel of bandwidth B and N SUs in an uplink setup. The bandwidth is divided into K subchannels, each of bandwidth B/K. At any given time, the PU system occupies either the entire bandwidth B or none at all (later in the paper, we provide some ideas on the extension of our model to more flexible primary scenarios). In contrast, a SU can occupy an arbitrary subset of K subchannels in a time instance. This means that SU $i \in \{1, \ldots, N\}$, at any given time, selects a set s_i of subchannels to use for transmission (i.e., $s_i \subseteq \{1, \ldots, K\}$). This kind of operation for the SUs captures the main characteristics of modern OFDM wireless systems [25].

In the SINR regime (i.e., interference is treated as noise), several SUs may initiate transmission at a given time, creating an added interference level on the transmissions of all the SUs using the same subchannels. This kind of setup then defines an adversarial interaction or game, because there exist conflicting interests for the SUs: the players want to access as many channels as possible to maximize their utility of added per-resource rewards, but there are possibly several other players sharing the same resources. Naturally, the latter reality results in a proportionally decreasing utility with the number of players occupying a resource. Situations like these have been called Congestion Games (CG) in the literature [17], [26].

For the CG considered here the set of players is the set $\mathcal{N}$ of SUs, and the strategy space $\mathcal{S}_i$ of SU i is the set of all subsets of $\mathcal{K}$ (i.e., $2^{\mathcal{K}}$, the power set of $\mathcal{K}$), where $\mathcal{K} = \{1, \ldots, K\}$. Clearly, a strategy of SU i is a set that contains a particular selection of subchannels to transmit over.

The reward or utility function of the SUs is defined such that it increases with the number of channels used when transmitting, but monotonically decreases with the number of users occupying the same subchannel. A natural definition of such a reward for the k-th subchannel corresponds to the achievable rate in an AWGN wireless environment, minus a cost due to power consumption when transmitting. This is as follows:

$$r^k(n_k) = \frac{B}{K} \log_2 \left(1 + \frac{P}{\sigma^2 + (n_k - 1)P}\right) - \gamma \log_2(1 + P) \ , \tag{1}$$

where:

- n_k is the number of SUs transmitting over the subchannel k. It is a function of the strategies of all SUs, since $n_k = \sum_{j \in \mathcal{N}} \mathbb{1}_{\{k \in s_j\}}$ (where $\mathbb{1}_{\{x\}}$ is 1 when x is true, and is 0 otherwise).
- σ^2 is the noise variance, assumed to be the same at the receiving base station for all SUs' transmissions.
- P is the fixed transmission power of the SUs. For the protection of the primary transmissions, the power levels P of all SUs satisfy a predefined maximum power constraint in each subchannel (e.g., $NP \leq P_{\max}$ for all $k \in \mathcal{K}$).
- γ is the constant price that accounts for the power consumption in a transmission.

The utility function of SU i using the strategy s_i, when the other SUs are using the strategy profile s_{-i} (i.e., vector of strategies of the other SUs), is then

$$u_i(s_i, s_{-i}) = \sum_{k \in s_i} r^k(n_k) \ . \tag{2}$$

The formulation of this CG is well known to be an exact potential game [17], [27], for which a real-valued potential function can be defined to capture the change in the strategy of one of the players. A potential function in this case can be constructed as

$$\Phi(s_i, s_{-i}) = \sum_{k \in \bigcup_{j \in \mathcal{N}} s_j} \sum_{l=1}^{n_k} r^k(l) \ . \tag{3}$$

This real-valued function captures, as stated in the previous section, the reward of all SUs in the system when a particular strategy profile is selected. More specifically, it represents the reward obtained over the subchannels that are being used by all SUs, when we consider from 1 to n_k SUs to be sharing a particular subchannel k. It is straightforward to show that the sufficient condition for the existence of exact potential game, i.e. $u_i(s_i, s_{-i}) - u_i(s_i', s_{-i}) = \Phi(s_i, s_{-i}) - \Phi(s_i', s_{-i})$ for all s_{-i} and $s_i \neq s_i'$, holds in this case.

The Nash equilibria of the potential games are found in the local maxima of the potential function, since they represent the best responses for each player to the strategies of the other players in terms of the rewards [17], [27]. Therefore, our task consists of finding the local maxima of the defined potential function of the CG. In this case, however, maximizing the potential function is a combinatorial problem, as the strategy spaces of the SUs are not continuous sets of real values. In the following sections we provide a procedure with a time-reversible Markov chain to approximate the optimal value of the Φ function with explicit optimality bounds.

