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Brief paper

Control of discrete time nonlinear systems with a time-varying structure☆

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Abstract

In this paper, we present a control methodology for a class of discrete time nonlinear systems that depend on a possibly exogenous scheduling variable. This class of systems consists of an interpolation of nonlinear dynamic equations in strict feedback form, and it may represent systems with a time-varying nonlinear structure. Moreover, this class of systems is able to represent some cases of gain scheduling control, Takagi–Sugeno fuzzy systems, as well as input–output realizations of nonlinear systems which are approximated via localized linearizations. We present two control theorems, one using what we call a "global" approach (akin to traditional backstepping), and a "local" approach, our main result, where backstepping is again used but the control law is an interpolation of local control terms. An aircraft wing rock regulation problem with varying angle of attack is used to illustrate and compare the two approaches.

Keywords: Discrete-time nonlinear systems; Time-varying structure; Scheduling variable; Local control; Backstepping; Wing rock

1. Introduction

The problem of control of nonlinear systems presents fundamental difficulties due to the structural complexity arbitrary nonlinear systems may possess. In order to perform stability analysis, it is of basic importance to first determine a *class* of systems that is representative of real problems, and yet amenable to analysis. One such class is that of strict feedback systems, which have been studied in continuous time under the backstepping methodology (Krstić, Kanellakopoulos, & Kokotović, 1995) (see also the references therein, Freeman & Kokotović (1996) and Marino & Tomei (1998), where tracking is addressed). Backstepping currently appears to be the most systematic method for nonlinear control design through step-by-step construction of quadratic Lyapunov functions. The stability analysis is constructive, and it generates stabilizing control laws. In addition, backstepping provides some guidelines on the effects of design parameters on transient performance of the closed loop system.

Here, we develop a theoretical framework and a control methodology which are inspired by backstepping results and which seek to encompass some aspects of gain scheduling control and the results in Tanaka and Sugeno (1992) and Johansen (1994). We present a class of discrete time nonlinear systems that have a time-varying structure. This class of systems is a generalization of the class of strict feedback discrete time systems traditionally considered in the literature (Kanellakopoulos (1994), Yeh & Kokotović (1995) and Zhang, Wen, & Soh (1999a,b)). The time-varying structure of the plant is affected by a possibly exogenous scheduling variable (which may include a subset of the states), and may be thought of as an interpolation between nonlinear "subsystems". The subsystems, or components of the plant, are in strict feedback form, which allows us to methodically develop a control law that guarantees asymptotic stability (which is global under some conditions). We show that there are two ways to construct stabilizing control laws for this class of systems. The first one is developed by treating the system as a whole, and directly applying the backstepping method to it. The second control law is constructed from laws that are tailored to each of the dynamic components of

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the nonlinear system (i.e., "local" laws within the scheduling variable space). In addition to a class of nonlinear systems with a time-varying structure, other systems that may be represented with the class proposed in this paper include a subclass of the systems considered in Tanaka and Sugeno (1992), as well as those in Johansen (1994). Moreover, the ideas developed in this paper have been successfully applied to the problems of direct (Ordóñez & Passino (2001a)) and indirect (Ordóñez & Passino (2001b)) adaptive backstepping in continuous time, but encompassing a larger class of systems than those studied in Krstić et al. (1995) and Polycarpou and Mears (1998).

The paper is organized as follows: In Section 2 we introduce the class of systems in discrete time. In Section 3 we briefly describe the first control design methodology for the class of systems considered. Since this method follows from a direct application of backstepping, its proof is omitted. Next, in Section 4 we show an alternative methodology which relies on "local" controllers to achieve asymptotic stability. This is the main result of the paper, and we apply it in Section 5 to the problem of wing rock regulation, where the angle of attack is allowed to vary with time. Moreover, we present a comparison between the global and local approaches to illustrate the methods.

2. A class of discrete time systems with a time-varying structure

In this section, we consider the development of a control methodology for a class of nonlinear discrete time systems that may represent a wide variety of control problems, including gain scheduling control, interpolations of linearized input–output dynamics, and systems with a time-varying structure. This class of systems and the stability proof are motivated by existing results on discrete time backstepping, in particular Kanellakopoulos (1994), Yeh and Kokotović (1995), Zhang et al. (1999a) and Zhang, Wen, and Soh (1999b). Here, we present general control results that can be applied to the three cases mentioned. For simplicity, we concentrate on the regulation problem, although the results herein can be readily applied to tracking.

