Sensor Selection Under Correlated Shadowing in Cognitive Radio Networks

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***Abstract*—In this letter, we study optimal sensor selection for cooperative sensing in cognitive radio networks using a linear soft fusion approach. Our goal is to find a set of K sensors and its optimal linear combination rule that maximizes the expected system capacity of secondary users while meeting the requirement on the protection of primary transmission. We formulate the problem as a cardinality-constrained convex optimization problem and propose approximation algorithms that account for the effect of pathloss and correlated shadowing. Simulation results show that the proposed algorithms achieve comparable performance to the optimal solution.**

***Index Terms*—Cognitive radio, cooperative spectrum sensing, sensor selection.**

I. Introduction

COOPERATIVE sensing has been extensively studied to improve sensing precision in Cognitive Radio (CR) via multiple secondary users (SUs) sensing the primary user (PU) channel and combining local sensing results at a fusion center. Therefore, main research issues in cooperative sensing are determination of nodes to sense the channel (sensor selection) and how to combine local sensing data to make a final decision on channel availability (fusion rule). In sensor selection, a subset of sensors must be selected at a time to reduce energy consumption [1]. Finding an optimal set of sensors is closely related to the correlation structure between sensors due to shadowing as correlation has an adverse impact on cooperation gain [2]–[4]. To tackle this problem, most of existing sensor selection algorithms focus on minimizing correlation effect based on the distance between sensors. However, since different sensors may also experience different pathloss based on their distance from PU transmitter, it should be also taken into account together with distance between sensors.

Given a fixed set of sensors, we still need to determine a fusion rule of local sensing results. If the distributions of received signals at SU side are known to fusion center, an optimal solution can be obtained by using likelihood ratio test (LRT) of binary hypothesis testing. However, the solution from LRT has a mathematically tractable form only for some limited cases, such as the measurement distributions under two hypothesis having the same covariance matrices or same mean vectors for multivariate Gaussian distributions. As an approximation, a linear fusion and a linear-quadratic fusion based on deflection criteria are studied in [5] and [6], respectively.

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In this letter, we find a set of SUs to sense the channel and its corresponding weight vector for linear soft fusion, which maximizes expected SU system capacity with a constraint on the number of SUs selected at a time, while meeting tolerable level of interference on PU transmissions. Since we assume each SU is equipped with a sensor, sensors and SUs are used interchangeably.

II. System Model and Problem Formulation

We consider a centralised CR network with N SUs. A fusion center selects a set of SUs to sense PU channel and collects local sensing data to make final decision on channel availability. Let us denote by y_i the distribution of received signal strength of PU at SU i. Using the same signal model from [4], $\mathbf{y} = \{y_1, y_2, \cdots, y_N\}$ has multivariate Gaussian distribution.

$$\mathbf{y} \sim \begin{cases} \mathcal{N}(\boldsymbol{\mu}_0, \Sigma_0) & H_0 \\ \mathcal{N}(\boldsymbol{\mu}_1, \Sigma_1) & H_1 \end{cases} \tag{1}$$

where H_0 (H_1) represents idle (busy) channel. i-th component of $\boldsymbol{\mu}_0$ and $\boldsymbol{\mu}_1$ represent the mean of received signal power determined by PU transmit power and distance-dependent pathloss between PU transmitter and SU i under H_0 and H_1 respectively. Under H_1, (i,j)-th component of Covarinace matrix Σ_1 is $\sigma_i \sigma_j \rho_{ij}$, where σ_i^2 is total variance of noise and shadowing observed at SU i and ρ_{ij} is correlation between SU i and SU j. If we adopt Gudmunson's correlation model as in [4], $\rho_{ij} = e^{ad_{ij}}$, where d_{ij} is geographical distance between SU i and SU j and a is a constant value representing decorrelation distance. Due to absence of PU signal under H_0, $\Sigma_0 = \sigma_0^2 I$, where σ_0^2 is noise variance.