B. Problem Formulation

Let $\mathbf{s}$ represent the strategy profile of all SUs in the system (accordingly, $\boldsymbol{\mathcal{S}}$ represents the strategy space of all SUs, so that $\mathbf{s} \in \boldsymbol{\mathcal{S}}$). Also, with a slight abuse of notation, we use $f(\mathbf{s})$ and $f(s_i, s_{-i})$ interchangeably to represent a function of the strategies of all the SUs. Our objective is thus defined as

$$\max_{\mathbf{s}\in\boldsymbol{\mathcal{S}}} \Phi(\mathbf{s}) \ . \tag{4}$$

As highlighted in the previous subsection, (4) is a combinatorial problem due to the fact that the optimization is carried out over the strategy sets of the SUs, which are composed of discrete sets of subchannels.

Based on the ideas developed for the CSMA-like algorithms in solving combinatorial network problems [28], in [29] the authors provide a detailed methodology to approximate combinatorial network problems using a Markov model with the time-reversibility property and the concept of conjugate function. We employ that methodology here to approximate the solution to (4), as its combinatorial nature prevents us from using a direct analytical procedure. In our case, however, we construct a distributed algorithm that allows to reach an approximate solution with an explicit optimality gap, while retaining the relationship with the equilibrium concepts of the proposed game.

To this end, let $p_{\mathbf{s}}$ represent the percentage of time for which each strategy profile $\mathbf{s}$ is employed. We obtain the following equivalent optimization problem:

$$\begin{aligned} \max_{\mathbf{p}\geq 0} \ & \sum_{\mathbf{s}\in\boldsymbol{\mathcal{S}}} p_{\mathbf{s}}\Phi(\mathbf{s}) \\ \text{s.t.} \ & \sum_{\mathbf{s}\in\boldsymbol{\mathcal{S}}} p_{\mathbf{s}} = 1, \end{aligned} \tag{5}$$

where $\mathbf{p} = [p_{\mathbf{s}}]_{\mathbf{s}\in\boldsymbol{\mathcal{S}}}$.

An interpretation of the equivalence between (4) and (5) is that both problems yield the same optimal value, but (5) is defined over correlated strategies (i.e., a probability distribution over $\boldsymbol{\mathcal{S}}$ [30]) instead of pure strategies. As it can be inferred from the developments that we describe later, with this kind of formulation each SU uses a methodology that offers individual recommendations to play particular strategies with certain probabilities.

We observe that the linear program in problem (5) is still hard to solve since it cannot be analytically tackled with standard derivative-based techniques. The solution in this case would be defined by corner points of the constraint set, which represent the pure strategy solutions that compose the original combinatorial problem. Additionally, interior point methods or simplex methods do not offer fruitful solution alternatives either because they are difficult to implement distributively. To overcome these hurdles, consider now the following fact from optimization theory [31, Section 3.1.5]:

$$\max_{\mathbf{s}\in\boldsymbol{\mathcal{S}}} \Phi(\mathbf{s}) \approx \frac{1}{\beta}\log\left(\sum_{\mathbf{s}\in\boldsymbol{\mathcal{S}}} \exp\left(\beta\Phi(\mathbf{s})\right)\right) \ , \tag{6}$$

where β is an strictly positive parameter that controls the approximation gap.

The idea is to reformulate the problem to include the (differential) *log-sum-exp* approximation, so that the solution procedure is more accessible. Using the same arguments provided in [29], we obtain the conjugate function of our *log-sum-exp* function, which is the entropy function $\frac{1}{\beta}\sum_{\mathbf{s}\in\boldsymbol{\mathcal{S}}} p_{\mathbf{s}}\log p_{\mathbf{s}}$. Here, $p_{\mathbf{s}}$ is just a quantity that needs to satisfy $\sum_{\mathbf{s}\in\boldsymbol{\mathcal{S}}} p_{\mathbf{s}} = 1$ [31, Example 3.25]. For our proposed method, however, we have a particular interpretation of the variable $p_{\mathbf{s}}$ in terms of probabilities (problem (5) and the subsequent lines).