Consider the class of systems (where $i = 1,\ldots,n-1$)

$$\begin{aligned} x_i(k+1) &= \phi_i^c(X_i(k), v(k)) + \psi_i^c(X_i(k), v(k))x_{i+1}(k), \\ x_n(k+1) &= \phi_n^c(X_n(k), v(k)) + \psi_n^c(X_n(k), v(k))u(k), \end{aligned} \tag{1}$$

where k is the discrete time index, $v \in \mathbb{R}^q$ is a scheduling variable, $u \in \mathbb{R}$ is the control input, $X_i(k) = [x_1(k),\ldots, x_i(k)]^\top \in \mathbb{R}^i$ is a partial state vector, and the nonlinearities are given by $\phi_i^c(X_i(k), v(k)) = \sum_{j=1}^R \rho_j(v(k))\phi_{i,j}(X_i(k))$ and $\psi_i^c(X_i(k), v(k)) = \sum_{j=1}^R \rho_j(v(k))\psi_{i,j}(X_i(k))$, and we assume $\phi_{i,j}(0) = 0$, (so that $\phi_i^c(0, v) = 0$, for all $v \in \mathbb{R}^q$), $\psi_{i,j}(\cdot) \neq 0$, and $\psi_i^c(\cdot) \neq 0$. The R nonlinear functions $\rho_j : \mathbb{R}^q \to \mathbb{R}$ are assumed to be piecewise continuous in their arguments and bounded for bounded v, and they are assumed to satisfy

$$\sum_{j=1}^R \rho_j(v(k)) < \infty \tag{2}$$

for any $v \in \mathbb{R}^q$, with $v(k)$ a vector of possibly exogenous scheduling inputs. We assume the state vector $X_n(k)$ is measurable. Moreover, the scheduling vectors $v(k), v(k+1),\ldots, v(k+n-1)$ are assumed to be available for measurement and bounded. This assumption may imply that $v(k)$ is given by a stable (but possibly unknown) exogenous dynamical system, and this is the point of view we take here. The assumptions we make on this exogenous system are mild: we do not need to know anything about its dynamics (which may be subjected to bounded disturbances), other than whether the system is internally stable. In this way, with the continuity of the functions ρ_j and assumption (2) we may guarantee that ϕ_i^c and ψ_i^c, $i = 1,\ldots,n$, do neither vanish nor tend to infinity for any bounded v.

3. Direct application of backstepping

Here, we approach the problem of controlling (1) by considering it as a unit, i.e., a strict feedback system with a time-varying structure. In this way, we may directly apply backstepping to (1) and develop a control law. For convenience, we use the notation $v_i(k) = [v(k), v(k+1),\ldots, v(k+i-1)] \in \mathbb{R}^{q\times i}$, $i = 1,\ldots,n$.

Theorem 1. *System* (1) *with the state vector $X_n(k)$ and $v_n(k)$ measurable, and satisfying the assumptions given in Section* 2 *about ϕ_i^c and ψ_i^c and* (2), *together with the recursive equations* (*letting $\alpha_0(k) = 0$*) $\alpha_i(X_i(k), v_i(k)) = (1/\psi_i^c(k))(-\phi_i^c(k)+\alpha_{i-1}(k+1))$, *for* $i = 1,\ldots,n-1$; $z_i(k) = x_i(k) - \alpha_{i-1}(k)$, *for* $i = 1,\ldots,n$; *and the control law* $u(X_n(k), v_n(k)) = (1/\psi_n^c(k))(-\phi_n^c(k)+\alpha_{n-1}(k)+c(k)z_1(k))$ with $c(k)$ such that $c(k) = c_1/\prod_{i=1}^{n-1}|\psi_i^c(k+n-i)|$ and $0 \leqslant |c_1| < 1$ *a design constant, has an asymptotically stable equilibrium at the origin under the scheduling variable $v(k)$. If, in addition, the functions ψ_i^c satisfy the conditions $|\psi_i^c(X_i(k))| \geqslant \beta_i > 0$ for some positive constants β_i, $i = 1,\ldots,n$, the stability results become global. If this condition is not satisfied, the closed loop system is asymptotically stable* (*i.e., a local result*).

Proof. Omitted for brevity.

4. Control via local control laws

Here, we are interested in developing a control strategy for system (1) by treating it as if it were a collection of pieces, or nonlinear dynamical structures, that can be "pulled out" of system (1) for control design purposes. We consider stabilizing control laws for the R pieces, which we later combine to systematically construct a control law for system (1). In this way, we are adopting a "local" point of view (local within the space of the scheduling vector), in that we act as if we were to control each of the R dynamical structures individually with a corresponding control law. As the structure of system (1) changes with the scheduling variable $v(k)$, the controller also changes to the corresponding local control law plus an extra term to eliminate the effects of cross couplings.

For simplicity of the notation, we restrict the class of systems (1) in that now we require the functions ρ_j to form a convex combination, i.e., they must satisfy

$$0 \leqslant \rho_j(\cdot) \leqslant 1 \quad \text{and} \quad \sum_{j=1}^{R} \rho_j(\cdot) = 1, \tag{3}$$

instead of the more general condition (2).

Consider the jth "piece" of (1) (where $i = 1,\dots,n-1$),

$$\begin{aligned} x_{i,j}(k+1) &= \phi_{i,j}(X_{i,j}(k)) + \psi_{i,j}(X_{i,j}(k))x_{i+1,j}(k) \\ x_{n,j}(k+1) &= \phi_{n,j}(X_{n,j}(k)) + \psi_{n,j}(X_{n,j}(k))u_j(k). \end{aligned} \tag{4}$$

Note that we have labeled the "state" $X_{n,j}(k)$, $j = 1,\dots,R$, of this piece of system (1) for design purposes. Although this labeling may be taken literally (i.e., by considering a system that in fact consists of R subsystems, each with its own state, which could be the case in a hierarchical system with a supervisor), here we use this separation into subsystems simply as a conceptual tool.

