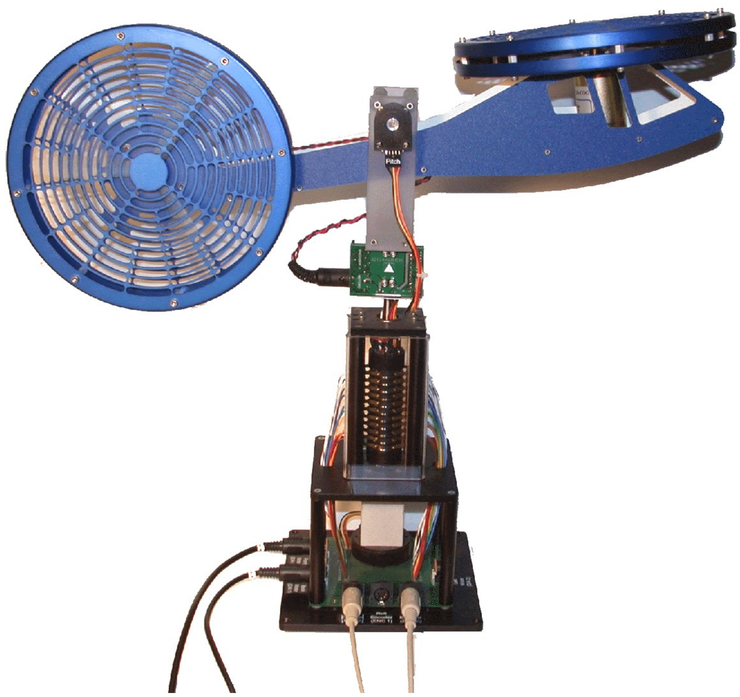
Aerospace Plant: 2-DOF Helicopter

Position Control

2-DOF Helicopter



Reference Manual

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1. Introduction

The Quanser 2 DOF Helicopter experiment, shown in Figure 1, consists of a helicopter model mounted on a fixed base with two propellers that are driven by DC motors. The front propeller controls the elevation of the helicopter nose about the pitch axis and the back propeller controls the side to side motions of the helicopter about the yaw axis. The pitch and yaw angles are measured using high-resolution encoders. The pitch encoder and motor signals are transmitted via a slipring. This eliminates the possibility of wires tangling on the yaw axis and allows the yaw angle to rotate freely about 360 degrees.

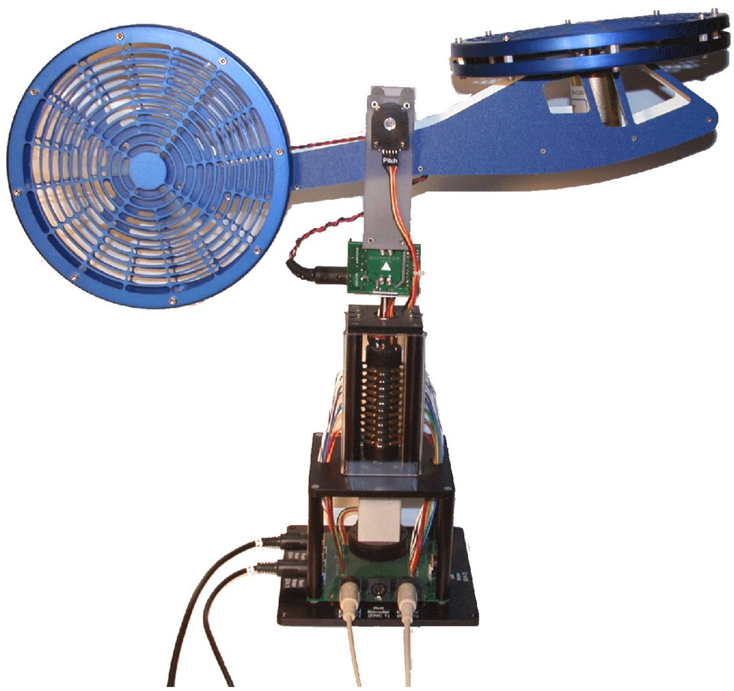


Figure 1: Quanser 2 DOF Helicopter

In Section 4 the components composing the 2 DOF Helicopter are described and the system specifications are given. Section 5 explains how to setup the system and gives the wiring procedure. The modeling and position control design of the helicopter are summarized in Section 6. In Section7, several procedures are outlined that show how to simulate the position controller and how to run this controller on the actual helicopter plant. Further, this section explains how to use the joystick to manually control the helicopter.

2. Prerequisites

In order to successfully carry out this laboratory, the user should be familiar with the following:

- 2-DOF Helicopter main components (e.g. actuator, sensors), the data acquisition card (e.g. Q8), and the power amplifier (e.g. UPM), as described in Section 4, Reference [1], and Reference [4], respectively.
- Wiring the 2-DOF Helicopter plant with the UPM and DAC device, as discussed in Section 5.
- Designing a state-feedback control using Linear-Quadratic Regulator, i.e. LQR.
- Using QuaRC, or another equivalent software, to control and monitor a plant in real-time and in designing a controller through Simulink.