Then, by finding the conjugate function of the entropy function, we obtain an expression that is equivalent to our *log-sum-exp* function. This corresponds precisely to the following optimization problem:

$$\begin{aligned} \max_{p\geq 0} \ & \sum_{\mathbf{s}\in\boldsymbol{\mathcal{S}}} p_{\mathbf{s}}\Phi(\mathbf{s}) - \frac{1}{\beta}\sum_{\mathbf{s}\in\boldsymbol{\mathcal{S}}} p_{\mathbf{s}}\log p_{\mathbf{s}} \\ \text{s.t.} \ & \sum_{\mathbf{s}\in\boldsymbol{\mathcal{S}}} p_{\mathbf{s}} = 1 \ . \end{aligned} \tag{7}$$

In summary, the optimal value of (7) is an approximation to the optimal value of (4), and the parameter β controls the approximation gap. This is the case because the approximation gap is upper bounded by $\frac{1}{\beta}\log|\boldsymbol{\mathcal{S}}|$, and it goes to 0 as $\beta \to \infty$.

This is then the first step in the construction of what is our main contribution: a new distributed multiple access algorithm with analytically characterized bounds on the optimality gap and convergence time.

C. Exact solution of the approximation

The Lagrangian of the problem in (7) is given by $\mathcal{L}(\mathbf{p},\lambda) = \sum_{\mathbf{s}\in\boldsymbol{\mathcal{S}}} p_{\mathbf{s}}\Phi(\mathbf{s}) - \frac{1}{\beta}\sum_{\mathbf{s}\in\boldsymbol{\mathcal{S}}} p_{\mathbf{s}}\log p_{\mathbf{s}} + \lambda\left(\sum_{\mathbf{s}\in\boldsymbol{\mathcal{S}}} p_{\mathbf{s}} - 1\right)$. Applying KKT conditions, we obtain

$$\Phi(\mathbf{s}) - \frac{1}{\beta}\left(1 + \log(p^*_{\mathbf{s}}(\Phi(\mathbf{s}))\right) + \lambda = 0, \ \forall \mathbf{s} \in \boldsymbol{\mathcal{S}} \tag{8}$$

$$\lambda = \frac{1}{\beta}\log\left(\sum_{\mathbf{s}\in\boldsymbol{\mathcal{S}}} \exp(1 - \beta\Phi(\mathbf{s}))\right) \ . \tag{9}$$

With this we solve for $p^*_{\mathbf{s}}(\Phi(\mathbf{s}))$:

$$p^*_{\mathbf{s}}(\Phi(\mathbf{s})) = \frac{\exp(\beta\Phi(\mathbf{s}))}{\sum_{\mathbf{s}'\in\boldsymbol{\mathcal{S}}} \exp(\beta\Phi(\mathbf{s}'))}, \ \forall \mathbf{s} \in \boldsymbol{\mathcal{S}} \ . \tag{10}$$

The expression in (10) represents a probability distribution over the strategy profiles of our CG, as a function of the corresponding achieved potential level. In the next section we present an algorithm that allows SUs to reach $p^*_{\mathbf{s}}(\Phi(\mathbf{s}))$ in a distributed manner.

III. Distributed Solution

The first step in constructing an algorithm that reaches $p^*_{\mathbf{s}}(\Phi(\mathbf{s}))$ is to make the following observation: the solution above for the probability distribution of the subchannel strategy profiles can be interpreted as the steady-state distribution of an ergodic continuous-time Markov chain. Our task then is to design an algorithm that operates distributively over the Markov chain model, so that when the algorithm converges to the stationary distribution in (10), we are sure that we have found an approximate solution to (4) (with an approximation gap of $\frac{1}{\beta}\log|\boldsymbol{\mathcal{S}}|$). Such Markov chain, though, should meet two conditions: (i) it must be irreducible, and (ii) must be time reversible [28], [29].

A. Particular characteristics of the Markov chain in the CG

Before we delve into the design of the algorithm, we need to analyze the specifics of the Markov model in our CG.

First, about the states of the Markov chain: the steady state distribution in (10) suggests that one state should represent a particular strategy profile $\mathbf{s}$. In that case, one state is formed by the whole set of individual strategies that are being employed by all SUs in a particular situation. Consequently, the state space of the Markov chain is $\boldsymbol{\mathcal{S}}$.

Now, recall that, as any potential function of a potential game, $\Phi(\mathbf{s})$ is able to track the change in the strategy of only one SU. Thus, a simple way to define any two adjacent states is to consider the ones that represent the change in the strategy of only one SU. Moreover, we consider two states to be adjacent if the SU changing its strategy is either terminating transmission on all of the channels it was using, or initiating transmission over a set of subchannels from an inactive state. These procedures represent the only two valid transitions for any SU in the system (note that, since the Markov model we are considering here operates in continuous time, the change in the strategy of more than one SU cannot occur simultaneously).