It may be shown that system (4) can be stabilized with the diffeomorphism (letting $\alpha_0^j(k) = 0$ for all k)

$$\begin{aligned} z_{1,j}(k) &= x_{1,j}(k), \\ z_{i,j}(k) &= x_{i,j}(k) - \alpha_{i-1,j}(k), \end{aligned} \tag{5}$$

where $\alpha_{i-1,j}(k) = (1/\psi_{i-1,j}(X_{i-1,j}(k)))(-\phi_{i-1,j}(X_{i-1,j}(k)) + \alpha_{i-2,j}(k+1))$, $i = 2,\dots,n$ and we choose the control $u_j(k) = (1/\psi_{n,j}(X_n(k)))(-\phi_{n,j}(X_n(k)) + \alpha_{n-1,j}(k+1) + c_j(k)z_{1,j}(k))$, with $c_j(k)$ to be defined below. Now we study a combination of this diffeomorphism and control laws u_j with the purpose of controlling system (1), and so we replace $X_{n,j}(k)$ by $X_n(k)$, the real state vector.

Theorem 2. *System (1) with the state vector $X_n(k)$ and $v_n(k)$ measurable, and satisfying the assumptions given in Section 2 about ϕ_i^c and ψ_i^c and (3), together with the diffeomorphism (5) and the auxiliary functions (letting $\alpha_0(k) = 0$ and $\delta_{1,2}(k) = 0$) $\alpha_m(k) = \alpha_m(X_m(k), v_m(k)) = \sum_{j=1}^{R} \rho_j(k)\alpha_{m,j}(k) + \alpha_m^s(k), m = 1,\dots,n-1$; stabilizing terms $\alpha_m^s(k) = (-1/\psi_m^c(k))(\delta_{m,1}(k) + \delta_{m,2}(k))$, $m = 1,\dots,n-1$; $\delta_{m,1} = \sum_{i=1}^{R} \sum_{j=1, j\neq i}^{\rho_i}(k)\rho_j(k)(\phi_{m,i}(k) - (\psi_m^i(k)/\psi_{m,j}(k))\phi_{m,j}(k)), m = 1,\dots,n$; $\delta_{m,2}(k) = \sum_{i=1}^{R} \rho_i(k) \sum_{j=1}^{R} \alpha_{m-1,j}(k+1)(\rho_j(k)(\psi_{m,i}(k)/\psi_{m,j}(k)) - \rho_j(k+1)) - \alpha_{m-1}^s(k+1)$, $m = 2,\dots,n$; $z_m(k) = x_m(k) - \alpha_{m-1}(k)$, $m = 1,\dots,n$; and the control law*

$$u(k) = u(X_n(k), v_n(k)) = \sum_{j=1}^{R} \rho_j(k)u_j(k) + u^s(k) \tag{6}$$

with the definitions $u^s(k) = (-1/\psi_n^c(k))(\delta_{n,1}(k) + \delta_{n,2}(k) + \delta_{n,3}(k))$, where we use the stabilizing term $\delta_{n,3}(k) = \sum_{i=1}^{R} \sum_{j=1, j\neq i}^{R} \rho_i(k)\rho_j(k)(\psi_{n,i}(k)/\psi_{n,j}(k))c_j(k)z_1(k)$ with $c_j(k) = c_{1,j} / \prod_{i=1}^{n-1} |\psi_i^c(k+n-i)|$, has an asymptotically stable equilibrium at the origin under the scheduling variable $v(k)$, where $0 \leqslant c_{1,j}$ are chosen such that[1]

$$\left(\sum_{i=1}^{R} \rho_i^2(k)c_{1,i}\right)^2 < 1. \tag{7}$$

If, in addition, the functions ψ_i^c satisfy the condition $|\psi_i^c(X_i(k))| \geqslant \beta_i > 0$ for some positive constants β_i, $i = 1,\dots,n$, the stability is global. If this condition is not satisfied, the closed loop system is asymptotically stable (i.e., a local result).

Proof. We start by letting $z_1(k) = \sum_{j=1}^{R} \rho_j(k)z_{1,j}(k)$ so that $z_1(k) = x_1(k)$. Notice that this equality holds because of assumption (3) and due to the fact that we are replacing the conceptual "state" $x_{1,j}$ in (5) by the actual state, x_1. Furthermore, let

$$z_2(k) = \sum_{j=1}^{R} \rho_j(k)z_{2,j}(k) - \alpha_1^s(k) = x_2(k) - \alpha_1(k) \tag{8}$$

with $\alpha_1(k) = \sum_{j=1}^{R} \rho_j(k)\alpha_{1,j}(k) + \alpha_1^s(k)$, where $\alpha_1^s(k)$ is a stabilizing term canceling the effects of cross-terms. Note that $z_1(k+1) = \phi_1^c(k) + \psi_1^c(k)(z_2(k) + \alpha_1(k)) = \psi_1^c(k)(z_2(k) + \alpha_1^s(k)) + \sum_{i=1}^{R} \sum_{j=1}^{R} \rho_i(k)\rho_j(k)(\phi_{1,i}(k) + \psi_{1,i}(k)(1/\psi_{1,j}(k)))(-\phi_{1,j}(k))) = \psi_1^c(k)z_2(k) + \delta_{1,1}(k) + \psi_1^c(k)\alpha_1^s(k)$, with $\delta_{1,1}(k)$ and $\alpha_1^s(k)$ given in the theorem, yielding the dynamic equation

