A Control-Theoretic Assessment of Interventions During Drinking Events

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***Abstract*—This paper employs control-theoretic tools to provide guidelines for *in-situ* interventions aimed at reducing high-risk alcohol consumption at drinking events. A dynamical directed network model of a drinking event with external intervention, suitable for mathematical analysis and parameter estimation using field data is proposed, with insights from pharmacokinetics and psychology. Later, a characterization of a bound on blood alcohol content (BAC) trajectories is obtained via Lyapunov stability analysis, and structural controllability guarantees are obtained via a graph-theoretic method. We use the degree of controllability, given to be the trace of the system's controllability Gramian, as a metric to compare the viability of network nodes for intervention based on theoretic and heuristic centrality measures. Results of numerical examples of bars and parties, informed by field data, and the stability and controllability results, suggest that intervening in the environment in *wet* bars, while targeting influential individuals with high alcohol consumption motivations in private parties efficiently yield lower peak BAC levels in individuals at the drinking events.**

***Index Terms*—Controllability, drinking events, feedback control, intervention, social systems, stability.**

I. Introduction

DRINKING in college settings has become a central health concern in the U.S. with more than 1800 fatalities per year [1], with more than 40% of college students reporting being drunk in the past month [2]. On a global scale, 25% of all unintentional and 10% of intentional injuries in the world are attributable to drinking events [3]. Considering such grim statistics, observing and intervening events where an individual or group of people engage in alcohol-consumption activities in order to reduce high-risk behaviors is an important goal of social scientists. Our goal is to provide guidelines to help design efficient *in-situ* interventions at drinking events using the tools provided by dynamical systems and control theory.

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Drinking events have been the subject of field studies in the social sciences. Via statistical analyses, these studies found that individual characteristics (motivation and drinking history) [4], [5], peer influence and social norms [6], and environmental factors (e.g., dancing and drink specials) [7], [8] are correlated with high-risk drinking. The data obtained in these studies were collected using environment observation, questionnaires, and breathalyzer readings of blood alcohol content (BAC) measured during drinking events. Nonetheless, there exists a need for studies that capture the dynamic nature of drinking events to guide intervention design [9]. Recent studies have incorporated the dynamics in their formulations using epidemic models [10] or agent-based methodologies [11]. However, none of these models lend themselves to a mathematical analysis that would identify performance guarantees, and provide an easy way to estimate its parameters, with experiment data, using firm connections with pharmacokinetics and psychosocial models.

Interventions to reduce high-risk behavior have been almost exclusively implemented *offline* with respect to drinking events. In the context of college drinking, educational programs, and advertising campaigns have been implemented, although rates of binge drinking have not been reduced [12]. More recently, social norm-based interventions [13] have been employed to provide feedback about individuals' and their peers drinking behavior, but no significant reduction of alcohol misuse has been found [14]. There exists a gap in the analysis of interventions at the event level to reduce high-risk alcohol consumption. Motivated by recent advances in data collection technologies [15] and the ubiquity of mobile devices, studies have begun to employ periodically sampled physiological measures to gain a better understanding of the dynamics of health processes [16]. These same drivers can provide the tools for *in-situ* interventions at drinking events, and clear guidelines are needed to efficiently design them.

In [17], a dynamical model of a drinking event is presented, where the drinking behavior of the individuals in the event is a result of the interaction between their group members, the environment, and their personal motivations and characteristics. The mathematical formulation of the model allowed the analysis of the effect of the model parameters in the individual's intoxication employing Lyapunov stability theory. A refined model for an individual agent was introduced in [18], providing a characterization of the individual's decision making process and a representation of the alcohol metabolism dynamics, with a methodology to estimate the parameters of the model using experimental data. The main objective of this paper then is to employ the previous models to expand the analysis toward behavioral interventions during drinking events that use the individuals' current drinking behavior (feedback), allowing for mathematical analysis and model parameter estimation using field data. Furthermore, we seek to identify leverage points concerning where to intervene in the drinking event using controllability results, with the goal to efficiently reduce high-risk behavior.

In Section II, we present the model of a drinking event that allows external intervention from a bottom-up perspective, starting from the alcohol pharmacokinetics in each individual's body to the environment "wetness" dynamics. Using Lyapunov stability analysis, in Section III we provide conditions for boundedness of intoxication trajectories for individuals as well as a characterization on those bounds using model parameters. In Section IV, we take advantage of the linearity of the model to employ well-known results for controllability. Via a scalar metric, the controllability degree, we draw conclusions on where to intervene in the drinking event. The theoretical insights learned are tested with numerical examples using real experiment data in Section V. We finalize this paper with a discussion and identification of possible future research.

II. Model of Drinking Event

In this section, we will establish a dynamical model of a drinking event based on the authors' previous work [17], [18], starting from the individual's dynamics. The model that describes the processes involving ethanol, the type of alcohol present in beverages, and the human body are derived from the pharmacokinetics literature. With a slight abuse of the terminology, in the remainder of this paper we will refer to ethanol as alcohol.

A. Alcohol Metabolism

In the field of pharmacokinetics, compartmental models are used to represent the processes of absorption, distribution, metabolism and elimination of alcohol in the human body [19], [20]. In [21], alcohol flows between two compartments: 1) the liver water and 2) the body water, connected by hepatic blood flow, while in [22], the central (which includes the blood) and peripheral compartments where considered. In [20], a higher dimensional model is considered by including the stomach, gastrointestinal, liver, central, and muscle compartments. Here we present a simplified two-compartment model of alcohol pharmacokinetics in the human body modeled as a second degree linear system. Even though we are not considering the nonlinearities present in the alcohol elimination process, often modeled with Michaelis–Menten kinetics [22], the model remains valid for our application, which is to model the behavior of individuals during the duration of a drinking event. Alcohol is ingested and flows to the first compartment, with volume V_g and with alcohol concentration at time t as $x_g(t) \geq 0$. The alcohol is later absorbed by the blood compartment, with volume V_b, where the state $x_b(t) \geq 0$ corresponds to the alcohol concentration in this compartment at time t. Alcohol returns to the first compartment to be metabolized and a fraction of it will be eliminated from the system. The following differential equations that govern the dynamics of the concentration of alcohol in the compartments are formulated using mass balance, with $u(t) \geq 0$ as the alcohol input rate:

$$\begin{aligned} V_g\dot{x}_g(t) &= -\gamma_1 x_g(t) + \gamma_2 x_b(t) + u(t) \\ V_b\dot{x}_b(t) &= \gamma_2 x_g(t) - \gamma_2 x_b(t) \end{aligned} \tag{1}$$

where $\gamma_1 \geq 0$ corresponds to the elimination rate and $\gamma_2 \geq 0$ is the flow rate between the two compartments, modeled in [21] as the flow rate in the hepatic vein. In this paper, we will use minutes as our time scale.