Let us denote a subvector of $\mathbf{y}$ on a set S by $\mathbf{y}_S = \{y_i\}_{i \in S}$. Using linear soft fusion, local sensing results from a sensing set S are linearly combined at a fusion center to obtain y as weighted sum of local measurements, i.e. $y = \sum_{i \in S} w_i y_i = \mathbf{w}^T \mathbf{y}_S$. Since y_i is Gaussian, its weighted sum y also has a Gaussian distribution as follows.

$$y \sim \begin{cases} \mathcal{N}(\mathbf{w}^T \boldsymbol{\mu}_{0_S}, \mathbf{w}^T \Sigma_{0_S} \mathbf{w}) & H_0 \\ \mathcal{N}(\mathbf{w}^T \boldsymbol{\mu}_{1_S}, \mathbf{w}^T \Sigma_{1_S} \mathbf{w}) & H_1 \end{cases} \tag{2}$$

where $\boldsymbol{\mu}_{1_S}$ ($\boldsymbol{\mu}_{0_S}$) and Σ_{1_S} (Σ_{0_S}) are subvector of $\boldsymbol{\mu}_1$ ($\boldsymbol{\mu}_0$) and submatrix of Σ_1 (Σ_0) on the set S, respectively.

After collecting local sensing results from a sensor set S, fusion center makes a final decision on channel availability using the following threshold test.

$$y \underset{H_0}{\overset{H_1}{\gtrless}} \eta \tag{3}$$

Given η, from the distribution of y in (2), probability of detection and false alarm of (3) can be evaluated as follows.

$$P_f = Q\Big(\frac{\eta - \mathbf{w}^T \boldsymbol{\mu}_{0_S}}{\sqrt{\mathbf{w}^T \Sigma_{0_S} \mathbf{w}}}\Big), \quad P_d = Q\Big(\frac{\eta - \mathbf{w}^T \boldsymbol{\mu}_{1_S}}{\sqrt{\mathbf{w}^T \Sigma_{1_S} \mathbf{w}}}\Big) \tag{4}$$

where $Q(\cdot)$ is complementary cumulative distribution function of standard normal. Now we formulate SU capacity maximising sensor selection problem.

$$\max_{S\subseteq V,\mathbf{w}\geq\mathbf{0}} \; r_0 P(H_0)P(y \leq \eta|H_0) \tag{5}$$
$$\text{s.t. } r_1 P(H_1)P(y \geq \eta|H_1) \geq \tau \tag{6}$$
$$|S| \leq K \tag{7}$$

where r_0 and r_1 is system capacity of SU and PU networks. We assume that concurrent transmissions will lead to a failure of both PU and SU communications. Our objective is to find an optimal SU set S from total SU set $V = \{1, 2, \ldots, N\}$ which maximizes SU capacity. The first constraint is for the purpose of PU protection; PU capacity should remain greater than or equal to the predetermined threshold τ. The second constraint is sensor set size constraints; the maximum number of SUs that can be selected at a time should be limited to a predetermined number K. Note that $P(y \leq \eta|H_0)$ and $P(y \geq \eta|H_1)$ correspond to $1 - P_f$ and P_d in (4) and they both change as sensor set S and weight vector $\mathbf{w}$ change. Assuming constant r_0, r_1, $P(H_0)$ and $P(H_1)$, (5)-(7) can be rewritten as follows:

$$\max_{\mathbf{w}} \; \frac{Q^{-1}(\tau')\sqrt{\mathbf{w}^T\Sigma_1\mathbf{w}} + \mathbf{w}^T(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)}{\sqrt{\mathbf{w}^T\Sigma_0\mathbf{w}}} \tag{8}$$
$$\text{s.t. } Card(\mathbf{w}) \leq K \tag{9}$$