Second, regarding the transition rates between any two adjacent states $\mathbf{s}$ and $\mathbf{s}'$, we have the following:

- To retain the Markov nature (as discussed later), we set the transmission duration for any SU to be random, following an exponential distribution. Thus, if one SU is making a transition of the type $active \rightarrow inactive$, it means that the associated transition rate is the reciprocal of the average transmission time. We assume that this average transmission time is the same for all SUs, and we use T to denote it. Then, any state transition from $\mathbf{s}'$ to $\mathbf{s}$ that represents a transmission termination of one SU has a transition rate $q_{\mathbf{s}'\mathbf{s}} = 1/T$.
- If one SU (call it i) is making a transition of the type $inactive \rightarrow active$ (i.e., initiating transmission over a set s'_i of subchannels) from a state $\mathbf{s}$ to a state $\mathbf{s}'$, our designed rate for becoming active is

$$q_{\mathbf{s}\mathbf{s}'} = \frac{1}{T}\exp\left(\beta\sum_{k\in s'_i} r^k(n'_k)\right) , \tag{11}$$

where n'_k denotes the number of SUs transmitting over subchannel k in state $\mathbf{s}'$. The structure of this rate is based on the time reversibility that needs to hold between any two adjacent states of the system.

Protection for the PU - Restrictions on T: We just implied that any secondary transmission lasts for a random time with fixed mean T. Call this random transmission duration Λ_i for SU i. Now, suppose that any active primary transmission tolerates interference from the whole set of SUs for up to $\mathcal{I}$ time units. We further assume that $\mathcal{I}$ is fixed in time. Additionally, let δ represent the fraction of time that this $\mathcal{I}$ constraint can be violated, where δ is very close to 0.

With this, the way of protecting the primary transmission from the secondary transmissions is to consider the worst case scenario of transmission times from all SUs. This goes as shown in Figure 1: the transmission of one SU is immediately followed by the transmission of a different SU without overlapping, and this repeats for the complete set of SUs. At the same time, the primary transmission becomes active over the whole bandwidth B at the beginning of the first secondary transmission.

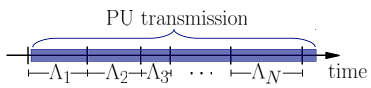

Figure 1. Worst case situation of interference to primary system.

In Figure 1, Λ_i represents the random transmission time of SU i (whose mean is T). Since Λ_i is exponentially distributed for all $i \in \mathcal{N}$, the worst case scenario is represented by the sum of N i.i.d. exponential random variables with rate $1/T$, which is Erlang distributed. Let Ω represent this Erlang random variable of N stages. The protection for the PU is achieved by

$$\begin{aligned}\Pr[\Omega > \mathcal{I}] &\leq \delta \\ \exp(-\mathcal{I}/T)\sum_{n=0}^{N-1}\frac{(\mathcal{I}/T)^n}{n!} &\leq \delta \\ \frac{\exp(\mathcal{I}/T)\Gamma(N,\mathcal{I}/T)}{\Gamma(N)} &\leq \delta\exp(\mathcal{I}/T) .\end{aligned}$$

Suppose that this restriction is met with equality. We then obtain $\Gamma(N,\mathcal{I}/T) = (N-1)!\delta$, where $\Gamma(N)$ represents the Gamma function and $\Gamma(N,\mathcal{I}/T)$ represents the incomplete Gamma function. Although we can not solve analytically for T in this equation, we can use standard numerical techniques.

Note here that this protection mechanism for the primary system can be readily extended to the case in which the operation of the PU has the flexibility of occupying subsets of subchannels, instead of the entire bandwidth B when it is active. It suffices to establish an average value of tolerable interference in time, and carry out the procedures for Ω, δ and $\mathcal{I}$ accordingly. Such scenario would allow to extend our entire formulation to the case where there is no assumption obligating the PU to occupy the entire set of subchannels when active.

B. Distributed algorithm

We now focus on the distributed algorithm that achieves the desired steady-distribution for the Markov chain of the problem defined in the previous section.

A particular situation in the operation of the system is as follows: the system is in state $\mathbf{s}$ at time t with SU i being inactive (i.e., $s_i = \emptyset$). SU i plans to initiate transmission at a random time Θ_i units in the future. Then, denote t_i the scheduled time for transmission initiation of SU i from a given time period. Recall that $\mathcal{K}$ denotes the entire set of subchannels (i.e., $\mathcal{K} = \{1, \ldots, K\}$).