3. Experiment Files Overview

Table 1 below lists and describes the various files supplied with the 2-DOF Helicopter experiment.

File Name	***Description***
2-DOF Helicopter Reference Manual.pdf	This manual is both the user and laboratory guide for the Quanser 2-DOF Helicopter specialty aerospace plant. It contains information about the hardware components, specifications, information to setup and configure the hardware, system modeling, control design, as well as the experimental procedure to simulate and implement the controller.
2-DOF Heli Equations.mws	Maple worksheet used to analytically derive the state-space model involved in the experiment. Waterloo Maple 9, or a later release, is required to open, modify, and execute this file.
2-DOF Heli Equations.html	HTML presentation of the Maple Worksheet. It allows users to view the content of the Maple file without having Maple 9 installed. No modifications to the equations can be performed when in this format.
quanser.ind and quanser.lib	The *Quanser_Tools* module defines the generic procedures used in Lagrangian mechanics and resulting in the determination of a given system's equations of motion and state-space representation. It also contains data processing routines to save the obtained state-space matrices into a Matlab-readable file.
setup_lab_heli_2d.m	The main Matlab script that sets the model, control, and configuration parameters. **Run this file only to setup the laboratory.**
setup_heli2d_configuration.m	Returns the 2-DOF Helicopter model parameters and encoder calibration constants.
HELI2D_ABCD_eqns.m	Matlab script file generated using the Maple worksheet 2-*DOF*

File Name	***Description***
	Heli Equations.mws. It sets the *A*, *B*, *C*, and *D* matrices for the state-space representation of the 2-DOF Helicopter open-loop system.
d_heli2d_lqr.m	Matlab script that generates the position-velocity controller gain K using LQR.
d_heli2d_lqr_i.m	Matlab function that the position-integral-velocity controller gain K using LQR.
heli_2d_lib.mdl	The supplied Simulink models are linked to the blocks in this Simulink library.
s_heli_2d_ff_lqr_i.mdl	Simulink file that simulates the open-loop or closed-loop 2-DOF Helicopter using a nonlinear model of the system.
q_heli_2d_ff_lqr_i.mdl	Simulink file that implements the real-time position controller for the 2 DOF Helicopter system.
q_heli_2d_open_loop.mdl	Simulink file that runs the 2 DOF Helicopter in open-loop, i.e. allows user to command voltage directly to motors.

Table 1: Files supplied with the 2-DOF Helicopter experiment.

4. System Description

The following is a listing of the major hardware components used for this experiment:

- **Power Amplifier**: Quanser UPM-1503 and UPM-2405, or equivalent.
- **Data Acquisition Board**: Quanser Q4, Q8, or equivalent.
- **Helicopter Plant**: Quanser 2-DOF Helicopter aerospace experiment.
- **Real-time control software**: PC equipped with QuaRC-Simulink configuration.
- **Joystick**: *Logitech Attack-3 USB* joystick, or another windows-enabled joystick.

See the references listed in Section 8 for more information on these components.

4.1. Components

Section 4.1.1 lists the components on the 2-DOF Helicopter plant and Section 4.2 summarizes the system specifications.

4.1.1. 2-DOF Helicopter Components

The components comprising the 2-DOF Helicopter system are labeled in figures 2, 3, 4, 5 and 6, are described in Table 2. The motors, propeller assemblies, and encoders are described in more detail below.

ID #	Description	ID #	Description
1	Back propeller	13	Metal shaft (rotates about yaw axis)
2	Back propeller shield	14	Slip ring
3	Yaw/back motor	15	Yaw encoder
4	Pitch encoder	16	Base platform
5	Yoke	17	Front motor connector
6	Helicopter body	18	Right motor connector (not used)
7	Front propeller	19	Back motor connector
8	Pitch/front motor	20	Yaw encoder connector
9	Front propeller shield	21	Roll encoder connector (not used)
10	Encoder/motor circuit	22	Pitch encoder connector
11	Encoder connector on circuit (not used)	23	Left motor connector (not used)
12	Motor connector on circuit		

Table 2: 2-DOF Helicopter components.