where $\tau' = \frac{\tau}{r_1 P(H_1)}$. We use the fact that given P_d, $\eta = Q^{-1}(P_d)\sqrt{\mathbf{w}^T\Sigma_1\mathbf{w}} + \mathbf{w}^T\boldsymbol{\mu}_1$ and $1 - P_f$ is maximized at $P_d = \frac{\tau}{r_1 P(H_1)}$ for any τ. Plugging this into P_f formula in (4), we obtain $P_f = Q\Big(\frac{Q^{-1}(\tau')\sqrt{\mathbf{w}^T\Sigma_1\mathbf{w}}+\mathbf{w}^T(\boldsymbol{\mu}_1-\boldsymbol{\mu}_0)}{\sqrt{\mathbf{w}^T\Sigma_0\mathbf{w}}}\Big)$. Since Q-function is monotonically decreasing, taking Q^{-1} on P_f, we obtain the objective function in (8) while satisfying the first constraint. Note that $\boldsymbol{\mu}_1$, $\boldsymbol{\mu}_0$, Σ_1 and Σ_0 in (8) is of the entire SU set, not of a specific subset S. The cardinality constraint in (9) will zero out i-th element of $\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0$ and i-th row and column of Σ_0 and Σ_1 if i-th SU is not selected for sensing, i.e. $i \notin Supp(\mathbf{w})$ where $Supp(\mathbf{w})=\{i : w_i \neq 0\}$.

We replace $Q^{-1}(\tau')$ with $-\alpha$ and $\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0$ with $\boldsymbol{\mu}_1$ by setting $\boldsymbol{\mu}_0$ to $\mathbf{0}$ without loss of generality for simplicity. If we further assume $P_f < \frac{1}{2}$, (8)-(9) is equivalent to the following.

$$\max_{\mathbf{w}} \; -\alpha\sqrt{\mathbf{w}^T\Sigma_1\mathbf{w}} + \mathbf{w}^T\boldsymbol{\mu}_1 \tag{10}$$
$$\text{s.t. } \mathbf{w}^T\Sigma_0\mathbf{w} \leq 1 \tag{11}$$
$$Card(\mathbf{w}) \leq K \tag{12}$$

Now we have convex objective function when $\alpha \geq 0$, i.e. $P_d \geq \frac{1}{2}$, which is true for most practical CR networks. It is worth to note that the first term, $\sqrt{\mathbf{w}^T\Sigma_1\mathbf{w}}$ and the second term, $\mathbf{w}^T\boldsymbol{\mu}_1$, of the objective function represent standard deviation and mean of y, respectively. Therefore, the objective (10) can be interpreted as choosing SU set S which minimises the variance of y while maximising its mean. Because standard deviation of y tends to increase as correlation between y_i increases, it can be seen that choosing the most independent set of SUs is more desirable. At the same time, the selection should also account for resulting mean, which may or may not grow with increasing correlation. The coefficient α determines the tolerance level to variance change. Although the objective is now formulated as a convex function, (10)-(12) are not a convex program due to non-convexity of its constraint set.

III. Solution

A. Exact Solution

We rewrite (10)-(12) by introducing $\mathbf{z} = \Sigma_0^{1/2}\mathbf{w}$.

$$\max_{\mathbf{z}} \; -\alpha\sqrt{\mathbf{z}^T\Sigma\mathbf{z}} + \mathbf{z}^T\boldsymbol{\mu} \tag{13}$$
$$\text{s.t. } \mathbf{z}^T\mathbf{z} \leq 1 \tag{14}$$
$$Card(\mathbf{z}) \leq K \tag{15}$$

Where $\Sigma = \Sigma_0^{-\frac{1}{2}}\Sigma_1\Sigma_0^{-\frac{1}{2}}$ and $\boldsymbol{\mu} = \Sigma_0^{-\frac{1}{2}}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)$. Cardinality constraint is not affected by this change of variable since Σ_0 is a diagonal matrix. By introducing an auxiliary variable $s = \sqrt{\mathbf{z}^T\Sigma_1\mathbf{z}}$ and a 0-1 variable u_i to enforce $z_i = 0$ or $z_i \neq 0$, (13)-(15) can be reformulated as the follows.