A widespread metric for alcohol intoxication is the BAC, which is easily measurable via breath, blood and urine tests, and more recently using wearable transdermal biosensors [15]. Via a change of coordinates on the system in (1), we align it with the model in [18] to obtain a controllable canonical form with the BAC $x_b(t) \in \mathbb{R}$ and the BAC rate of change $v_b(t) \in \mathbb{R}$ as state variables

$$\begin{aligned} \dot{x}_b(t) &= v_b(t) \\ \dot{v}_b(t) &= -ax_b(t) - bv_b(t) + cu(t) \end{aligned} \tag{2}$$

where

$$a = \frac{\gamma_1\gamma_2}{V_g V_b},\ b = \frac{Vb(\gamma_1+\gamma_2)+\gamma_1 V_g}{V_g V_b},\ c = \frac{\gamma_1}{V_g V_b}. \tag{3}$$

We will fix $V_g = \gamma_3 V_d$, with $\gamma_3 \geq 0$ and the volume of blood $V_b = (1/9)V_d$, where V_d is the total body water. In Appendix C in the supplementary file, it is detailed how the parameters γ_i, for $i = 1, 2, 3$ are chosen in order to provide the best fit to the experimental data based on weight and gender found in [23]. In what follows, we will drop the subscript b and replace it with the subscript i, denoting the ith individual at the drinking event.

B. Drinking Decision Making

A relevant model for human behavior control during alcohol consumption is the self-regulation model, found in the field of social psychology [24], [25]. In it, it is hypothesized that the person manages her actions to achieve some predetermined goals, commonly referred as goal-oriented behavior. Cognitive scientists have linked the self-regulation theory with the set of cognitive processes occurring mostly in the prefrontal cortex of the brain. This region deals with the control of behavior, called "executive functions," and include the inhibitory control of impulsive responses and working memory, where goals are stored [26], [27]. Furthermore, the self-regulation model has been represented via feedback control [28], [29], where an error signal is computed by comparing the goal or reference value and the perceived status of the environment. The individual then acts based on this error to alter her environment. Using these concepts in a drinking event setting, an individual computes the mismatch between her desired intoxication (goal) and the perception of her actual intoxication at a time t and decides on her drinking rate level to achieve the goal. Employing the individual decision making model in [18], we can model the drinking rate at time t, $p_i(t)$, that results from the assessment of her personal goal as

$$p_i(t) = k_i\Big[\Big(x_i^d(t) - x_i(t)\Big) + \alpha_i\Big(v_i^d(t) - v_i(t)\Big)\Big]. \tag{4}$$

Individuals perceive their own drinking behavior employing interoceptive, proprioceptive and behavioral cues, and compare them with expectations and norms regarding intoxicated status [30]. In [31], it is reported that individuals overestimate their BAC when it is growing while drinking, i.e., $v_i(t) > 0$ and underestimate it when their BAC is decaying after the drinking stops, or $v_i(t) < 0$. Furthermore in [32], the rate of change of the state variable is employed in the control of locomotion in humans steering toward a goal, providing further argument for the use of the variable $v_i(t)$ in (4). Thus, the parameter $\alpha_i \geq 0$ is associated with how the individual perceives and reacts to the rate of change on her intoxication. This behavioral parameter will also be employed when we model how the individual processes the rate of change on the different external stimuli to be discussed in the rest of this section. The variable $x_i^d(t)$ is the desired BAC for the ith individual at time t and $v_i^d(t) = \dot{x}_i^d(t)$ is the desired rate of change on her BAC. It is assumed that the trajectory $x_i^d(t)$ has continuous and bounded derivatives for all t. An example of a desired trajectory can be seen in Fig. 2, where the individual plans to maintain a drinking rate of 0.7 g/min of alcohol, or three standard drinks (12 fl. oz of regular beer) per hour for close to 2.5 h, reaching a peak BAC of slightly above 0.12 g/dL. The parameter $k_i \in \mathbb{R}$ can be viewed as the commitment or motivation strength of the individual to self regulate her drinking behavior, where a highly committed individual will have $k_i >> 0$, while a small or negative value of k_i will signal lack of self control.

The acute effects of alcohol intoxication in decision making have been studied in the literature on *alcohol myopia* [33]. In [34], it is noted that alcohol consumption reduces the ability of individuals to compute the mismatch between their desired and actual trajectories, while in [33], it is argued that the ingestion of alcohol leads individuals to focus only on immediate salient environmental cues, reducing their ability to consider future consequences. These effects could be modeled by making the commitment and the rate of change awareness parameters, k_i and α_i, respectively, to be nonincreasing functions of the current BAC at time t, $x_i(t)$. This choice of modeling introduces nonlinearities in the self-regulation variable $p_i(t)$ that will limit our intended analysis. On the other hand, in a more recent publication, it is hypothesized that the effects of alcohol in self-regulation are associated with the ability of alcohol to stimulate alcohol-seeking behavior [35]. Employing this approach, we model the effects of alcohol intoxication on the individual's choice of the drinking rate as an additive signal $e_i(t)$ to $p_i(t)$, with

$$e_i(t) = m_i(x_i(t) + \alpha_i v_i(t)) \tag{5}$$

where $m_i \geq 0$ represents the weight of the alcohol-seeking behavior in the ith individual.

External stimulus-driven influences often compete with self-regulation for attentional resources [26] and eventually can alter the decision significantly. Social perception [36], which involves nonverbal, visual and body gestures cues, is the main process involving perception of intoxication and drinking rate of the group and the drinking event's environment. In drinking events, social influence to comply is often cited as a cause of overdrinking in college students [6] and can be viewed as an external and often desirable stimuli for the individual. To model how the peer pressure affects the individual in the decision of how much to drink, we employ the model in [17]

$$g_i(t) = \sum_{j=1}^{N} w_{ij}\big[\big(x_j(t) - x_i(t)\big) + \alpha_i\big(v_j(t) - v_i(t)\big)\big] \tag{6}$$

where the number of individuals in the drinking event is N and $w_{ij} \geq 0$ represents the strength of the influence of individual j on individual i. The parameter w_{ij} also includes the intoxication misperception due possibly to descriptive social norms identified in [6]. The term $\sum_{j=1, j\neq i}^{N} w_{ij}[x_j(t) + \alpha_i v_j(t)]$ will be referred as *group wetness* as seen from individual i, and represents the average drinking behavior of the group of peers that influence her directly.

Another external influence in the decision to drink comes from the drinking event's environment [8], via the *environment wetness*, i.e., the whole environment's behavior influence. We assume that the environment wetness dynamics are governed by the same second-order structure of (2). The rationale behind this choice comes from the fact that, from the individuals' perspective, the environment wetness not only represents the physical and social settings, but it also aggregates the drinking behavior of all the individuals at the drinking event [8], [37]. Furthermore, the environment wetness is not static and cannot be changed immediately, thus the parameters $a, b, c > 0$ will be assumed to reflect this fact. Also, in some situations, the environment wetness can be controlled and steered toward a desirable goal as in (4), e.g., goals designed by the staff of the bar or house owner, where possible interventions include modifying the price of drinks, restricting drinking games, among others, to increase or decrease the wetness. Thus, we model the environment as the $(N+1)$th member in the drinking event, and its wetness at time t as $x_{N+1}(T)$, with $w_{(N+1),i} > 0$, for all $i = 1, \ldots, N$, meaning that the environment wetness is influenced by all the members in the drinking event. As the environment's "metabolic" and behavioral parameters must be indirectly inferred from information in data sets such as number of individuals at the event, presence of food, music, beverage prices, among others; in Appendix C in the supplementary file, we describe how we obtain these parameters using two constants: 1) the environmental risk and 2) size factors. The parameters w_{ij} for $i, j = 1, \ldots, N+1$ could be employed to define a weighted, directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, \ldots, N+1\}$ is the set of nodes representing agents in the drinking event, $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of directed links that connect the individuals, and $W = [w_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ is the associated weighted adjacency matrix. Fig. 1 shows the topology of a drinking event in a bar with $N = 9$ and the environment node in the center as the 10th agent.