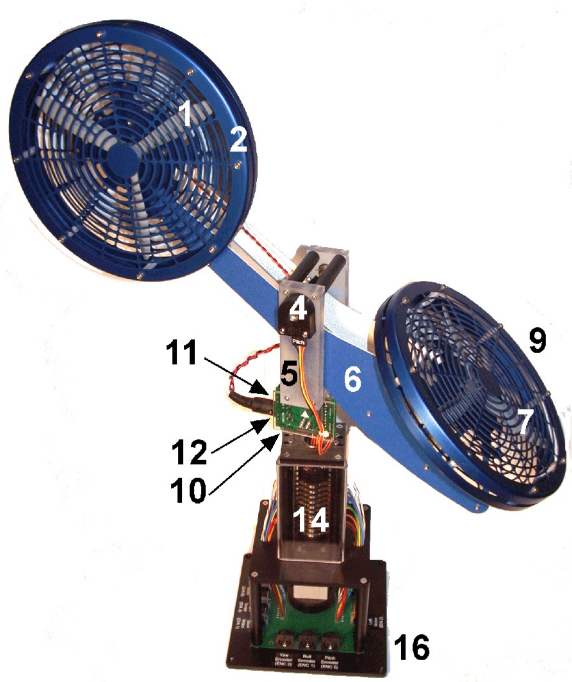


Figure 2: 2 DOF Helicopter components

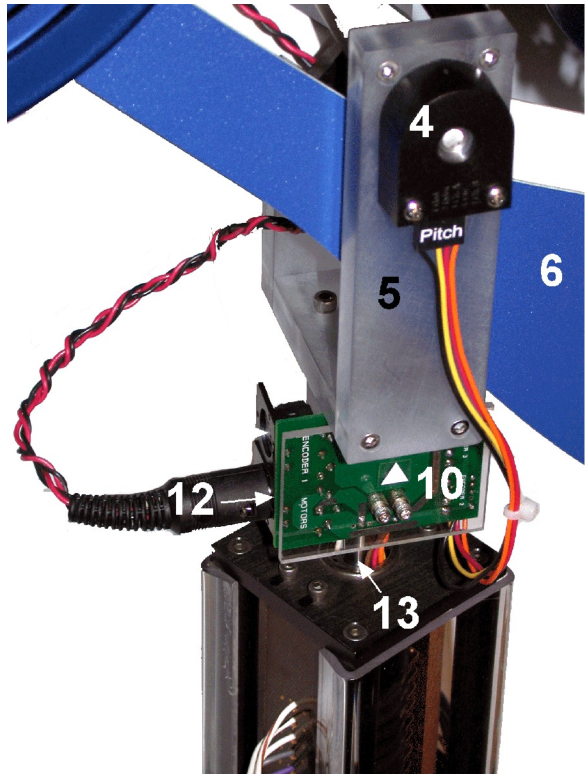


Figure 3: 2 DOF Helicopter yoke components.

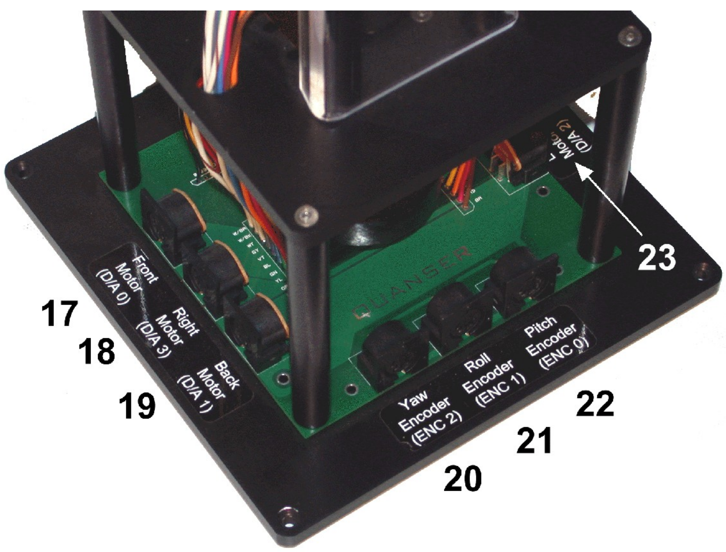


Figure 4: 2 DOF Helicopter base components.

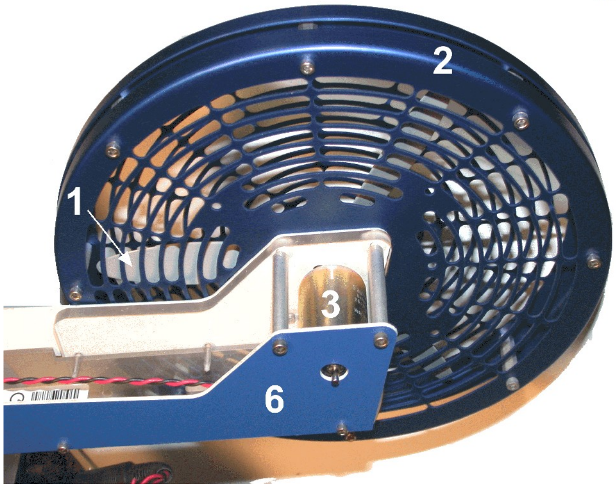


Figure 5: 2 DOF Helicopter tail components.