$$\max_{\mathbf{z},s\geq 0,\mathbf{u}\in\{0,1\}^N} \; -\alpha s + \boldsymbol{\mu}^T\mathbf{z} \tag{16}$$
$$\text{s.t. } \|A\mathbf{z}\| \leq s \tag{17}$$
$$\|\mathbf{z}\| \leq 1 \tag{18}$$
$$\mathbf{1}^T\mathbf{u} \leq K \tag{19}$$
$$0 \leq z_i \leq M_i u_i, \quad \forall i \in \{0, 1, \ldots, N\} \tag{20}$$

where $\|\cdot\|$ denotes the Euclidean norm and $\mathbf{1}$ is an all one column vector. M_i are the upper bound of z_i and can simply be set to $M_i = M$ for a sufficiently large M. A is a Cholesky decomposition of Σ, i.e. $\Sigma = A^T A$. (16)-(20) is a mixed integer convex problem, more specifically, MISOCP (Mixed Integer Second Order Cone Programming). Due to integer variables, this family of problems usually become combinatorial and known to be NP-hard [7]. Convex (continuous) relaxation on $\mathbf{u}$ such that $\mathbf{u} \in [0, 1]^N$ will provide a lower bound with fractional solution. A straightforward way to solve this problem is basic branch and bound based algorithms with interior point method to solve SOCP at each search tree node. However, solving SOCP for $\binom{N}{K}$ possible choices is not practical unless N and K are small.

B. Approximation Algorithms

In this section, we provide a simple approximation algorithm that consists of two steps; Fix and SOCP. We first select the sensor set and optimize the weight vector $\mathbf{w}$ on the fixed set by solving SOCP (16)-(18). In the following Max Mean algorithm, we simply pick SUs with K top element of $\boldsymbol{\mu}$. Then on receipt of sensing results from them, fusion center uses threshold test in (3) to decide channel availability using $\mathbf{w} = \Sigma_0^{-\frac{1}{2}}\hat{\mathbf{z}}$.

Let us denote by $\mathbf{z}^*$ the weight vector that optimizes (13)-(15) or (16)-(20) and λ_i is i-th largest eigenvalue of Σ and $\mathbf{v}_i$ is its corresponding eigenvector. The following theorem characterizes the performance of the approximation Algorithm 1.

Algorithm 1 Max Mean

Input: $(\boldsymbol{\mu}_1, \Sigma_1)$, $(\boldsymbol{\mu}_0, \Sigma_0)$, P_d, K

1: $\boldsymbol{\mu} \leftarrow \Sigma_0^{-\frac{1}{2}}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)$, $\Sigma \leftarrow \Sigma_0^{-\frac{1}{2}}\Sigma_1\Sigma_0^{-\frac{1}{2}}$, $\hat{\mathbf{z}} \leftarrow \mathbf{0}$, $\alpha \leftarrow Q^{-1}(P_d)$

STEP I *(Fix)* :

2: $\hat{S} \leftarrow$ indice of K top elements of $\boldsymbol{\mu}$

STEP II *(SOCP)* :

3: $A \leftarrow$ Cholesky decomposition of $(\Sigma_{\hat{S}})$, $\boldsymbol{\mu} \leftarrow \boldsymbol{\mu}_{\hat{S}}$

4: $\hat{\mathbf{z}}(\hat{S}) = \underset{\mathbf{z}}{\text{argmax}}\{-\alpha s + \boldsymbol{\mu}^T\mathbf{z} : \|A\mathbf{z}\| \leq s, \|\mathbf{z}\| \leq 1, (s, \mathbf{z}) \succcurlyeq \mathbf{0}\}$