External interventions at the drinking event level have as objective to reduce risky drinking behavior of individuals

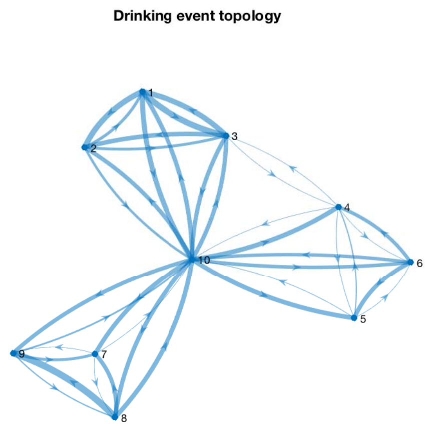

Fig. 1. Topology of drinking event with environment as the 10th node. Data from [37].

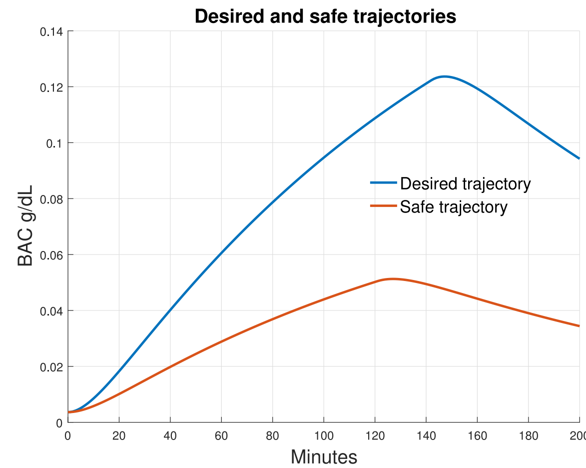

Fig. 2. Desired and safe intoxication trajectories for an individual in a drinking event.

at the event. Examples of such interventions include party monitors [38], and intoxication feedback interventions [39]. At the environment level, they could include drink pricing schemes, and responsible serving training and implementation. An important element of the interventions is to consider what is a safe drinking behavior and how to influence it on individuals and environment. We define the *safe trajectory* for individual i at time t as the signal $x_i^r(t) \geq 0$, where $x_i^r(0) = x_i(0)$ and $v_i^r(0) = v_i(0)$, to be the desired intoxication trajectory of individual i from an intervention perspective. We assume that there exists known and constant bounds for the time derivatives of $x_i^r(t)$. We will employ the safe trajectories as the reference for intervention efforts. An example of a safe trajectory is given in Fig. 2 where it is stipulated that the individual should safely drink at a rate of 0.2 g/min, approximately one standard drink per hour, during 120 min without going above a BAC level of 0.05 g/dL. It must be true that $x_i^d(t) \geq x_i^r(t)$, for all $t \geq 0$ in order to avoid trying to enforce a safe trajectory as the one shown in Fig. 2 to an individual that does not want to drink. We can model the effect of an intervention on an individual deciding on how much to drink as

$$z_i(t) = s_i\left[\left(x_i^r(t) - x_i(t)\right) + \alpha_i\left(v_i^r(t) - v_i(t)\right)\right] \tag{7}$$

where $s_i \geq 0$ measures how susceptible the ith individual is to change its drinking rate based on the intervention. An individual's susceptibility to the intervention depends on how persuasive the message being delivered by the intervention is. Interventions may have different degrees on how to enforce this safe trajectory to individuals, from a few messages with information about the safe trajectory level at time t, to provide strong enforcement proportional to the mismatch between the safe and actual intoxication as measured via BAC sensing hardware. However, a discussion on this topic is beyond the scope of this paper and the reader is referred to [40].

Finally, given that we must modify (6) to include in the sum the $(N+1)$th member, the environment, we can state that the amount of alcohol to ingest by individual i at time t is

$$u_i(t) = p_i(t) + e_i(t) + g_i(t) + z_i(t). \tag{8}$$

It is important to note that, in the case of drinking events, individuals cannot arbitrarily decrease their BAC by choosing $u_i(t) < 0$, with the exception of the environment node which may have more control over its wetness. A saturation nonlinearity was considered in [18] to ensure that $u_i(t) \geq 0$ for all $t \geq 0$. However, we will not impose such nonlinearity as we will assume that we will have $u_i(t) > 0$ in (8) for all $i = 1, \ldots, N$ and for all $t \geq 0$. We do so accounting for the fact that we are interested in modeling the behavior of individuals during the actual drinking period, i.e., when the drinking rate is positive, which leads to the risky behavior we are interested in reducing. Extensive simulations with parameters tuned with data from experiments in [8] and [37] showed that this assumption is valid in the time framework we are considering. Furthermore, if we assume that the following actions are available for each individual: fructose ingestion, known for increasing the metabolic rate [41], or emesis (vomiting), which eliminates the alcohol in the stomach before being absorbed by the blood stream, they could be considered as actions taken by the individual to decrease their BAC, hence, being modeled by $u_i(t) < 0$. This assumption allows us to employ the stability and controllability results from linear systems theory.

The effects of the internal and external influences in (8) are depicted in Fig. 3, where the personal desire to get intoxicated adds to the influence by peers, environment, and myopia effect and would result in a higher drinking rate if they were not countered by the influence of the intervention. Note that as the individual intoxication rises, the influence of peers and environment decreases while the myopia effect increases. When the individual reaches her peak desired intoxication around time $t = 120$, instead of setting her drinking rate to zero, it continues with a lower but strictly positive drinking rate, influenced by her peers, the environment wetness, and the myopia effect. Note that the resulting drinking rate is nonnegative for all the time period considered.

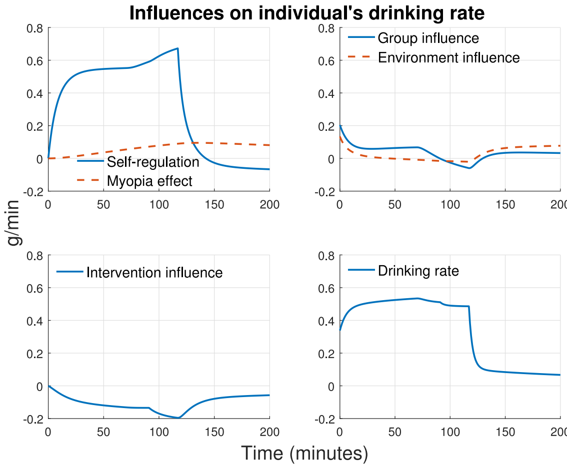

Fig. 3. Influences on individual's decision to drink in a drinking event.