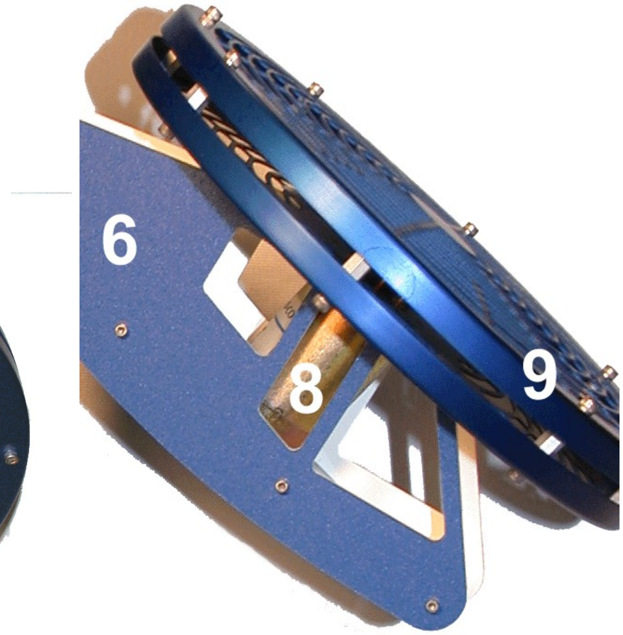


Figure 6: 2 DOF Helicopter front propeller assembly components.

4.1.1.1. DC Motors (Component #3 and #8)

The 2-DOF Helicopter has two DC motors: the yaw motor, component #3, actuating the back propeller and the pitch motor, component #8, rotating the front propeller.

The yaw motor is a *Faulhaber Series 2842 Model 006C* motor. It has a terminal resistance of 1.6 Ω and a current-torque constant of 0.0109 N.m/A. See Reference [6] for the full specifications of this motor. The larger pitch motor is a *Pittman Model 9234*. It has an electrical resistance of 0.83 Ω and a current-torque constant of 0.0182 N·m/A. The rated voltage of the motor is 12 V but its peak voltage can be brought up to 22 V without damage. See Reference [5] for the full specifications of this motor.

4.1.1.2. Propellers (Component #1 and # 7)

The pitch and yaw propeller assemblies are composed of the actual propeller, which is directly mounted to the motor shaft, and the aluminum propeller shield. The propellers used for both the pitch and yaw motors are Graupner 20/15 cm or 8/6". The pitch motor/propeller has an identified thrust-force constant of 0.104 N/V and the yaw motor/propeller has a thrust-force constant of 0.43 N/V..

4.1.1.3. Encoders (Components #4 and #15)

The 2-DOF Helicopter experiment has two encoders: the encoder measuring the pitch angle, component #4, and the encoder measuring the yaw angle, component #15. In quadrature mode, the pitch encoder has a resolution of 4096 counts per revolution and the yaw encoder has a resolution of 8192 counter per revolution. Thus the effective position resolution is 0.0879 degrees about the pitch axis and 0.0439 degrees about the yaw axis

4.2. System Specifications

Table 3 below lists the main parameters associated with the Quanser 2-DOF Helicopter experiment. The parameters that are *not* used directly in the mathematical model of the system or in the lab files given are shaded.

Symbol	*Matlab Notation*	*Description*	*Value*	*Unit*
$R_{m,p}$	R_m_p	Armature resistance of pitch motor.	0.83	Ω
$R_{m,y}$	R_m_y	Armature resistance of yaw motor.	1.60	Ω
$K_{t,p}$	K_t_p	Current-torque constant of pitch motor.	0.0182	N.m/A
$K_{t,y}$	K_t_y	Current-torque constant of yaw motor.	0.0109	N.m/A
$J_{m,p}$	J_m_p	Rotor moment of inertia of pitch motor.	1.91E-006	$kg.m^2$
$J_{m,y}$	J_m_y	Rotor moment of inertia of yaw motor.	1.37E-004	$kg.m^2$
$K_{f,p}$	K_f_p	Pitch propeller force-thrust constant (found experimentally).	0.1037	N/V
$K_{f,y}$	K_f_y	Yaw propeller force-thrust constant (found experimentally).	0.428	N/V
K_{pp}	K_pp	Thrust force constant of yaw motor/propeller.	0.204	N.m/V
K_{yy}	K_yy	Thrust torque constant acting on yaw axis from yaw motor/propeller	0.072	N.m/V
K_{py}	K_py	Thrust torque constant acting on pitch axis from yaw motor/propeller.	0.0068	N.m/V
K_{yp}	K_yp	Thrust torque constant acting on yaw axis from pitch motor/propeller	0.0219	N.m/V
$B_{eq,p}$	B_eq_p	Equivalent viscous damping about pitch axis.	0.800	N/V
$B_{eq,y}$	B_eq_y	Equivalent viscous damping about yaw axis.	0.318	N/V
m_{heli}	m_heli	Total moving mass of the helicopter (body, two propellers assemblies, etc.).	1.3872	kg
$m_{m,p}$	m_motor_p	Mass of pitch motor.	0.292	kg
$m_{m,y}$	m_motor_y	Mass of yaw motor.	0.128	kg
m_{shield}	m_shield	Mass of propeller shield.	0.167	kg