Output: $\hat{\mathbf{z}}$

Theorem 1: For any $1 \leq K \leq N$, Algorithm 1 outputs $\hat{\mathbf{z}}$, where $\|\hat{\mathbf{z}}\| = 1$, $\text{Card}(\hat{\mathbf{z}}) \leq K$ and

$$-\alpha\sqrt{\hat{\mathbf{z}}^T\Sigma\hat{\mathbf{z}}} + \boldsymbol{\mu}^T\hat{\mathbf{z}} \geq (1-\epsilon)\big(-\alpha\sqrt{\mathbf{z}^{*T}\Sigma\mathbf{z}^*} + \boldsymbol{\mu}^T\mathbf{z}^*\big)$$

with $\epsilon = (1 - \sqrt{\frac{\lambda_N}{\lambda_1}})$, if $\mu_i \geq 2\alpha\sigma_i$, $\forall i \in \{1, 2, \cdots, N\}$

Proof: Define $Z = \{\mathbf{z} : \|\mathbf{z}\| = 1, \text{Card}(\mathbf{z}) \leq K\}$ and $f(\mathbf{z}) = -\alpha\sqrt{\mathbf{z}^T\Sigma\mathbf{z}} + \boldsymbol{\mu}^T\mathbf{z}$. We first find an upper bound on the optimal solution.

$$f(\mathbf{z}^*) \leq \max_{z\in Z} -\alpha\sqrt{\mathbf{z}^T\Sigma\mathbf{z}} + \max_{z\in Z}\boldsymbol{\mu}^T\mathbf{z} \leq \|\boldsymbol{\mu}^{(k)}\| - \alpha\sqrt{\lambda_N} \tag{21}$$

By Cauchy-Schwartz inequality, $\max_{z\in Z}\boldsymbol{\mu}^T\mathbf{z}$ has optimal solution $\|\boldsymbol{\mu}^{(k)}\|$, where $\boldsymbol{\mu}^{(k)}$ denotes the vector obtained from $\boldsymbol{\mu}$ by replacing N-K elements of $\boldsymbol{\mu}$ with smallest magnitude by 0. Since $\{\mathbf{z}^T\Sigma\mathbf{z}; \|\mathbf{z}\| = 1\} \in [\lambda_N, \lambda_1]$ by Rayleigh-Ritz theorem, it can be easily seen that $\max_{z\in Z} -\alpha\sqrt{\mathbf{z}^T\Sigma\mathbf{z}} \leq -\alpha\sqrt{\lambda_N}$. Now, we find a lower bound on $f(\hat{\mathbf{z}})$. Since $Supp\{\boldsymbol{\mu}^{(k)}\} = \hat{S}$ from Algorithm 1,

$$\|\boldsymbol{\mu}^{(k)}\| - \alpha\sqrt{\lambda_1^{(k)}} \leq f\Big(\frac{\boldsymbol{\mu}^{(k)}}{\|\boldsymbol{\mu}^{(k)}\|}\Big) \leq f(\hat{\mathbf{z}}) \tag{22}$$

where $\lambda_1^{(k)}$ is the largest eigenvalue of zeroed-out version of Σ except on $Supp\{\boldsymbol{\mu}^{(k)}\}$. Combining (21) and (22), we get

$$f(\hat{\mathbf{z}}) \geq \|\boldsymbol{\mu}^{(k)}\| - \alpha\sqrt{\lambda_1^{(k)}} \geq f(\mathbf{z}^*) + \alpha(\sqrt{\lambda_N} - \sqrt{\lambda_1^{(k)}})$$

Therefore,

$$\frac{f(\mathbf{z}^*) - f(\hat{\mathbf{z}})}{f(\mathbf{z}^*)} \leq \frac{1}{\frac{\|\boldsymbol{\mu}^{(k)}\|}{\alpha\sqrt{\lambda_1^{(k)}}} - 1}\Big(1 - \sqrt{\frac{\lambda_N}{\lambda_1^{(k)}}}\Big) \leq 1 - \sqrt{\frac{\lambda_N}{\lambda_1}}$$