C. Model Formulation

Considering (2)–(7), the drinking event dynamics can be characterized by the following differential equations for each agent i:

$$\begin{aligned}\dot{x}_i(t) &= v_i(t)\\ \dot{v}_i(t) &= -\left(a_i + c_i\left[k_i - m_i + s_i + \sum_{j=1}^{N+1} w_{ij}\right]\right)x_i(t)\\ &\quad - \left(b_i + \alpha_i c_i\left[k_i - m_i + s_i + \sum_{j=1}^{N+1} w_{ij}\right]\right)v_i(t)\\ &\quad + c_i\left(\sum_{j=1}^{N+1} w_{ij}\left[x_j + \alpha_i v_j\right] + k_i\left(x_i^d + \alpha_i v_i^d\right)\right.\\ &\quad \left. + s_i\left(x_i^r + \alpha_i v_i^r\right)\right). \end{aligned} \tag{9}$$

Equation (9) can be written as an affine linear dynamical system in matrix form as

$$\dot{x}(t) = Ax(t) + B_d x^d(t) + B_r x^r(t) \tag{10}$$

where the state vector is $x(t) = [x_1(t), v_1(t), \ldots, x_{(N+1)}(t), v_{(N+1)}(t)]^\top$, and the matrix $A \in \mathbb{R}^{2(N+1)\times 2(N+1)}$ is given by $A = \hat{A} + B_w$, where $\hat{A} = \mathrm{diag}\{\hat{A}_1, \ldots, \hat{A}_{(N+1)}\}$ is a block diagonal matrix with ith block

$$\hat{A}_i = \begin{bmatrix} 0 & 1 \\ -\hat{a}_i & -\hat{b}_i \end{bmatrix}$$

with

$$\hat{a}_i = a_i + c_i\left(k_i - m_i + s_i + \sum_{j=1}^{N+1} w_{ij}\right)$$

$$\hat{b}_i = b_i + \alpha_i c_i\left(k_i - m_i + s_i + \sum_{j=1}^{N+1} w_{ij}\right)$$

and $B_w = [B_{w1}^\top, \ldots, B_{w(N+1)}^\top]^\top \in \mathbb{R}^{2(N+1)\times 2(N+1)}$ with $B_{wi} = B_i \otimes W_i$ where for each agent i

$$B_i = \begin{bmatrix} 0 & 0 \\ c_i & \alpha_i c_i \end{bmatrix}$$

and W_i is the ith row of $W \in \mathbb{R}^{(N+1)\times(N+1)}$, the weighted adjacency matrix of the graph $\mathcal{G}$. The external input vectors are $x^d(t) = [x_1^d(t) + \alpha_1 v_1^d(t), \ldots, x_{(N+1)}^d(t) + \alpha_{(N+1)} v_{(N+1)}^d(t)]^\top$ and $x^r(t) = [x_1^r(t) + \alpha_1 v_1^r(t), \ldots, x_{(N+1)}^r(t) + \alpha_{(N+1)} v_{(N+1)}^r(t)]^\top$. The input matrices are $B_d = BK_d$ and $B_r = BK_r$, where $K_d,\ K_r \in \mathbb{R}^{(N+1)\times(N+1)}$ are diagonal matrices containing the gains k_i and s_i, respectively, and $B = \mathrm{diag}\{B_1, \ldots, B_{N+1}\}$.

The objective of the following sections will be to provide guidelines in designing the trajectories $x_i^r(t)$, for all $i = 1, \ldots, N+1$ to efficiently reduce high-risk drinking behavior for individuals in a drinking event.

III. Stability of Drinking Event With External Intervention

In this section, we will be concerned with the dynamic behavior of the drinking event as well as the sensitivity to some of the most relevant model parameters. We will start with a stability analysis to learn under which conditions the BAC trajectories will remain bounded with respect to the safe trajectories.

We define the change of coordinates $\hat{x}_i(t) = x(t) - x^r(t)$ with $\hat{x}(t) = [\hat{x}_1(t), \hat{v}_1(t), \ldots, \hat{x}_{(N+1)}(t), \hat{v}_{(N+1)}(t)]^\top$ which is a measure of how far the intoxication trajectory is from the safe trajectory for each agent. The dynamics of the system under this new set of coordinates can be derived as follows:

$$\begin{aligned}\dot{\hat{x}}(t) &= Ax + B_d x^d(t) + B_r x^r(t) - \dot{x}^r(t)\\ &= A\hat{x} + B_d x^d(t) + (A + B_r)x^r(t) - \dot{x}^r(t).\end{aligned} \tag{11}$$

The following theorem establishes the conditions under which the trajectory of $\hat{x}$ will remain bounded around the origin $\hat{x}^* = 0$, which in the original coordinates are trajectories that satisfy $x(t) = x^r(t)$ for all $t \geq 0$. Under these conditions, the intoxication trajectories $x(t)$ will not diverge away from the safe trajectories $x^r(t)$ in the long run. In what follows, we will drop the time dependency argument on the variables to simplify the notation.

Theorem 1: For the drinking event in (11), assume that for all agents $i = 1, \ldots, N+1$, the commitment to the desired BAC trajectory k_i, the strength of the intervention r_i, and the influences the peers and environment as measured by w_{ij} are such that we have

$$0 < \hat{b}_i - \hat{a}_i \leq 1 \tag{12}$$

$$\left(2\hat{a}_i - (1+\alpha_i)c_i\sum_{j=1}^{N+1} w_{ij}\right) > \rho_i \sum_{j=1}^{N+1}\left(1 + \hat{b}_j - \hat{a}_j\right)c_j w_{ji} \tag{13}$$

where

$$\rho_i = \max\left\{\frac{1}{\left(\hat{b}_i - \hat{a}_i\right)}, \max_j \alpha_j\right\}. \tag{14}$$

Then the trajectories of $\hat{x}(t)$ in (11) remain uniformly ultimately bounded around the origin $\hat{x}^* = 0$, with ultimate bound given by

$$\eta = \sqrt{\operatorname{cond}(P) \sum_{i=1}^{N+1} \bar{\phi}_i^2 \left(\frac{c_i^2}{\left(\theta_i^x\right)^2} + \frac{1}{\left(\theta_i^v\right)^2} \right)} \tag{15}$$

where $\theta_i^x, \theta_i^v \in (0, 1)$, and $\bar{\phi}_i$ is the supremum over all $t \geq 0$ of the time varying expression

$$\begin{aligned}\phi_i = -a_i x_i^r - b_i v_i^r + c_i \Bigg[& k_i\Big(x_i^d - x_i^r + \alpha_i\Big(v_i^d - v_i^r\Big)\Big) \\ & + \sum_{j=1}^{N+1} w_{ij}\Big(x_j^r - x_i^r + \alpha_i\Big(v_j^r - v_i^r\Big)\Big) \\ & + m_i\big(x_i^r + \alpha_i v_i^r\big)\Bigg] - \dot{v}_i^r \end{aligned} \tag{16}$$

and $\operatorname{cond}(P)$ is the condition number of the positive definite block diagonal matrix P with ith block

$$P_i = \frac{1}{2}\begin{bmatrix} 2\hat{a}_i^2 + \hat{b}_i^2 - 3\hat{a}_i\hat{b}_i & \hat{b}_i - \hat{a}_i \\ \hat{b}_i - \hat{a}_i & 1 \end{bmatrix}$$

for $i = 1, \ldots, N+1$.

Remark 1: Note that the ultimate bound depends on the expression $\phi_i(t)$, given in (16). The safe trajectory could be designed using the individual's metabolism and behavioral parameters as

$$\dot{x}_i^r = v_i^r \tag{17}$$

$$\dot{v}_i^r = -a_i x_i^r - b_i v_i^r + c_i\big(u_i^r + m_i\big(x_i^r + \alpha_i v_i^r\big)\big) \tag{18}$$

with u_i^r being the safe drinking rate. Note that under this choice of model, the expression $\phi_i(t)$ becomes

$$\begin{aligned}\phi_i(t) = -u_i^r(t) & + k_i\Big(x_i^d - x_i^r + \alpha_i\Big(v_i^d - v_i^r\Big)\Big) \\ & + \sum_{j=1}^{N+1} w_{ij}\Big(x_j^r - x_i^r + \alpha_i\Big(v_j^r - v_i^r\Big)\Big).\end{aligned} \tag{19}$$

We could make the value of the expression $\bar{\phi}_i^2$ smaller by choosing $u_i^r(t)$ close to the drinking rate obtained by the ith individual following her desired intoxication trajectory $x_i^d(t)$. Furthermore, we could have $\phi_i(t) = 0$ for all $t \geq 0$ choosing $u_i^r = k_i(x_i^d - x_i^r + \alpha_i(v_i^d - v_i^r)) + \sum_{j=1}^{N+1} w_{ij}(x_j^r - x_i^r + \alpha_i(v_j^r - v_i^r))$ as the safe drinking rate, rendering the system exponentially stable with $\dot{V}(x) \leq -\lambda\|x\|^2$. However, this choice might not lead to an actual *safe* trajectory, as the desired trajectories $x_i^d(t)$ for $i = 1, \ldots, N+1$ might reach higher intoxication values than what is recommended to avoid risky behaviors in individuals at the drinking event.