The main steps of the access algorithm, for SU i, are then: **(Steps 1 and 2 in Algorithm 1)** Obtain from the base station the number n_k of active SUs in each subchannel $k \in \mathcal{K}$ (this information can be periodically broadcasted by the base station or provided to the SU by request). With that, compute the transition rates from the current state of the system to any other possible state when only the strategy s_i changes (i.e, compute $q_{\mathbf{ss}^{(l)}}$ for $l = 1, 2, \ldots, 2^K - 1$). **(Steps 3-6 in Algorithm 1)** For each transition rate, generate an exponentially distributed random time. Then, wait until one of these times expire. **(Step 7 Algorithm 1 and transmission)** Initiate transmission over the set of subchannels contained in the state whose timer expired first. Transmit for a period given by an exponentially distributed time with mean T. When transmission terminates, initiate the algorithm again from state 1.

Algorithm 1 Distributed Cognitive Spectrum Access Algorithm *(SU i)*

1: **Inputs**: T, β, K, $\mathbf{n}_{\mathcal{K}} := [n_k]_{k \in \mathcal{K}}$
Outputs: s'_i
2: $\mathbf{q}_{\mathbf{s_i s'_i}} := \left[\frac{1}{T} \exp(\beta \sum_{k \in s_i^{(l)}} r^k(n_k^{(l)}))\right]_{l=1}^{2^K-1}$
3: $\mathbf{\Theta_i} := \mathrm{EXP}(\mathbf{q}_{\mathbf{s_i s'_i}})$
4: $t_i := \min\ \mathbf{\Theta_i}$, $l^* := \arg\min_l \mathbf{\Theta_i}$
5: wait t_i
6: $s'_i = s_i^{(l^*)}$
7: **return** s'_i

In Algorithm 1, EXP$(\cdot)$ represents the function generating exponentially distributed random numbers from each rate included in the vector of its argument. It returns a vector of exponentially distributed random values, each one generated as $-\log(1-u)/\lambda_l$ (where $u \sim \mathcal{U}(0, 1)$, and λ_l is the rate).

Also, note that in Algorithm 1 we are using $\mathbf{q}_{\mathbf{s_i s'_i}}$ instead of $q_{\mathbf{ss}^{(l)}}$ to represent transition rates for SU i. The difference lies in that $\mathbf{q}_{\mathbf{s_i s'_i}}$ accounts for a vector of transition rates from strategy s_i of SU i to every possible other strategy s'_i (the strategies of all other SUs remain unchanged), while $q_{\mathbf{ss}^{(l)}}$ represents the individual transition rate from state $\mathbf{s}$ to state $\mathbf{s}^{(l)}$.

From the point of view of one SU, the outlined algorithm is of polynomial complexity in all system parameters except for the number of subchannels. The algorithm scans the power set of all subchannels to assess their valuation in case they are used. However, this fact does not represent a decreasing performance of our method since, for a given communication system, the number of subchannels does not change and has a constant contribution to the general complexity of the algorithm.

1) Convergence analysis:

Proposition 1. *The proposed algorithm converges to the steady-state distribution defined by (10) when β is finite.*

Proof. The proof basically follows from the fact that, for the proposed algorithm, three properties hold (Theorem 7.6.1 of [32]): **(i)** Our algorithm forms an irreducible continuous-time Markov chain, since the time spent in each state of the system is exponentially distributed and the underlying discrete-time Markov chain has one recurrent class. **(ii)** If $\nu_{\mathbf{s}}$ represents the rate at which the Markov chain leaves the state $\mathbf{s}$ and $p_{\mathbf{s}}$ denotes the state probability of state $\mathbf{s}$, then $\sum_{\mathbf{s} \in \boldsymbol{\mathcal{S}}} p_{\mathbf{s}} \nu_{\mathbf{s}} < \infty$ for any finite β. **(iii)** For any two adjacent states $\mathbf{s}$ and $\mathbf{s}'$, $p_{\mathbf{s}} q_{\mathbf{ss}'} = p_{\mathbf{s}'} q_{\mathbf{s}'\mathbf{s}}$, since the fact that only one SU's strategy changes makes equal the ratios $p^*_{\mathbf{s}}(\Phi(s))/p^*_{\mathbf{s}'}(\Phi(s'))$ and $q_{\mathbf{s}'\mathbf{s}}/q_{\mathbf{ss}'}$. Please see the technical report provided in [33] for the details. □

2) Convergence time discussion: Having shown that the proposed algorithm converges to the desired steady state distribution, the analysis on the mixing time (convergence time) of the algorithm is in order.