Symbol	*Matlab Notation*	*Description*	*Value*	*Unit*
m_{props}	m_props	Mass of pitch and yaw propellers, propeller shields, and motors.	0.754	kg
$m_{body,p}$	m_body_p	Mass moving about pitch axis.	0.633	kg
$m_{body,y}$	m_body_y	Mass moving about yaw axis.	0.667	kg
m_{shaft}	m_shaft	Mass of metal shaft rotating about yaw axis.	0.151	kg
L_{body}	L_body	Total length of helicopter body.	0.483	m
l_{cm}	l_cm	Center of mass length along helicopter body from pitch axis.	0.186	m
L_{shaft}	L_shaft	Length of metal shaft rotating about yaw axis.	0.280	m
$J_{body,,p}$	J_m_p	Moment of inertia of helicopter body about pitch axis.	0.0123	$kg.m^2$
$J_{body,,y}$	J_m_y	Moment of inertia of helicopter body about yaw axis.	0.0129	$kg.m^2$
J_{shaft}	J_shaft	Moment of inertia of metal shaft about yaw axis end point.	0.0039	$kg.m^2$
J_p	J_p	Moment of inertia of front motor/shield assembly about pitch pivot.	0.0178	$kg.m^2$
J_y	J_y	Moment of inertia of back motor/shield assembly about yaw pivot.	0.0084	$kg.m^2$
$J_{eq,p}$	J_eq_p	Total moment of inertia about pitch axis.	0.0384	$kg.m^2$
$J_{eq,y}$	J_eq_y	Total moment of inertia about yaw axis.	0.0432	$kg.m^2$
g	g	Gravitational constant.	9.81	m/s^2
$K_{EC,LN,T}$	K_LN_EC_T	Yaw encoder resolution (in quadrature mode).	8192	counts/rev
$K_{EC,LN,P}$	K_LN_EC_P	Pitch encoder resolution (in quadrature mode).	4096	counts/rev
$K_{EC,Y}$	K_EC_Y	Yaw encoder calibration gain.	7.67E-04	rad/counts
$K_{EC,P}$	K_EC_P	Pitch encoder calibration gain.	1.50E-03	rad/counts

Table 3: 2-DOF Helicopter system specifications.

5. System Setup and Wiring

Section 5.1 describes how to assemble and setup the Quanser 2-DOF Helicopter specialty plant. The cables used to connect the helicopter system are summarized in Section 5.2 and the standard wiring procedure is given in Section 5.3. Lastly, the joystick that can be used to control the helicopter is discussed in Section 5.4.

5.1. System Setup

Follow these steps for the mechanical setup of the Quanser 2-DOF Helicopter device:

1. Place the support base, component #16 shown in Figure 2, on a table or on the floor.
2. By holding the front and back of helicopter body, component #6, place the yoke, component #5 in Figure 2, on the top of the base. Ensure the white arrow labels on the circuit board, ID #10 in in Figure 4, and the frame are aligned. Once fitted, tighten the two thumb screws.
3. Connect the 6-pin-DIN to 6-pin-DIN motor cable protruding from the helicopter body to the connector labeled “MOTOR” on the helicopter circuit, component #12 shown in Figure 3.

⚠ **CAUTION: Once the body installed on the support base, never lift the system from the helicopter body. Always carry the system from the base with one hand and stabilize the body with the other hand. Never apply extreme loads in the vertical direction!**

4. The standard setup and starting position for the 2-DOF Helicopter system is depicted in Figure 2, above.

⚠ **CAUTION: Exposed moving parts. Ensure all obstructions that may interfere with the complete 360-degree axial motion of the helicopter are removed before performing any experiment.**

5.2. Cable Nomenclature

Table 4, below, provides a description of the standard cables used in the wiring of the 2-DOF Helicopter system.

Cable	Designation	Description
 *Figure 7 "From Digital-To-Analog" Cable*	5-pin-DIN to RCA	This cable connects an analog output of the data acquisition terminal board to the power module for proper power amplification.
 *Figure 8: "To Load" Cable of gain 3.*	4-pin-DIN to 6-pin-DIN	This cable connects the output of the power module, after amplification, to the desired actuator (e.g., propeller motor). One end of this cable contains a resistor that sets the amplification gain. When carrying a label showing "3", at both ends, the cable has that particular amplification gain.
 *Figure 9 "To Load" Cable Of Gain 5*	4-pin-DIN to 6-pin-DIN	This cable connects the output of the power module, after amplification, to the desired actuator (e.g., propeller motor). One end of this cable contains a resistor that sets the amplification gain. When carrying a label showing "5", at both ends, the cable has that particular amplification gain.