Remark 2: The assumption in (12) is valid in the general case, considering the time constants in the model. However, it introduces bounds to the intervention parameter s_i that we are interested in design. For instance, if the rate of change awareness parameter α_i is large, with a large s_i the upper bound on (12) might be violated. This could be interpreted in the same way as the case of diverging trajectories produced by high derivative gain in a PD controller. The ith agent with high α_i will over-react to the change in any of the stimuli she is receiving, producing undesirable responses. On the other hand, for agents that respond slowly to the rate of change of stimuli, $\alpha_i < 1$, high values of s_i may cause the lower bound on (12) to be violated, possibly leading to instability.

Remark 3: The assumption in (13) introduces upper bounds on the strength on external stimuli from peers and environment, measured by w_{ij}. This is reflected in Fig. 4, where the mean of the average peak BAC of drinking events with $N = 50$ is computed along 500 runs of a Monte Carlo simulation for various values of the environmental risk factor and strength of external influence. We define the environmental risk factor $\kappa \in [0, 1]$ as the parameter defining the environment wetness' initial condition $\{x_{N+1}(0), v_{N+1}(0)\}$, and desired trajectories, with $\kappa = 0$ indicating a *protective*, or dry environment. We also define the environmental size factor $\zeta \in [0, 1]$ as the parameter defining the environment wetness' "metabolism" parameters a_{N+1}, b_{N+1}, and c_{N+1}, and its influence on individuals $w_{i(N+1)}$ for all $i = 1, \ldots, N$, where $\zeta = 0$ indicates a small drinking event setting. The behavioral parameters are obtained via a combination of these factors. A detailed account on the calculation of these parameters is given in Appendix C in the supplementary file. For this simulation, we adopted $\zeta = 0.6$, corresponding to a small bar or big party. The strength of external influence on individuals is the linear combination parameter $\delta \in [0, 1]$ in $(1-\delta)k_i + \delta\sum_{j=1}^{N+1} w_{ij}$ for all $i = 1, \ldots, N$, and at $\delta = 0$, the individuals are not influenced by their peers or the environment. In this simulation, we considered no interventions, thus $s_i = 0$ for all $i = 1, \ldots, N+1$. High values of peak BAC are obtained in events where groups have a strong influence between their members and at the same time, are greatly influenced by a *wet* environment, i.e., large values in $w_{i(N+1)}$ and $x_{N+1}(t)$ compared to k_i and $x_i(t)$ at time t for all $i = 1, \ldots, N$. This creates a reinforcement to continue drinking, which is higher than any self-regulation mechanism. On the other hand, the peer influence could act as *protective* from risky behaviors in cases with low environment wetness factors. This could be explained by an environment with low initial conditions and high susceptibility to the individuals' low intoxication at the beginning of the drinking event, leading to a reinforcement of low drinking rates. The values employed in Fig. 4 can be found in Appendix C in the supplementary file. The data employed to tune the parameters is from the bars and parties surveyed in [8] and [37].

In summary, the individuals' metabolic and behavioral parameters and their desired intoxication trajectories should be employed in the design of the safe trajectories as seen in (19). In Fig. 4, we learned that the topology and strength of the external influence network could have risky and protective effects depending on the environment wetness. It can also be seen in (19) that determining which individuals to target in interventions with strictly positive s_i could further lower the bound in (15) by targeting members with higher mismatch between their desired and safe intoxication trajectories. In Section IV, we will take a more careful look at the decision of which agent to intervene in a drinking event.

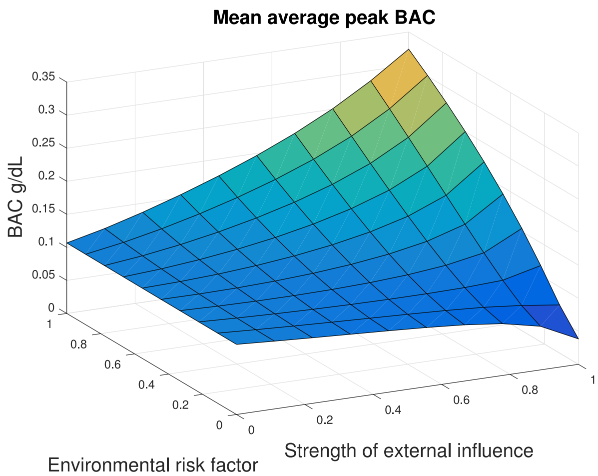

Fig. 4. Mean average peak BAC for individuals in DE for various values of strength of external stimuli w_{ij} for all $i, j \in \mathcal{V}$ and environmental risk factor $\kappa \in [0, 1]$.

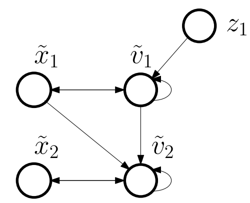

Fig. 5. Unweighted directed graph of the system in (20) with two individuals.

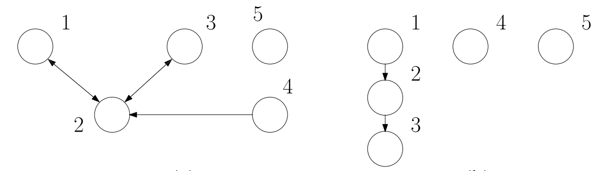

Fig. 6. (a) Directed graph $\mathcal{G}$ and (b) its longest forward paths with origins at nodes 1, 4, and 5.

IV. Controllability of Drinking Events

A. Structural Controllability

In this section, we address the conditions under which it is possible to design an intervention that will drive the intoxication state of the drinking event $x(t)$ in (10) to the safe trajectory $x^r(t)$ and how much effort it will require. We do this by assessing the structural controllability of the system and its degree of controllability for different intervention allocations in the graph $\mathcal{G}$. Before introducing these concepts, we must transform the system in (10) to model the system before the intervention, i.e., $s_i = 0$ for all $i = 1, \dots, N+1$. We employ the assumption introduced in Section II-B that the desired trajectory and its rate of change are bounded with $\|[x_i^d(t), v_i^d(t)]\| \le [d_i^x, d_i^v]$ with $d_i^x, d_i^v \ge 0$ for all $i = 1, \dots, N+1$. Using the change of coordinates $\tilde{x}(t) = x(t) - d$ with $d = [d_1^x, d_i^v, \dots, d_{N+1}^x, d_{N+1}^v]^\top$, we can rewrite the system in (10) with the new coordinates as a linear system

$$\dot{\tilde{x}}(t) = \bar{A}\tilde{x}(t) + \bar{B}z(t) \tag{20}$$

where $\bar{A}$ corresponds to the matrix A in (10) with $s_i = 0$ for all $i = 1, \dots, N+1$, the vector $z(t) = [z_1(t), \dots, z_{N+1}(t)]^\top$ where $z_i(t)$ is the intervention effect on the ith agent as seen from (7), and $\bar{B}$ is a block diagonal matrix with ith block $\bar{B}_i = [0, c_i]^\top$.