For this, we base our analysis on the *total variation distance*, as provided in [34]. The mixing time, from the total variation perspective, is defined as

$$t_{mix}(\epsilon) \triangleq \inf \left\{ t \geq 0 : \max_{\mathbf{s} \in \boldsymbol{\mathcal{S}}} \|\mathbf{H}_t(\mathbf{s}) - \mathbf{p}^*\| \leq \epsilon \right\}, \tag{12}$$

where $\mathbf{H}_t(\mathbf{s})$ is the probability distribution of all states in $\boldsymbol{\mathcal{S}}$ at time t for any initial state $\mathbf{s}$ of the system. Recall that $\mathbf{p}^* = [p^*_{\mathbf{s}}(\Phi(\mathbf{s}))]_{\mathbf{s} \in \boldsymbol{\mathcal{S}}}$.

Proposition 2. *A lower bound of the mixing time for the Distributed Cognitive Spectrum Access Algorithm is given by*

$$\frac{1}{2\theta} \log\left(\frac{1}{2\epsilon}\right) . \tag{13}$$

An upper bound of the mixing time for the Distributed Cognitive Spectrum Access Algorithm is given by

$$\frac{\theta 2^{2KN+1} \exp(2\beta(\Phi_{\max} - \Phi_{\min}))}{[(1/T)\exp(\beta \Phi_{\min})]^2} \times \left[\log\left(\frac{1}{2\epsilon}\right) + \frac{KN}{2}\log(2) + \frac{1}{2}\beta(\Phi_{\max} - \Phi_{\min})\right], \tag{14}$$

where

$$\theta = \frac{(2^K - 1)N}{T} \exp(\beta K r(1))$$
$$\Phi_{\max} = NKr(1)$$
$$\Phi_{\min} = \begin{cases} r(N), & \text{if } r(N) \geq 0 \\ NKr(N), & \text{if } r(N) < 0 . \end{cases}$$

Proof. The proof of Proposition 2 is based on the following general procedures: **(i)** We use the uniformization technique to map the information of the continuous-time Markov chain of our algorithm (transition rates) to a transition probability matrix P of a discrete-time Markov chain. **(ii)** We employ the arguments provided in Chapter 20 of [34] to find bounds for the gap between $\mathbf{p}^*$ and the empirical state distribution obtained by the algorithm at time t (in terms of the second largest eigenvalue of P). **(iii)** We use Cheeger's inequality [35] to bound the eigenvalue involved in the previous step. **(iv)** We find bounds for the involved transition rates and the arising state probabilities of the discrete-time Markov chain. Please refer to the technical report provided in [33] for the details. □

We have just found the bounds for the mixing time (convergence time) of the proposed algorithm. An important analysis to be done at this point is to determine how these bounds are affected by the values of T, ϵ, β, N and K.

In the following, to facilitate the interpretation, we assume that T is not dependent on N, and we take it as an independent variable.

Now, with a careful look at the expressions in (13) and (14), we can construct a table for the bounds of the mixing time when each parameter is considered separately. This information is presented in Table I for the worst case of the upper bound (i.e., when analizing one parameter, the other parameters yield nonnegative exponential behavior).

Table I
WORST CASE ASYMPTOTIC ANALYSIS OF THE MIXING TIME OF THE ALGORITHM FOR DIFFERENT SYSTEM PARAMETERS

Parameter	$\Omega(\cdot)$	$O(\cdot)$
ϵ	$-\log(\epsilon)$	$-\log(\epsilon)$
T	T	T
β	$\exp(-\beta K r(1))$	$\beta\exp(c_2\beta)$
K	$\exp(-c_1 K)$	$K\exp(c_3 K)$
N	1	$N\exp(c_4 N)$

where $c_1 = \beta r(1) + \log(2)$, $c_2 = 2NKr(1) + Kr(1)$, $c_3 = 2\beta N r(1) + r(1) + \log(2) + 2N log(2)$, $c_4 = 2\beta K r(1) + 2K\log(2)$. In the table each entry assumes all other variables fixed.

Our analysis in Table I focuses on the cases where the combination of different values of the parameters yield a feasible system. From there it is easy to see that, as ϵ decreases (i.e., decreases the closeness between empirical distribution and $\mathbf{p}^*$), the convergence time is expected to increase at a logarithmic rate. Also, when the transmission time T increases, the convergence time increases linearly.