The second inequality follows from the fact that $\lambda_1^{(k)} \leq \lambda_1, \forall k$ and the condition that $\mu_i \geq 2\alpha\sigma_i$, $\forall i \in \{1, 2, \cdots, N\}$. Under the condition the following inequality holds for all $\mathbf{z} \in Z$.

$$0 \leq (\boldsymbol{\mu} - 2\alpha\boldsymbol{\sigma})^T\mathbf{z} \leq \boldsymbol{\mu}^T\mathbf{z} - 2\alpha\sqrt{\mathbf{z}^T\Sigma\mathbf{z}}$$

by setting $\mathbf{z}$ to the eigenvector correponding to the largest eigenvalue $\lambda_1^{(k)}$, it can be shown that $\|\boldsymbol{\mu}^{(k)}\| \geq 2\alpha\sqrt{\lambda_1^{(k)}}$, resulting in $\frac{1}{\frac{\|\boldsymbol{\mu}^{(k)}\|}{\alpha\sqrt{\lambda_1^{(k)}}} - 1} \leq 1$. ■

Remark 1: As the difference between maximum and minimum eigenvalue of Σ becomes smaller, $f(\hat{\mathbf{z}})$ approaches the optimal solution, which occurs when Σ_1 becomes uncorrelated and the variance of each SU becomes similar. As expected, when $\Sigma = \sigma^2 I$, Max Mean algorithm yields the optimal solution.

Since Algorithm 1 has better performance as the covariance matrix becomes uncorrelated, we propose another algorithm that works better for the opposite case where the local sensings are highly correlated. In this case, we can expect the impact of Σ term to become more important and should be taken into account. In the following Low Rank algorithm, we approximate Σ to a rank-1 matrix $\lambda_1\mathbf{v_1}\mathbf{v_1}^T$ in its one dimensional eigen subspace and find a set of SUs that maximizes $-\alpha\sqrt{\mathbf{z}^T(\lambda_1\mathbf{v_1}\mathbf{v_1}^T)\mathbf{z}} + \boldsymbol{\mu}^T\mathbf{z}$. After the set is fixed, we solve SOCP on the set. We only need to modify STEP I in Algorithm 1 as follows. In line 2, $\Lambda = \{\lambda_i\}_{i=1}^N$

Algorithm 2 Low Rank

1: *STEP* I *(Fix)* :

2: $[\Lambda, V] \leftarrow$ eigen decomposition of Σ

3: $\hat{S} \leftarrow$ indice of K top non-negative elements of $\boldsymbol{\mu} - \alpha\sqrt{\lambda_1}\mathbf{v_1}$

and $V = \{\mathbf{v}_i\}_{i=1}^N$. We note that the computation complexity of the algorithm is polynomial in N, which is $O(N^3)$ from eigen-decomposition in STEP 1. SOCP solver in STEP 2 has polynomial-time complexity $O(K^3)$ [8].

Theorem 2: For any $1 \leq K \leq N$, Algorithm 2 outputs $\hat{\mathbf{z}}$, where $\|\hat{\mathbf{z}}\| = 1$, $\text{Card}(\hat{\mathbf{z}}) \leq K$ and

$$-\alpha\sqrt{\hat{\mathbf{z}}^T\Sigma\hat{\mathbf{z}}} + \boldsymbol{\mu}^T\hat{\mathbf{z}} \geq (1-\epsilon)\big(-\alpha\sqrt{\mathbf{z}^{*T}\Sigma\mathbf{z}^*} + \boldsymbol{\mu}^T\mathbf{z}^*\big)$$

with $\epsilon = \sqrt{\frac{\lambda_2}{\lambda_1}}$, if $\mu_i \geq \alpha(1 + \sqrt{\frac{N}{K}})\sigma_i$, $\forall i \in \{1, 2, \cdots, N\}$