The concept of *structural controllability* was introduced in [42] and it is useful when dealing with large scale systems where model parameters are difficult to assert with high precision. A pair $(\bar{A}, \bar{B})$ has elements which are zero or constants, and we say that the pair is structurally controllable if and only if there exists a completely controllable pair (A_c, B_c) that has the same structure as $(\bar{A}, \bar{B})$, that is, for every zero entry of the matrix $(\bar{A}\bar{B})$, the corresponding entry of the matrix $(A_c B_c)$ is zero, and vice versa.

Before proceeding with the statement of the theorem of this section, we introduce a graph representation of the pair $(\bar{A}, \bar{B})$ and graph theory concepts that will aid in the proof. Given the system in (20), we define its unweighted directed graph $\mathcal{G}_s = \{\mathcal{V}_s, \mathcal{E}_s\}$ with the states of the system as the set of nodes $\mathcal{V}_s = \{\tilde{x}_1(t), \tilde{v}_1(t), \dots, \tilde{x}_{(N+1)}(t), \tilde{v}_{(N+1)}(t)\}$, and set of edges $\mathcal{E}_s$ defined by the nonzero entries of the matrix A. Fig. 5 shows the graph of the system in (20) with two individuals, where individual 1 influences individual 2. The matrix $\bar{B}$ defines edges to the intervened nodes from "origin" or input nodes, e.g., $z_1(t)$ in Fig. 5. To avoid confusion with the terminology, we employ the word *agent* or individual to denote the nodes from the event topology graph $\mathcal{G}$ from Section II-B, leaving the word nodes to the ones from the graph of the system $\mathcal{G}_s$.

For the directed graph $\mathcal{G}_s$, we define the *inaccessible nodes*, as the set $\mathcal{D} \subset \mathcal{V}_s$ where, for every $i \in \mathcal{D}$, there are no directed paths reaching i from any $j \in \mathcal{V}_s$. Clearly, if we do not intervene in the inaccessible agents, the system will not be controllable. This can be seen with the two-agent example in Fig. 5, if the individual 2 had an intervention, but individual 1 is an inaccessible agent as it is not influenced by individual 2. The matrices of this system are

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\bar{a}_1 & -\bar{b}_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ c_2 w_{21} & \alpha_2 c_2 w_{21} & -\bar{a}_2 & -\bar{b}_2 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ c_2 \end{bmatrix}$$

and the controllability matrix for this system is

$$R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & c_2 & -\bar{b}_2 c_2 & (\bar{b}_2^2 - \bar{b}_1)c_2 \\ c_2 & -\bar{b}_2 c_2 & (\bar{b}_2^2 - \bar{b}_1)c_2 & (\bar{b}_1\bar{b}_2 - \bar{b}_2(\bar{b}_2^2 - \bar{b}_1))c_2 \end{bmatrix}$$

which has rank $r = 2 < n = 4$. However, by intervening in the inaccessible agent 1 instead, the same analysis concludes that the system is controllable. Next, we define the concept of *distinguished agents* in the graph $\mathcal{G}$ as the set of agents $\mathcal{S} \subset \mathcal{V}$ which are the origins of the longest forward paths in the graph such that any agent $j \in \mathcal{V}$ is included in one and only in one of the mentioned forward paths. Fig. 6(b) shows the longest forward paths for the graph in Fig. 6(a), along with their origins, which constitute the distinguished agents. Note that there could be more than one choice of origin for the longest path. In the case of Fig. 6, all three agents 1, 2, or 3 could have been selected as the origin, and thus,

Algorithm 1 Distinguished Agents

```
for i = 1 to N + 1 do
    Calculate the set of agents in the forward path of agent i, P_i
end for
Sort nodes with respect to number of agents in their forward path in descend order
Fix first node in S as a distinguished agent and fix its path P_i to the list of influenced nodes F
for i = 2 to N + 1 do
    if P_i ⊄ F then
        S = S ∪ {i}
        F = F ∪ P_i
    end if
end for
```

as a distinguished agent. The following Algorithm 1 serves to identify the distinguished agents in the graph.

Finally, we say that the directed graph $\mathcal{G}_s$ presents a *dilation* if and only if there is a subset $S \subset \mathcal{V}_s$ such that its cardinality is strictly greater than the cardinality of its neighboring set, i.e., $|S| > |T(S)|$. The neighboring set of a set S can be defined as the set of all nodes $v_j \in \mathcal{V}_s$ for which there exists an oriented edge going from v_j to a node in S. The origins or input nodes are not allowed to belong to S but may belong to $T(S)$. We are now ready to state the theorem in this section.

Theorem 2: Consider the drinking event described by (20) and the intervention input vector $z(t)$. Assuming that the distinguished agents $\mathcal{S} \in \mathcal{V}$ in the directed graph $\mathcal{G}$ are targets of an intervention with $s_i > 0$ for all $i \in \mathcal{S}$. Then the system is structurally controllable.

Remark 4: Note that more practical implementations of drinking event interventions, such as targeting the leaders and stubborn, not influenceable agents in groups must include the set of distinguished agents. Also, under the assumption that the environment agent influences all the agents in the drinking event and is influenced by all of them, it suffices to intervene with only one agent in the network to achieve structural controllability.

B. Degree of Controllability

Theorem 2 provides us with a theoretical result that allows us to shift focus from when is it possible to intervene in the drinking event to which and how many agents should be targeted in the intervention, considering the cost required to produce a change in the event. For this, the degree of controllability [43] provides us with a metric to compute the energy required to steer the system in (20) to the desired state. Throughout this section, we assume that the matrix A is stable, with negative real part eigenvalues. Hence, the controllability Gramian is obtained as the unique positive definite solution of the Lyapunov equation

$$\bar{A}W_c + W_c\bar{A}^\top + \bar{B}\bar{B}^\top = 0.$$

Scalar quantitative metrics have been employed to measure the degree of controllability, and many of them are related to the controllability Gramian W_c via the minimum energy control, i.e., the control input that takes the system state to the origin x_0 in certain time t_f while minimizing

$$\min_u \int_0^{t_f} \|u(\tau)\|^2 d\tau$$

and the closed form expression of the minimum energy control is

$$u^*(t) = x_0^\top W_c^{-1}(t)x_0.$$

For instance, the minimum eigenvalue of the controllability Gramian, $\lambda_{\min}(W_c)$ is related to the worst controllable directions of the system, while $\det(W_c^{-1})$ is related to the volume of a hyperellipsoid containing the states that are reachable by employing a control input with one unit of energy [43]. The average value of the minimum control energy is proportional to the trace of the inverse of the controllability Gramian, $\text{trace}(W_c^{-1})$. Noting the fact that

$$\frac{\text{trace}\left(W_c^{-1}\right)}{N} \geq \frac{N}{\text{trace}(W_c)}$$

the results obtained by using $\text{trace}(W_c^{-1})$ are correlated to the ones obtained by using the more computationally amenable $\text{trace}(W_c)$. Thus we choose the later one as our measure of degree of controllability. The selection of agents to intervene to maximize degree of controllability has been approached as a combinatorial optimization problem, where the number of candidate input sets grows exponentially with the number of inputs considered [44]. Recently, some theoretical results with performance guarantees have been obtained in [45] and [46]. In [46], it was shown that the set valued function $f(D) = \text{trace}(W_c^D)$ is modular, where $D \subset \mathcal{V}$ is a set of agents selected for the intervention and W_c^D is the controllability Gramian resulting from the intervention of the agents in D. With this result, the selection of agents to intervene in is simplified to the evaluation and later sorting of the average controllability centrality