Not surprisingly, whenever the size of the system grows (i.e., K or N), the upper bound on the convergence time of the algorithm grows at a pace given by $x\exp(x)$. One interesting fact, though, is that our approximation parameter β also shows an influence of the type of $x\exp(x)$ on the convergence time. This indicates that although we need β to grow as large as possible to reach the solution of (4), the convergence speed to reach to $p^*_{\mathbf{s}}(\Phi(\mathbf{s}))$ seems to decrease almost exponentially. In the following section we provide some numerical examples of this tradeoff, and show that in some cases a good level of optimality can still be achieved with small β.

Worth noting in the case of β, however, is the fact that there could exist practical situations in which larger

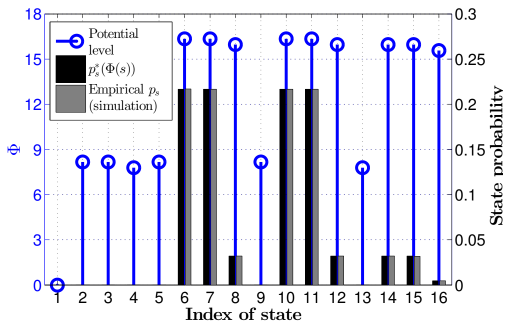

Figure 2. Potential levels for different states in simulation scenario and relationship with equilibria.

values give better convergence speeds. This for example occurs when all possible utilities for the SUs give a positive value, because the growth of β can only increase the initiation rates of the users, which translates into more frequent state changes. The convergence speed in those cases increases with β exponentially, as it can be seen looking at the asymptotic notation analysis of those cases.

IV. Numerical Analysis

In this section we present numerical results to assess the performance of our algorithm. We consider two sets of simulation scenarios, one with small N and K to analyze the details of our algorithm, and other cases with larger system size to analyze the convergence performance. In the first case we use a simulation scenario composed of 2 SUs and 2 subchannels (16 states), and with a mean primary transmission time of $1/\nu_{\text{PU}} = 0.1$ [sec]. The rest of the configuration parameters are the following: $\beta = 5$, $\gamma = 3$ [1/sec], $z = 0.1(1/\nu_{\text{PU}})$ [sec], $\delta = 0.01$, $P = 2$ [watts], $B = 10$ [Hz], $\sigma^2 = 0.4$ [watts].

In Figure 2 we plot the potential levels for each state of the system and the corresponding state probability distributions obtained with our algorithm. The darker bars shown in the plot correspond to the theoretical steady-state distribution $p^*_{\mathbf{s}}(\Phi(s))$, and the other bars correspond to the empirical distribution obtained from the operation of our algorithm. It is shown in the figure that, for all the states of the system, both distributions have virtually the same values. This is achieved by running the simulation for 3×10^4 [sec].

Regarding the potential levels of all the states, we can identify 4 local maxima in Figure 2, corresponding to an approximate value of 16.3399. These are located at states 6, 7, 10 and 11, which represent respectively: the case where the the last SU occupies all subchannels and the first SU occupies none, the case where the last SU occupies the last subchannel and the first SU occupies the first subchannel, and the cases formed by swapping the SUs in the subchannels being used. As mentioned in Section II, these 4 states attain the NE of the CG.

From the plot we can see that the probability distributions over the states follows closely the behavior of the equilibria, making clear that the secondary system randomizes mainly over the equilibrium states equiprobably. For $\beta = 5$, the average potential achieved in the secondary network with the empirical distribution

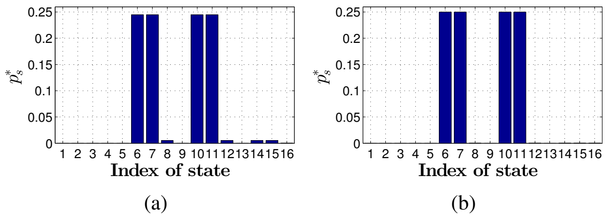

Figure 3. Steady-state distributions in simulation scenario for (a) $\beta = 10$ and (b) $\beta = 40$