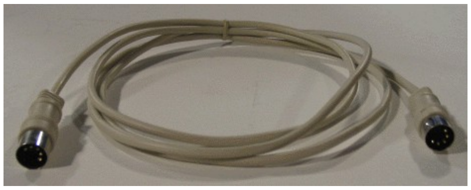
Cable	*Designation*	*Description*
 *Figure 10 "Encoder" Cable*	5-pin-stereo-DIN to 5-pin-stereo-DIN	This cable carries the encoder signals between an encoder connector and the data acquisition board (to the encoder counter). Namely, these signals are: +5VDC power supply, ground, channel A, and channel B.

Table 4 Cable Nomenclature

5.3. Typical Connections for the 2-DOF Helicopter

The yaw and pitch encoders are connected directly to the data-acquisition board. This provides the position feedback necessary to control the helicopter. The data-acquisition board, i.e. DACB, outputs a control voltage that is amplified and drives the front and back motors. The front motor is driven by a Quanser Universal Power Module 2405, i.e. UPM-2405, which is capable of delivering a maximum voltage of ±24V to the motors. The back motor is connected to a UPM-1503, which can deliver up to ±15V.

CAUTION: Pitch motor input: ±24V, 5A peak, 3A continuous.
Yaw motor: ±15V, 3A peak, 1A continuous.

This section describes the typical cabling connections that are used by default for the Quanser 2-DOF Helicopter system. Figures 11, 12, 13, and 14 illustrate, respectively, the wiring of the the Q8 Terminal Board, the Helicopter base, the UPM driving the front motor, and the UPM driving the back motor. The connections are described in detail in the procedure below and summarized in Table 5.

Follow these steps to connect the 2-DOF Helicopter system:

1. It is assumed that the Quanser Q4 or Q8 board is already installed as discussed in the Reference [1]. If another data-acquisition device is being used, e.g. NI M-Series board, then go to its corresponding documentation and ensure it is properly installed.
2. Make sure everything is powered off before making any of these connections. This includes turning off your PC and the UPMs.
3. Connect the 5-pin-DIN to RCA cable from the *Analog Output Channel #0* on the DAC board to the *From D/A Connector* on a UPM-2405. See cable #1 shown in Figure 11 and Figure 13. This carries the attenuated **front** motor voltage control signal, $u_p/K_{a,p}$, where $K_{a,p}$ is the UPM-2405 amplifier gain.
4. Connect the 5-pin-DIN to RCA cable from the *Analog Output Channel #1* on the DAC board to the *From D/A Connector* on a UPM-2405. See the cable #2 shown in Figure 11 and Figure 14. This carries the attenuated **back** motor voltage control signal, $u_y/K_{a,y}$, where $K_{a,y}$ is the UPM-1503 amplifier gain.
5. Connect the 4-pin-stereo-DIN to 6-pin-stereo-DIN that is labeled **Gain 5** from *To Load* on the

UPM-2405 to the *Front Motor* connector. See connection #3 shown in Figure 12 and Figure 13. This cable sets the gain of the amplifier to 5 and the connector on the UPM-side is gray in colour. The cable transmits the amplified voltage that is applied to the **front** motor, denoted $V_{m,p}$.

ATTENTION: The Quanser **UPM-2405** is capable of providing the required power to the 2-DOF Helicopter motors. However, it should be used in conjunction with a ***"To Load"*** **cable of gain 5** (i.e. 4-pin-DIN-to-6-pin-DIN cable), as described in Table 5, below. See Reference [4] for more detail.

6. Connect the 4-pin-stereo-DIN to 6-pin-stereo-DIN that is labeled **Gain 3** from *To Load* on the UPM-1503 to the *Back Motor* connector. See connection #4 shown in Figure 12 and Figure 14. The cable carries the amplified **back** motor voltage and is represented by the variable $V_{m,y}$.

ATTENTION: The Quanser **UPM-1503** is capable of providing the required power to the 2-DOF Helicopter back motor. However, it should be used in conjunction with a ***"To Load"*** **cable of gain 3** (i.e. 4-pin-DIN-to-6-pin-DIN cable), as described in Table 5, below. See Reference [4] for more detail.

7. Connect the 5-pin-stereo-DIN to 5-pin-stereo-DIN cable from the *Pitch Encoder* connector on the 2-DOF Helicopter base to *Encoder Input #0* on the terminal board. See connection #5 in Figure 11 and Figure 12. This carries the pitch angle measurement which is represented by variable θ.
8. Connect the 5-pin-stereo-DIN to 5-pin-stereo-DIN cable from the *Yaw Encoder* connector on the 2-DOF Helicopter base to *Encoder Input #1* on the terminal board, as depicted by connection #6 in Figure 11 and Figure 12. This carries the yaw angle measurement and is denoted by the variable ψ.

CAUTION: Any encoder should be directly connected to the Quanser terminal board (or equivalent) using a standard 5-pin DIN cable. **DO NOT connect the encoder cable to the UPM!**

9. If you are using are using the Logitech Attack 3 USB joystick shown in Figure 16, then connect the USB cable from the joystick to a USB port on the PC while it is running. The system should detect the joystick and automatically install the driver (you will be prompted). See the *Logitech Installation Manual* for more information on the setup procedure. See Section 5.4 for more information on system requirements of the Logitech joystick and how to use the *Rate Command* knob.