$$C_i = \text{trace}\left(W_c^i\right)$$

where W_c^i is the controllability Gramian resulting from the intervention of the ith agent only, i.e., $\bar{B} \in \mathbb{R}^{2(N+1)}$ with $\bar{B} = c_i e_{2i}$, where e_{2i} is the unit vector with non zero element in $2i$. The positive definitiveness of W_c^i results from Theorem 2, under the assumption that the environment influences and is influenced by all the agents in the drinking event. We will compare the results with a more heuristic centrality measure from a practical implementation perspective, that exploits the distinguished agents concept from Theorem 2. We define the intervention centrality for agent i

$$Q_i = \beta \frac{\sum_{j=1}^N w_{ji}}{\max_k \sum_{j=1}^N w_{jk}} + (1-\beta)\left(1 - \frac{\sum_{j=1}^N w_{ij}}{\max_k \sum_{j=1}^N w_{kj}}\right) \quad (21)$$

where $\beta \in [0, 1]$. The metric will rank agents in a descending order, allowing us to select the individuals with higher influence on others, and those that are less influenced by the others, thus intervening with a set of agents that may have as a subset the set of distinguished agents. We compare these

Algorithm 2 Selection of Agents to Intervene

for $i = 1$ to N **do**
    Compute the centrality measures C_i and Q_i
end for
for $j = 1$ to Ng (number of groups at DE) **do**
    Assign a rank to each agent in the group with respect to Q_i
end for
Create three lists of sorted agents: the first and second ones with respect to C_i and Q_i and the third with respect to the ranks assigned in the previous step for Q_i
Select the first K agents in each lists

selection methods with a third method that randomly selects individuals from the drinking event.

Given a number K of agents to intervene with, we select them according to Algorithm 2.

Fig. 7 shows results of a 300 run Monte Carlo simulation comparing the controllability degree, as measured by trace(W_c) in the vertical axis, of the three different approaches mentioned above and the result obtained by only controlling the environment agent, for different percentages of agents intervened in the drinking event, i.e., K/N. The drinking event setting was modeled with environmental risk and size factors $\kappa = 0.8$ and $\zeta = 0.8$ corresponding to a wet bar. We employ the intervention centrality with $\beta = 0.8$, considering the assumption that the environment influences and is influenced by all the agents. It is seen that, as expected, the selection using the metric C_i provides higher values for trace(W_c) than the other metrics, while the intervention centrality yields an intermediate result between the controllability centrality and the random selection method. This can be explained by the fact that the intervention centrality uses the interconnections between members of different groups to influence the safe trajectories. It is interesting to note that due to the relevance of the environment agent in each individual's decision making, controlling the environment yields higher degree of controllability than intervening less than 12% of individuals at the drinking event.

V. Numerical Examples

Employing the insights obtained in the previous sections, we proceed to simulate drinking events to observe the behavior of the individuals and the environment under interventions to reduce risky behaviors. We employ data sets from bars [37] and parties [8] to calibrate our model. Some parameters' distribution can be directly inferred from the data sets, such as initial and final intoxication, weight and gender, desired trajectory, environment wetness and size factor elements, and number of agents in each group. Others, mainly behavioral parameters, can be inferred indirectly as they did not belong to the original data sets, such as individual commitment, desired intoxication, group and environment influence on the individuals. A detailed account on the calibration of the model can be found in Appendix C in the supplementary file.

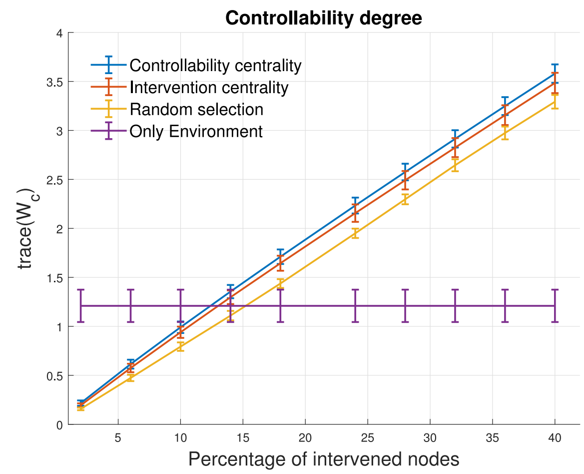

Fig. 7. Mean (lines) and standard deviation (vertical lines) of controllability degree, measured by the trace of the controllability Gramian W_c, for agent selection according to controllability centrality C_i, intervention centrality Q_i, group centrality, and intervention in the environment wetness agent for different numbers of intervened agents.

To illustrate the time trajectories of individuals' intoxication and environment wetness in a drinking event with intervention, we have in Fig. 8 the BAC trajectories (top plot) and corresponding drinking rates (bottom plot) for two groups, each of three individuals and the environment wetness (dashed line) of the bar. The data set in [37] indicates that the first group drank together for 120 min at the bar, and we depict their final intoxication with the star mark at the mentioned time, but we extended their BAC trajectories for completeness. The second group remained in the bar together for 180 min and their final intoxication is also marked with a star mark. The environment corresponds to a bar with 600 individuals, with presence of music and food, corresponding to environmental risk and size factors of $\kappa = 0.75$ and $\zeta = 0.9$, respectively, and we see that its wetness slightly increases with time driven by the intoxication of its patrons. The list of parameter values employed in this simulation can be found in Appendix C in the supplementary file. We see that the individuals maintained their intoxication and drinking rates close to each other, although influenced by the environment wetness, they did not drive their drinking rates to zero after they reached their desired peak intoxication. Fig. 9 shows the trajectories of the same drinking event, now with the intervention of the environment agent as well as the individuals at the bar. Note that their peak intoxication levels are lower than those in the no intervention case, represented by the star marks in the plot.

Next, we simulate individual and environment drinking behaviors in two different settings: bars and parties. We compare two intervention methods: 1) intervening only in the environment wetness agent and 2) intervening with 1/3 of the individuals using a modified version of the intervention centrality measure. With the insight gained in Section III, we modify the centrality measure in (21) to target the individuals whose desired trajectories are higher than their assigned safe trajectories. We model these trajectories as in (17), with constant drinking rates u_i^d and u_i^r, respectively, and add the term $(u_i^d - u_i^r)/\max_i(u_i^d - u_i^r)$ to the intervention centrality Q_i.

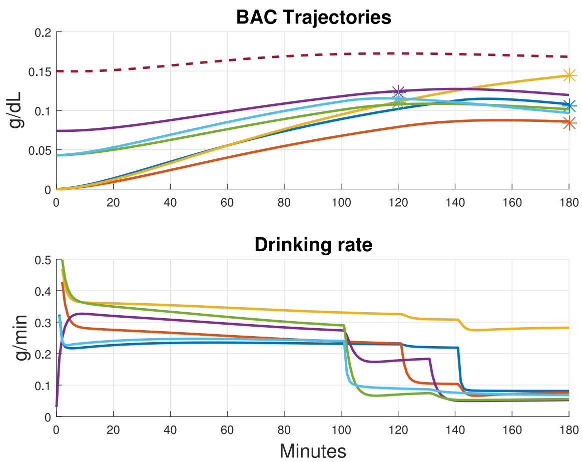

Fig. 8. BAC time trajectories (top) and drinking rate (bottom) for two groups of individuals in a bar. Markers are final measured BAC. Data is from [37].