Proof: Let $f_1(\mathbf{z}) = -\alpha\sqrt{\mathbf{z}^T\lambda_1\mathbf{v_1}\mathbf{v_1}^T\mathbf{z}} + \boldsymbol{\mu}^T\mathbf{z}$ and $\mathbf{z}_1 = \underset{z\in Z}{\text{argmax}}\, f_1(\mathbf{z})$. Then the following inequalities hold.

$$\begin{aligned} f_1(\mathbf{z}_1) &\geq f(\mathbf{z}^*) \geq f(\mathbf{z}_1) \\ &\geq -\alpha\sqrt{\mathbf{z}_1^T\lambda_1\mathbf{v}_1\mathbf{v}_1^T\mathbf{z}_1} - \alpha\sqrt{\mathbf{z}_1^T\Big(\sum_{2\leq i\leq N}\lambda_i\mathbf{v}_i\mathbf{v}_i^T\Big)\mathbf{z}_1} + \boldsymbol{\mu}^T\mathbf{z}_1 \\ &\geq f_1(\mathbf{z}_1) - \max_{z\in Z}\alpha\sqrt{\mathbf{z}^T\Big(\sum_{2\leq i\leq N}\lambda_i\mathbf{v}_i\mathbf{v}_i^T\Big)\mathbf{z}} \\ &\geq f_1(\mathbf{z}_1) - \alpha\sqrt{\lambda_2} \\ &\geq f(\mathbf{z}^*) - \alpha\sqrt{\lambda_2} \end{aligned}$$

Therefore,

$$\frac{f(\mathbf{z}^*) - f(\hat{\mathbf{z}})}{f(\mathbf{z}^*)} \leq \frac{f(\mathbf{z}^*) - f(\mathbf{z}_1)}{f(\mathbf{z}^*)} \leq \frac{\alpha\sqrt{\lambda_2}}{f(\mathbf{v}_1^{(k)})} \leq \sqrt{\frac{\lambda_2}{\lambda_1}}$$

The first inequality holds because $Supp\{\mathbf{z}_1\} = Supp\{\hat{\mathbf{z}}\}$ and $f(\hat{\mathbf{z}})$ is the optimal solution on the given $Supp\{\mathbf{z}_1\}$. The last two inequalities can be shown in a similar way to the proof of Theorem 1 using a lower bound on $f(\mathbf{z}^*) \geq f(\mathbf{v}_1^{(k)})$ and a lower bound on $(\mathbf{v}_1^{(k)})^T\Sigma\mathbf{v}_1^{(k)} \geq \frac{K}{N}\lambda_1$, where $\mathbf{v}_1^{(k)}$ denotes the vector obtained from $\mathbf{v}_1$ by replacing N-K elements of $\mathbf{v}_1$

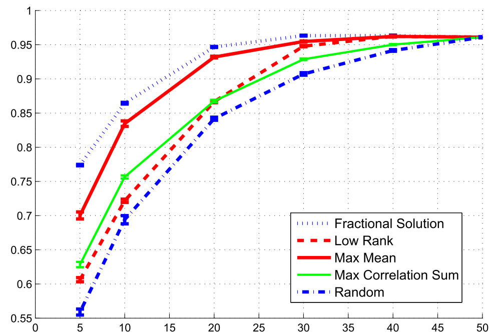

Fig. 1. $1 - P_f$ vs. K when 50 SUs are distributed within 1km range.