$$z_1(k+1) = \psi_1^c(k)z_2(k). \tag{9}$$

In the special case $\psi_{1,1}(k) = \dots = \psi_{1,R}(k)$, the stabilizing term is no longer necessary and it vanishes, i.e., $\alpha_1^s(k) = 0$. Note also that $\alpha_1(k)$ is a function of $x_1(k)$ and $v(k)$. To finish this analysis step, define the function $V_1(k) = z_1^2(k)$, so that its difference is given by $\Delta V_1(k) = \psi_1^{c^2}(k)z_2^2(k) - z_1^2(k)$.

In the second step, we let $z_3(k) = \sum_{j=1}^{R} \rho_j(k)z_{3,j}(k) - \alpha_2^s(k) = x_3(k) - \alpha_2(k)$, with $\alpha_2(k) = \sum_{j=1}^{R} \rho_j(k)\alpha_{2,j}(k) + \alpha_2^s(k)$. Then, $z_2(k+1) = \phi_2^c(k) + \psi_2^c(k)(z_3(k) + \alpha_2(k)) - \alpha_1(k+1) = \psi_2^c(k)z_3(k) + \sum_{i=1}^{R} \sum_{j=1}^{R} \rho_i(k)\rho_j(k)(\phi_{2,i}(k) + (\psi_{2,i}(k)/\psi_{2,j}(k))(-\phi_{2,j}(k) + \alpha_{1,j}(k+1))) - \sum_{j=1}^{R} \rho_j(k+1)$

[1] Note that the choice $0 \leqslant c_{1,j} < 1$, $j = 1,\dots,R$ satisfies (7) automatically due to assumption (3). This is, however, not a necessary condition, so that some $c_{1,j} > 1$ may still satisfy (7). This is illustrated in Section 5.

$\alpha_{1,j}(k+1)-\alpha_1^s(k+1)+\psi_2^c(k)\alpha_2^s(k)=\psi_2^c(k)z_3(k)+\delta_{2,1}(k)+\delta_{2,2}(k)+\psi_2^c(k)\alpha_2^s(k)$, where $\delta_{2,1}(k)$ and $\delta_{2,2}(k)$ are as given in the theorem statement, and the stabilizing term $\alpha_2^s(k)$ cancels out the effect of the cross terms and yields

$$z_2(k+1)=\psi_2^c(k)z_3(k). \tag{10}$$

Note that here, in the special case when $\psi_{2,1}(k)=\cdots=\psi_{2,R}(k)$, only $\delta_{2,1}(k)$ vanishes, whereas $\delta_{2,2}(k)$ depends on the difference between $\rho(k)$ and $\rho(k+1)$, which is not necessarily zero. Also note that $\alpha_2(k)=\alpha_2(X_2(k),v_2(k))$. To complete this step, let $V_2(k)=V_1(k)+\psi_1^{c^2}(k)z_2^2(k)$, whose difference is $\Delta V_2(k)=\Delta V_1(k)+\psi_1^{c^2}(k+1)\psi_2^{c^2}(k)z_3^2(k)-\psi_1^{c^2}(k)z_2^2(k)=(\prod_{i=1}^{2}\psi_i^{c^2}(k+2-i))z_3^2(k)-z_1^2(k)$.

We may continue this process until the nth state, where the system in z-coordinates is given by

$$\begin{aligned} z_i(k+1)&=\psi_i^c(k)z_{i+1}(k), \qquad i=1,\ldots,n-1,\\ z_n(k+1)&=\phi_n^c(k)+\psi_n^c(k)u(k)-\alpha_{n-1}(k+1), \end{aligned} \tag{11}$$

via the global diffeomorphism given by the recursive equations z_m, together with the auxiliary control laws α_m and the stabilizing terms α_m^s, $\delta_{m,1}$ and $\delta_{m,2}$. If we let $V_{n-1}(k)=V_{n-2}(k)+(\prod_{i=1}^{n-2}\psi_i^{c^2}(k+n-2-i))z_{n-1}^2(k)$ (with $V_1(k)$ and $V_2(k)$ as defined above, and $V_{n-2}(k)$ defined recursively), its difference at time k is given by $\Delta V_{n-1}(k)=(\prod_{i=1}^{n-1}\psi_i^{c^2}(k+n-1-i))z_n^2(k)-z_1^2(k)$.