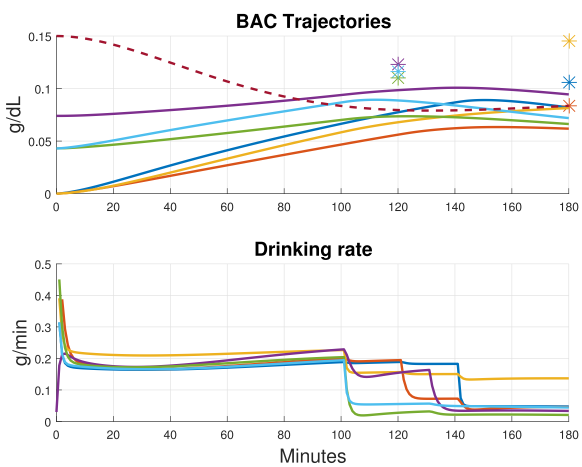

Fig. 9. BAC time trajectories (top) and drinking rate (bottom) for two groups of individuals in a bar after intervention on leaders of the groups and the environment.

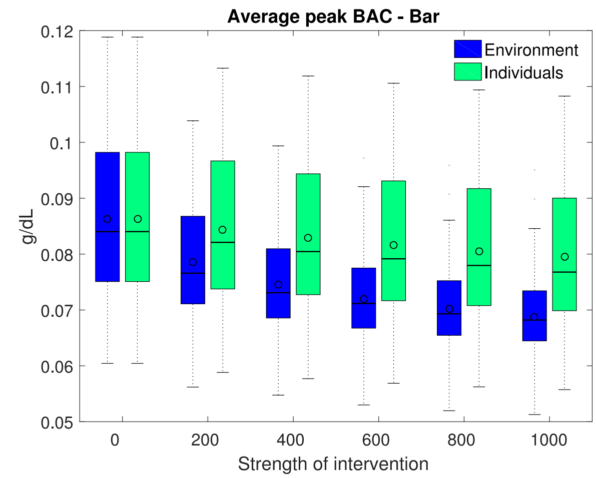

Fig. 10. Average peak BAC for DE in a bar with $N \sim \mathcal{N}(590, 340)$ with intervention in the environment wetness agent and subset of individuals. Parameters were informed by data collected in [37].

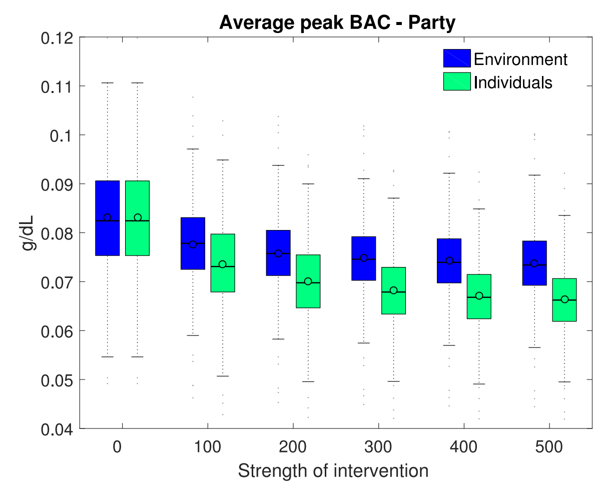

Fig. 11. Average peak BAC for DE in a party with $N \sim \mathcal{N}(40, 20)$ with intervention in the environment wetness agent and subset of individuals. Parameters were informed by data collected in [8].

Fig. 10 shows the results of a Monte Carlo simulation with 500 runs with the average (circles), median (horizontal lines), first and third quartiles (edges of the boxes) of peak BAC, in g/dL, of the individuals at a bar under the two proposed intervention methods for various strength of interventions, represented here as $\bar{s} = \sum_{i=1}^{\mathcal{K}} s_i$, where $\mathcal{K} \subset \mathcal{V}$ is the set of agents that had interventions. The distribution of the environmental risk and size factors was computed using [37]. It is seen that channeling all the resources to intervene in the environment yields lower levels of intoxication for the individuals at the drinking event. This result was expected after the results in Section IV-B, where the influence of the environment agent is exploited to reduce the levels of intoxication. Note that if we continue increasing the available strength of intervention we will be able to drive the environment to its safe trajectory, while further increasing $\bar{s} = s_{N+1}$ yields no improvement with respect to the environment wetness trajectory and the peak BAC values of the individuals. Thus, a *saturation* of the intervention in the environment occurs at some intervention strength $\bar{s}_e$. On the other hand, when intervening with the subset of individuals, by further increasing $\bar{s} = \sum_{i \in \mathcal{K}} s_i$, in this case beyond $\bar{s}_e$, we would have enough resources to drive these individuals to their safe trajectories before reaching the saturation point $\bar{s}_s$. At this point we could have the same or lower levels of peak BAC than the ones obtained at $\bar{s}_e$, because the set of individuals, with their intoxication matching their safe trajectories, could have a stronger influence than the environment at its safe trajectory. However, even though not carefully discussed in this paper, the use of higher values on the strength on the intervention would probably yield higher implementation costs, such that targeting the environment becomes the most efficient intervention method under this scenario. Fig. 11 shows results for parties setting, where lower peak BAC levels are achieved by intervening with the individuals. Here, the saturation point $\bar{s}_e$ is achieved by lower values of the strength of intervention $\bar{s}$ due to the smaller size of the environment agent with respect to bars.

VI. Conclusion

In this paper, we employed tools of control theory to provide suggestions for interventions at the event level in drinking events. We showed that the intoxication trajectories for the individuals and the environment wetness remain bounded under coupling constraints, and that the bound depends on the distance between each agent's desired and assigned safe trajectory. We also showed that by intervening with the agents that are not susceptible to social influence, and the agents that are the origins of the longest forward paths in the graph, we could ensure structural controllability of the system. The average controllability centrality, based on the trace of the controllability Gramian, provided a benchmark for the selection of agents to intervene based on a heuristic measure in which the more influential and less influenceable individuals are targeted. This measure, the intervention centrality, outperformed a simple random selection as measured by the degree of controllability. Finally, based on the ultimate bound found in the stability analysis, we added a third element to the intervention centrality metric: targeting individuals with higher mismatch between desired and safe trajectories. Simulations of bar and private party settings using experimental data showed that in the case of bars, it is more cost effective to intervene in the environment wetness agent, while in the case of parties, devoting the resources to intervene with a subset of individuals resulted in more efficient peak BAC levels reduction, and indirectly, risky individual behavior.

Besides the implementation of this approach in mobile apps or dedicated devices, future research directions could include improving the human behavior model to account for the dynamics of persuasion of interventions, which would further validate the usefulness of this approach in reducing risky alcohol consumption. Also, with some modifications, this model could prove useful in studying the dynamic effects of interventions to reduce the abuse of other recreational drugs. Furthermore, closing the loop by allowing feedback from the agents intoxication to be used in the design of the input signal $x^r(t)$ could lead to more efficient implementations, possibly reducing the peak BAC values found in Section V. These directions may result in nonlinear dynamics representing the drinking event, but we hope that this paper could serve as a starting point for the mentioned implementations.

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