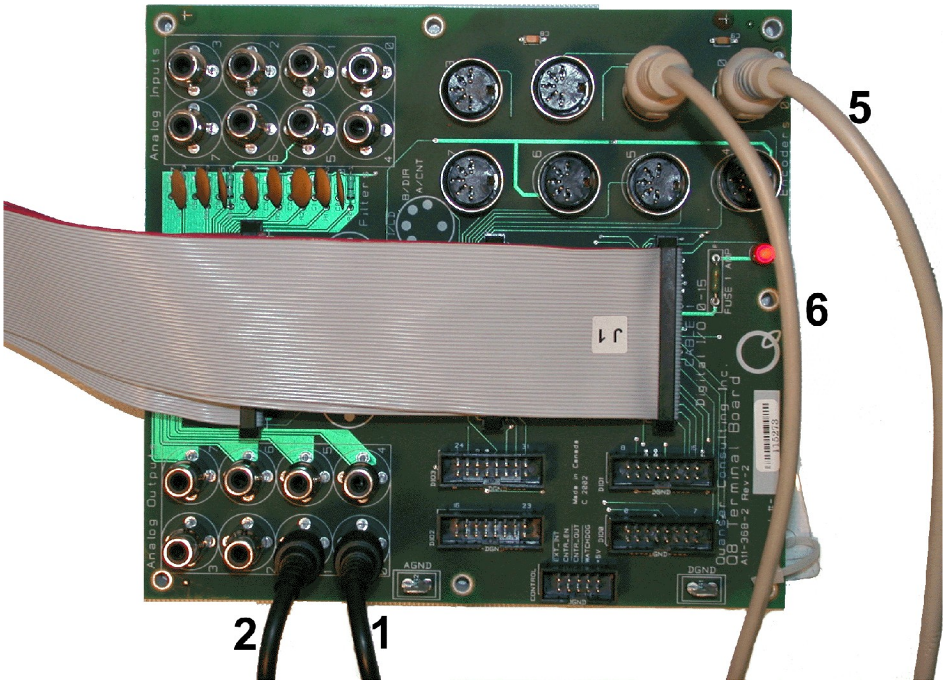


Figure 11: Connection on the terminal board.

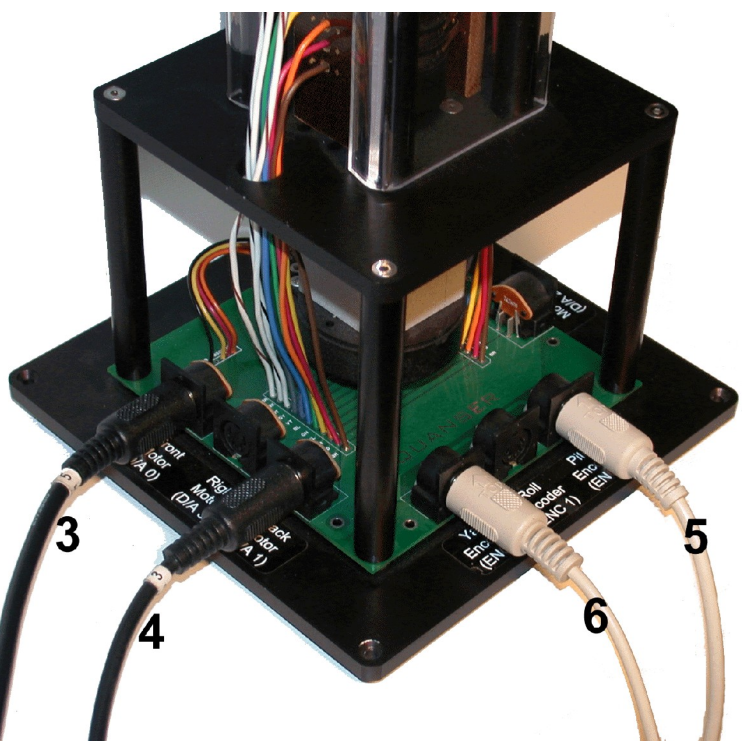


Figure 12: Connection on base of 2-DOF Helicopter.

Figure 13: Connection on UPM-2405 for pitch motor.

Figure 14: Connection on UPM-1503 for yaw motor.