To complete the proof, note that $z_n(k+1)=\phi_n^c(k)+\psi_n^c(k)u(k)-\alpha_{n-1}(k+1)=\sum_{i=1}^{R}\sum_{j=1}^{R}\rho_i(k)\rho_j(k)(\phi_{n,i}(k)+(\psi_{n,i}(k)/\psi_{n,j}(k))(-\phi_{n,j}(k)+\alpha_{n-1,j}(k+1)+c_j(k)z_{1,j}(k)))-\sum_{j=1}^{R}\rho_j(k+1)\alpha_{n-1,j}(k+1)-\alpha_{n-1}^s(k+1)+\psi_n^c(k)u^s(k)=\delta_{n,1}(k)+\delta_{n,2}(k)+\delta_{n,3}(k)+\sum_{i=1}^{R}\rho_i^2(k)c_i(k)z_1(k)+\psi_n^c(k)u^s(k)$, where we have used the fact that $z_{1,j}(k)=z_1(k)$ by construction, together with the definition of $\delta_{n,3}$. Using the definition of the design terms $c_j(k)$, the control choice (6) yields

$$z_n(k+1)=\frac{\sum_{i=1}^{R}\rho_i^2(k)c_{1,i}}{\prod_{m=1}^{n-1}|\psi_m^c(k+n-m)|}z_1(k). \tag{12}$$

Pick the Lyapunov candidate

$$V(k)=V_{n-1}(k)+\left(\prod_{i=1}^{n-1}\psi_i^{c^2}(k+n-1-i)\right)z_n^2(k),$$

and note that, with a design satisfying (7), one obtains

$$\Delta V(k)=\left(\left(\sum_{i=1}^{R}\rho_i^2(k)c_{1,i}\right)^2-1\right)z_1^2(k)\leqslant 0. \tag{13}$$

Thus, the system in z-coordinates is stable, and asymptotic stability follows from LaSalle's theorem (LaSalle, 1986). □

Remark 3. No assumptions are made on differentiability or continuity of the functions $\rho_j(k)$, so they can be anything that satisfies (3). In particular, these functions may specify the left-hand side of a TS fuzzy system, as has been considered in Tanaka and Sugeno (1992) and Johansen (1994). However, in this paper the right-hand side of the TS fuzzy system would be given by nonlinear systems in strict feedback form, whereas in the aforementioned works it is given by controllable linear systems. Moreover, the functions ρ_j may be piecewise continuous as well, so that the class (1) includes systems with a discontinuously time-varying structure.

5. Application: Wing rock regulation with varying angle of attack using local control laws

Subsonic wing rock is a nonlinear phenomenon experienced by aircraft with slender delta wings, in which limit cycle roll and roll rate oscillations or unstable behavior are experienced by aircraft with pointed fore bodies at high angles of attack. Wing rock may diminish flight effectiveness or even present a serious danger due to potential instability of the aircraft. Here, we apply the methods of global and local control laws of Theorems 1 and 2 to the problem of wing rock regulation and provide a comparison between the two methods. Please see Ordóñez and Passino (2001b) and the references therein for a survey of other approaches to this problem.

Most methods for wing rock regulation are developed at a *fixed* angle of attack, and then in some cases tested at another angle close to the design point, which serves to help claim robustness of the designs. Here, the problem is considered in a more general setting, where the angle of attack is allowed to vary with time according to the evolution of an external dynamical system (which may represent the commands of the pilot together with the aircraft dynamics).

The angle of attack can be used as a scheduling variable that allows us to perform gain scheduling over a set of *nonlinear* models of wing rock. The dependence on the angle of attack is itself a nonlinear relationship in the overall wing rock model. Note that traditional gain scheduling would prefer the use of local linear models. Here, however, a better representation of the phenomenon is obtained through the use of the *already available* nonlinear models.

There exist several analytical nonlinear models that characterize the phenomenon of wing rock (Hsu & Lan (1985), Nayfeh, Elzebda, & Mook (1989)). The model we use as a starting point is the one presented in Nayfeh et al. (1989), which has the advantage over the model in Hsu and Lan (1985) of being differentiable and, according to the authors, slightly more accurate. Like all wing rock models we are aware of, this model is formulated in continuous time, so we use an Euler approximation to convert it to discrete time. The continuous time equation is given by $\ddot\phi=-w_j^2\phi+\mu_{1,j}\dot\phi+b_{1,j}\dot\phi^3+\mu_{2,j}\phi^2\dot\phi+b_{2,j}\phi\dot\phi^2+g\delta_a$, where ϕ is the roll angle, δ_a is the output of an actuator with first-order dynamics, $g=1.5$ is an input gain, and $w_j^2=-c_1a_{1,j}$, $\mu_{1,j}=c_1a_{2,j}-c_2$,

Table 1
Parameters for the coefficients in the wing rock model

v	$a_{1,j}$	$a_{2,j}$	$a_{3,j}$	$a_{4,j}$	$a_{5,j}$
15	−0.01026	−0.02117	−0.14181	0.99735	−0.83478
17	−0.02007	−0.0102	−0.0837	0.63333	−0.5034
19	−0.0298	0.000818	−0.0255	0.2692	−0.1719
21.5	−0.04207	0.01456	0.04714	−0.18583	0.24234
22.5	−0.04681	0.01966	0.05671	−0.22691	0.59065
23.75	−0.0518	0.0261	0.065	−0.2933	1.0294
25	−0.05686	0.03254	0.07334	−0.3597	1.4681

Table 2
Centers and spreads for wing rock interpolation functions

j	1	2	3	4	5	6	7
v_j	15	17	19	21.5	22.5	23.75	25
s_j	1.5	1.5	1.5	2.0	1	1	1

$b_{1,j} = c_1 a_{3,j}$, $\mu_{2,j} = c_1 a_{4,j}$ and $b_{2,j} = c_1 a_{5,j}$ are system coefficients that depend on the parameters $a_{i,j}$, which in turn are functions of the angle of attack, denoted here by v [2]. From Nayfeh et al. (1989) we let $c_1 = 0.354$ and $c_2 = 0.001$, constants given by the physical parameters of a delta wing used in wind tunnel experiments in Levin and Katz (1984). In Nayfeh et al. (1989), four angles of attack are considered, at which the coefficients $a_{i,j}$ are given. In Table 1 we added three points (at $v = 17, 19$ and 23.75 degrees) by assuming that the functions passing through the points $a_{i,j}$ are approximately piecewise linear (a reasonable assumption, considering the plots presented in Nayfeh et al. (1989)).