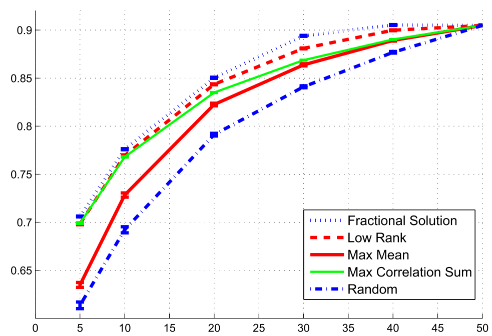

Fig. 2. $1 - P_f$ vs. K when 50 SUs are distributed within 200m range.

with smallest magnitude by 0 and normalizing it to have a unit norm. ■

Remark 2: $f(\hat{\mathbf{z}})$ approaches optimal solution as λ_2 decays faster, which occurs when Σ_1 becomes highly correlated and the variance of each SU becomes similar. As expected, when $\Sigma = \boldsymbol{\sigma}\boldsymbol{\sigma}^T$ and $\boldsymbol{\sigma} = \sigma\mathbf{1}$, Low Rank algorithm yields the optimal solution.

We note that the objective function in the original optimization problem is $1 - P_f$, which is $1 - Q(f(\mathbf{z}))$ for linear soft fusion. Let $g(x) = 1 - Q(x)$. Then $g(x)$ is monotonically increasing concave function over $x \geq 0$. Therefore, if $f(\hat{\mathbf{z}}) \geq (1-\epsilon)f(\mathbf{z}^*)$, $g(f(\hat{\mathbf{z}})) \geq g((1-\epsilon)f(\mathbf{z}^*)) \geq \epsilon g(0) + (1-\epsilon)g(f(\mathbf{z}^*)) \geq (1-\epsilon)g(f(\mathbf{z}^*))$.

IV. Performance Evaluation

In this section, the solution algorithms of Section III are evaluated numerically. We consider a single PU system with 50 SUs. P_d is fixed to 0.9. Local sensings are correlated according to Gudmundson's correlation model with $a = -5$ which corresponds to decorrelation distance of 200m, and shadow fading has fixed dB-spread $\sigma_1 = 2.0$. To evaluate the algorithms under different pathloss distribution and shadowing correlation, we consider two different SU placement settings; In the first setting, SUs are placed within 1km×1km region at a distance of 10km from PU transmitter. In the second setting, SUs are placed within 200m×200m region at the same distance from PU transmitter. The results under the two settings are shown in Fig. 1 and Fig. 2, respectively. Each point in both Figures are obtained by averaging over 100 samples of random SU placement and the lower and upper boundaries of the 95% confidence interval is shown for each point. In Fig. 1, Max Mean performs better than Low Rank. This is because under the first setting, SUs are distributed over a wide range and tend to be apart from each other. Hence, the mean received signal strength of top K SUs are higher than its average due to larger deviation in pathloss while correlation between SUs are low. On the other hand, Fig. 2 shows Low Rank performs better than Max Mean when SUs are closely placed under the second setting. In this case, Max Mean behaves more like random selection as the difference between mean values of received signal strength becomes small. However, Low Rank gives a good approximation to the optimal solution due to high correlation between SUs. The effect of correlation on the SU system performance can be also seen by comparing the results of Fig. 1 with Fig. 2. Since the correlation becomes higher as the SUs are more densely placed, the results shown in Fig. 1 is better than that of Fig. 2 for any fixed number of sensing SUs and the same algorithm. However, in both settings, the proposed algorithms outperform random selection and obtains good performance approximation to the optimal solution. We also compare the performance of our algorithms with Maximal Correlation Sum algorithm which minimizes correlation effect between SUs [1]. In Maximal Correlation Sum, the fusion center iteratively removes a sensor which has the greatest correlation sum with all the other remaining sensors until K sensors are left. Since the pathloss effect is not considered in Maximal Correlation Sum, its performance is much worse than Max Mean in the first setting.

V. Conclusion

We have developed sensor selection algorithms with optimal linear soft fusion for cooperative sensing in cognitive radio. The performance of proposed algorithms depend on the spectral profile of covariance matrix and the magnitude of mean vectors of the distribution of measurements of PU signal. The simulation results show that the proposed algorithm can be applied to different type of shadowing environments.

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