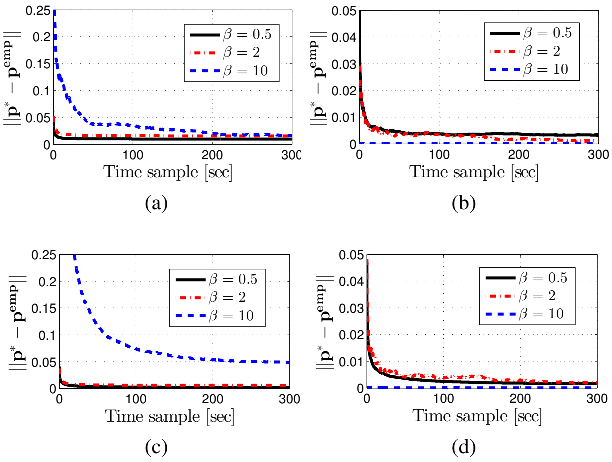

Figure 4. Convergence behavior in time for different β, with: (a) $N = 4$, $K = 2$ and negative rewards for crowded subchannels; (b) $N = 4$, $K = 2$ and always positive rewards; (c) $N = 4$, $K = 3$ and negative rewards for crowded subchannels; (d) $N = 4$, $K = 3$ and always positive rewards.

is 16.2872, which is fairly close to the theoretical one given the small value of the approximation parameter. Additionally, this is indicative of the existence of certain fairness in the access policy, since the states that represent only one SU transmitting are visited in the same proportion to the states that are beneficial for all SUs at the same time.

An additional observation can be made on the non-equilibrium states and their non-zero probabilities (e.g., state 8). This situation corresponds directly to the approximation gap between the solution of (4) and the solution of (7). Intuitively, a completely optimal algorithm would randomize only over the equilibrium states, since those are the ones that maximize the potential level of the network. Clearly, then, with a better approximation achieved by increasing β, the non-equilibrium states should exhibit probabilities closer to 0. This is precisely what Figure 3 presents for the same simulation scenario: larger values of β translate into almost zero probability for the non-equilibrium states.

We can infer from Figure 3 that the larger the β, the better performance in the sense of potential of the secondary network. However, as shown in Figure 4 and as discussed before, growing β carelessly harms the convergence speed.

The results depicted in Figure 4a correspond to a simulation scenario with $\gamma = 2$ ($\gamma = 0.3$ in Figure 4b), while the rest of the parameters are the same as the ones used in the previous plots. It is evident from Figure 4a that, when there exist negative rewards for crowded subchannels (worst case in the asymptotic analysis), increasing β produces a slow convergence to the theoretic steady-state distribution. The opposite occurs in Figure 4b, where always positive rewards translate into increased convergence speed for larger β. This fact confirms our discussion on the asymptotic analysis of the mixing time of the algorithm.

In the case of always positive rewards, the equilibrium solution represents the main issue, since the best thing for the SUs is to prioritize the transmission over all the subchannels at the same time, regardless of the number of interfering SUs. In that case, any access scheme becomes useless, since it is always better for the users to keep adding subchannels to their strategies, even in high interference. This situation calls for further study of the theoretical implications of the energy consumption cost and its relationship with the efficacy of the spectrum sharing methods.

Lastly, in Figures 4c and 4d we present the convergence behavior of a simulated system of 4096 states, with $\gamma = 1.5$ and $\gamma = 0.1$, respectively (without changing the other parameters). According to the plots, we have a similar trend to the ones in Figures 4a and 4b, but with higher convergence time due to a larger network scenario. A direct evidence of this is the line of $\beta = 10$ in Figure 4c, which strengthens the arguments provided in our discussion of the asymptotic worst-case performance for larger networks.

V. Concluding Remarks

In this work we developed a novel distributed algorithm for the multiple access of secondary users (SUs) in a multichannel OSA environment. By formulating the spectrum access problem as a congestion game, we showed that our multiple access algorithm achieves close-to-optimal equilibrium performance (with explicit approximation gap) in a SINR regime.

Additionally, we analytically derived explicit performance bounds on optimality and convergence speed of our proposed algorithm, and we provided insights into its asymptotic behavior for each parameter involved. To corroborate our analytical findings, we presented numerical results indicating that an increased value of β translates into a better approximation of the algorithm to the optimal equilibrium performance, which in turn allows for a fairer utilization of the resources.

Our numerical results, along with theoretical asymptotic analysis, also showed that a careless growth of the approximation parameter β leads to reduced convergence speed of the algorithm in some cases. In those cases a small value of β was found to provide a good optimality/convergence speed tradeoff in the sense of the achieved average network potential. This reality, however, was shown to be strictly dependent on the other parameters of the system and the nature of the user rewards: always positive rewards yielded better convergence speed with larger β, but at the expense of an access behavior where every SU always occupies all subchannels. This clearly configures a trivial and impractical distributed access scheme that offers no performance guarantees for secondary systems in reality.

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