To build a smooth, time-varying model of the wing rock that depends on the angle of attack v, we consider the interpolation functions

$$\rho_j(v) = \frac{\exp\left(\frac{-(v-v_j)^2}{s_j^2}\right)}{\sum_{l=1}^{7} \exp\left(\frac{-(v-v_l)^2}{s_l^2}\right)},$$

where the centers v_j and spreads $s_j, j = 1,\ldots,7$, are given in Table 2. Notice that these interpolation functions satisfy assumption (3).

In order to test the accuracy of the interpolations, let $a_i(v) = \sum_{j=1}^{7} \rho_j(v) a_{i,j}$, for $i = 1,\ldots,5$. Fig. 1(a) contains the plots of the interpolated coefficients $a_i(v)$ (dashed lines), and a linear interpolation between the points in Table 1 (solid lines). We see that the interpolations are generally close to the data points, so we may consider the resulting time-varying-structure model accurate enough.

We assume the control input u affects the wing through an actuator with linear, first-order dynamics. In order to express the model in the form (1), we let $x_1(t) = \phi(t)$, $x_2(t) = \dot{\phi}(t)$ and $x_3(t) = \delta_a(t)$, and $x_1(k), x_2(k)$ and $x_3(k)$ the corresponding sampled states. Then, assuming a sampling time T, the discrete time wing rock model is given by $x_1(k+1) = x_1(k) + Tx_2(k)$, $x_2(k+1) = x_2(k) + T\sum_{j=1}^{7} \rho_j(v)(-w_j^2 x_1(k) + \mu_{1,j}x_2(k) + b_{1,j}x_2^3(k) + \mu_{2,j}x_1^2(k)x_2(k) + b_{2,j}x_1(k)x_2^2(k)) + Tgx_3(k)$, $x_3(k+1) = (1 - T/\tau)x_3(k) + k_a u(k)$ where the actuator time constant is $\tau = 1/15$ and it has a gain $k_a = 100$. We select the sampling time $T = 0.01$ s, since simulation studies show this rate to be sufficient to keep the error between the discrete time approximation and the continuous time model small and bounded when the states are bounded (the error becomes unbounded when the system is allowed to become unstable, which feedback control attempts to prevent in the first place). We assume that the angle of attack $v(k)$ varies according to an exogenous dynamical system,

$$\begin{bmatrix} v_1(k+1) \\ v_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0.95 \end{bmatrix} \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} r(k),$$

where $v(k) = v_1(k)$ and $r(k)$ is a command input that can take values between −2.5 and 2.5. This system has its poles at 0 and 0.95, and its equilibrium is at $v_1(k) = 20, v_2(k) = 20$.

The wing rock system has a stable focus at the origin for angles of attack $v(k)$ less than approximately 19.5 (Nayfeh et al, 1989). For higher angles, the origin becomes an unstable equilibrium, and a limit cycle appears around it. In both cases, however, the system is unstable and may diverge to infinity if the initial conditions are large enough. The problem we consider here has the angle of attack varying within the range between 15 and 25 degrees, so the qualitative behavior of the discrete time model changes periodically, as $v(k)$ becomes, respectively, smaller or larger than 19.5. To gain a better insight into how the dynamic behavior of the wing rock phenomenon changes qualitatively with $v(k)$, consider Fig. 1(b), where we let the system start at the initial condition $X_3(0) = [-1, 0, 0], v_2(0) = [22, 22]$. Initially, we set $r = 1$, so the angle of attack remains fixed at 22, and we let the system run in open loop for 105 s. We observe that $x_1(k)$ and $x_2(k)$ approach a limit cycle, which would be reached if

[2] Aircraft notation conventions dictate the use of α as the angle of attack; however, to avoid confusions with the notation here, we use v instead.

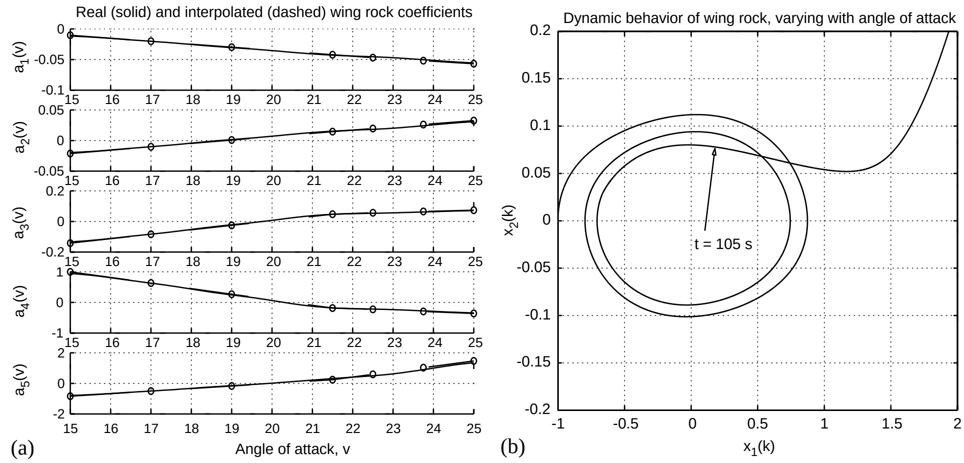

Fig. 1. (a) Interpolated coefficients for the time-varying wing rock model and (b) qualitative change in wing rock dynamics with varying angle of attack.

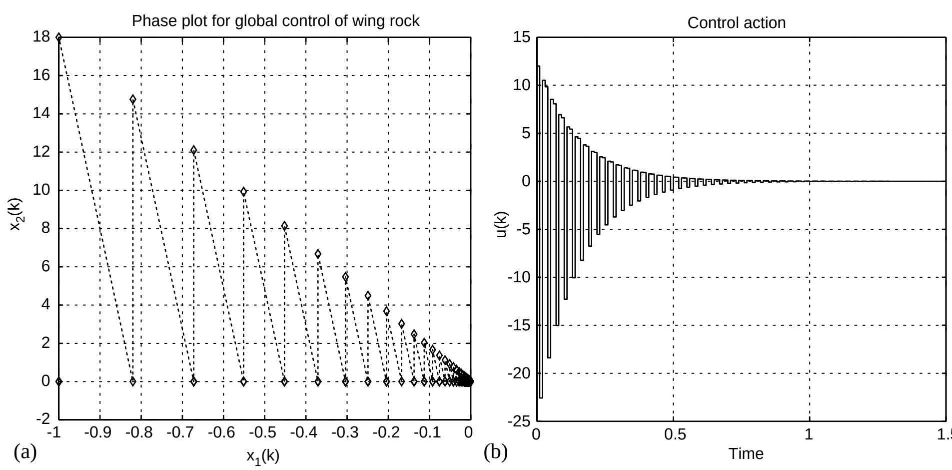

Fig. 2. (a) Wing rock regulation with global control: roll and roll rate and (b) control input and angle of attack.

the system were allowed to run for a longer time; however, at $t = 105$ we let $r = -1$ (this is marked by an arrow in Fig. 1(b)), so the angle of attack changes and after a short transient stabilizes at 18. Not being close enough to the origin to be attracted by the local stable focus, the system in fact diverges.

For our closed loop control tests, we let the reference to the angle of attack system alternate between -2.5 and 2.5 every 0.4 s (this may correspond to the aircraft experiencing some rough flying conditions). We use the wing rock problem as a test bed for comparison of the global and local control results in Theorems 1 and 2. Application of both theorems is greatly simplified by the fact that the only time-varying dynamics in the discrete time model occur in the difference equation of $x_2(k+1)$. For both cases we consider the initial conditions $X_3(0)=[-1,0,0]$, $v_2(0)=[22,22]$. We set the control objective to achieving a 1% settling time of 0.7 s for $x_1(k)$, the roll angle.

For the global design using Theorem 1 there is only one design parameter, c_1, which we set to 0.82 (this choice achieves the settling time objective, and significantly, roughly corresponds to the average control gain for the local approach as shown below). Figs. 2(a) and (b) show the results. In Fig. 2(a) we observe the phase plot for global closed loop control. We observe a typical dead-beat-like behavior (setting $c_1 = 0$ would yield an actual dead-beat controller). Fig. 2(b) shows the control input. The average control energy (measured as $(1/N)\sum_{i=1}^{N} u^2(k)$, with N the total number of time samples) is 15.24, and, as one would expect given the global nature of the controller, this level of control energy does not depend significantly on the evolution of the angle of attack. Another issue of interest is that, with only one parameter to tune, the basic time behavior of the closed loop system (the "dead-beat-like" feature of the phase plane plot) remains unchanged regardless of the choice for c_1.

Fig. 3 shows the results when Theorem 2 is used. We observe in Fig. 3(a) the phase plain plot using local control, where the dead-beat behavior of the global controller is less obvious. One can also note in Fig. 3(b) that the controller's

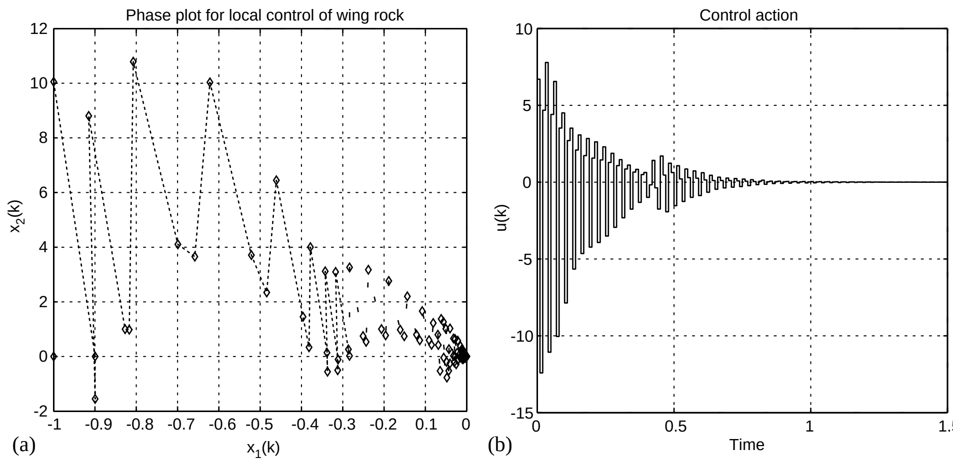

Fig. 3. (a) Wing rock regulation with local control: roll and roll rate and (b), control input and angle of attack.

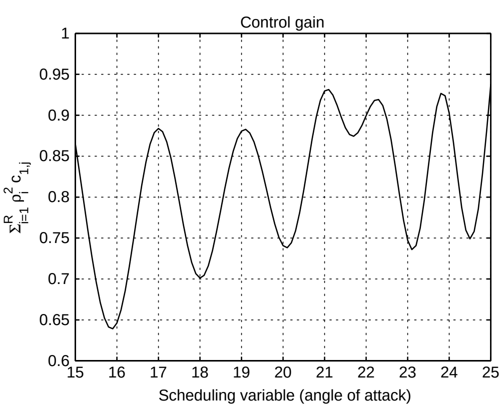

Fig. 4. Interpolated control gain for local scheme.

behavior depends to a greater degree than the global controller on the evolution of the angle of attack: at 0.4 s, when the reference input to the angle at attack system changes value and $v(k)$ starts to decrease, a sudden jump in control action is clearly visible. The seven control gains are chosen as $c_{1,1} = 1.14, c_{1,2} = 1.52, c_{1,3} = 1.57,\ c_{1,4} = 1.47,\ c_{1,5} = 2.57, c_{1,6} = 2.38$ and $c_{1,7} = 1.38$, and this design satisfies the settling time requirement. It is of interest to note that all these gains are larger than one, and yet they satisfy condition (7). This is illustrated in Fig. 4, which shows a plot of $\sum_{i=1}^{7} \rho_i^2(v)c_{1,i}$ where v takes values between 15 and 25, the range of interest for the angle of attack. The function $\sum_{i=1}^{7} \rho_i^2(v)c_{1,i}$ can be thought of as the interpolated control gain that determines the convergence rate of the Lyapunov function (see (13)). In the global case, this gain is constant for all values of v and equal to c_1. As mentioned above, the mean of $\sum_{i=1}^{7} \rho_i^2(v)c_{1,i}$ turns out to be approximately equal to the c_1 chosen in the global design for the settling time requirement.

The design rationale followed to choose the local control gains is the observation that, for angles of attack smaller than 19.5, the wing rock system diverges and becomes unbounded, whereas for larger angles a limit cycle appears, as illustrated in Fig. 1(b). Therefore, the interpolated control gain is chosen to be, in average, smaller for angles below 19.5 than otherwise (recall that a smaller gain yields faster convergence of the state) as can be seen in Fig. 4, so that the more unstable manifestations of wing rock are kept in check faster. This means that more control energy is used when the angle of attack is small, and also that the average control energy used depends on how $v(k)$ changes with time. For the simulation run presented here we obtain an average control energy equal to 5.82, roughly 2.6 times less than in the global case. However, as noted, when $v(k)$ is concentrated in the range below 19.5, the average control energy can be twice as large as in the global case.

The comparison between the global and the local schemes is not intended to produce a general winner, since such a judgment clearly depends on the application at hand and on the design approach. For the wing rock case, the global approach has the advantage of simplicity and consistence, but it is only able to produce one kind of behavior—its simplicity takes away some flexibility. In the local case, more tuning is necessary to meet a desired control objective, but a greater design flexibility is possible. In particular, the method allows the designer to allocate more control action to certain operating conditions and less to others, which may be useful in some instances. However, the control gain in Fig. 4 is difficult to manipulate uniformly, especially when the number of operating points is small as in this wing rock simulation.

6. Conclusions

In this paper, we have presented a control methodology for a class of discrete time nonlinear systems that depend on a possibly exogenous scheduling variable. This class of systems consists of an interpolation of nonlinear dynamic equations in strict feedback form, and it may represent systems with a time-varying nonlinear structure. In addition, the classes of systems considered allow for the representation of some cases of gain scheduling control, Takagi–Sugeno fuzzy systems, and input–output realizations of nonlinear systems which are approximated via localized linearizations. We introduce a so-called local control approach, which is contrasted with the global approach that corresponds to traditional backstepping design. The framework presented here establishes the foundations for methodologies that can be dedicated to a wide variety of applications. The ideas presented are applied to the problem of regulation of aircraft wing rock with a time-varying angle of attack, where a comparison between the global and local approaches is performed.

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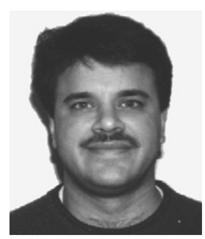

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