Cable #	*From*	*To*	*Signal*
1	Terminal Board: DAC #0	Front-UPM "From D/A" connector	Control signal to the front UPM
2	Terminal Board: DAC #1	Back-UPM "From D/A" connector	Control signal to the back UPM
3	UPM-2405 "To Load" connector	2-DOF Helicopter "Front Motor D/A 0" connector	Power leads to the 2-DOF Helicopter's front DC motor (propeller). **Cable of gain 5**.
4	UPM-1503 "To Load" connector	2-DOF Helicopter "Back Motor D/A 1" connector	Power leads to the 2-DOF Helicopter's back DC motor (propeller). **Cable of gain 3**.
5	2-DOF Helicopter "Yaw Encoder ENC 0" connector	Terminal Board: Encoder Channel #0	2-DOF Helicopter's yaw angle feedback signal to the data acquisition card
6	2-DOF Helicopter "Pitch Encoder ENC 1" connector	Terminal Board: Encoder Channel #1	2-DOF Helicopter's pitch angle feedback signal to the data acquisition card

Table 5 2-DOF Helicopter system wiring summary

5.4. Joystick Description

The Quanser 2-DOF Helicopter experiment is supplied with either an analog joystick, shown in Figure 15, or a Logitech Attack 3 USB joystick, shown in Figure 16. They are used to generate a desired position instead of commanding it via the Simulink model blocks (see lab procedure later).

Figure 15: Analog joystick.

Figure 16: Logitech Attack-3 USB joystick.

The setup procedure for the USB joystick is described in Section 5.3. The rate command knob shown in Figure 16 changes the rate at which a command is generated by the joystick. The rate is at its greatest when the knob is turned fully toward the joystick handle.

The system requirements for the Logitech Attack-3 USB joystick are:

- PC with Pentium Processor or compatible
- 64 MB RAM
- USB port
- Windows 98, 2000, Me, or Xp

6. Modeling and Control Design

The mathematical model developed for the 2-DOF Helicopter system is summarized in Section 6.1. In Section 6.2, the feedback system used to control the position of the helicopter is described.

6.1. Modeling

The free-body diagram of the 2-DOF Helicopter is illustrated in Figure 17 and it accompanies the Maple worksheet named 2-*DOF Helicopter Equations.mws* or its HTML equivalent 2-*DOF Helicopter Equations.html*. The equations can be edited and re-calculated by executing the worksheet using Maple 9.

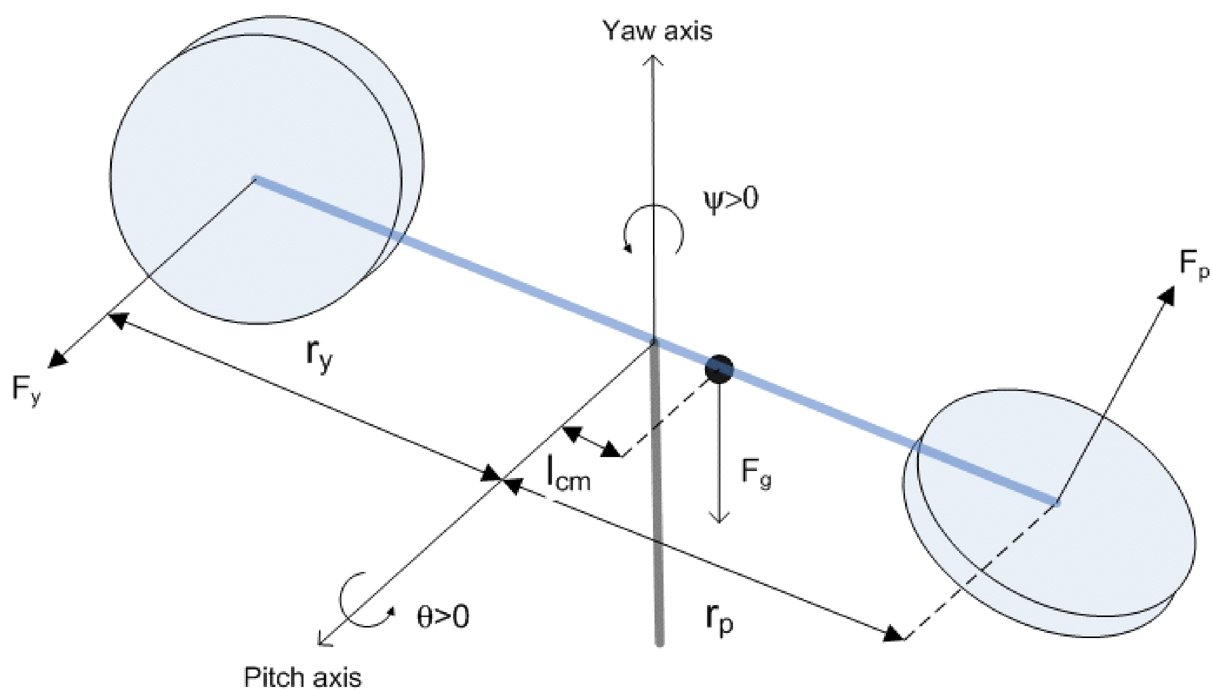


Figure 17: Simple free-body diagram of 2-DOF Helicopter.

The worksheet goes through the kinematics of the system. Thus the Cartesian coordinates of the center-of-mass are expressed relative to the base coordinate system, as shown in Figure 18. These resulting equations are used to find the potential energy and translational kinetic energy.

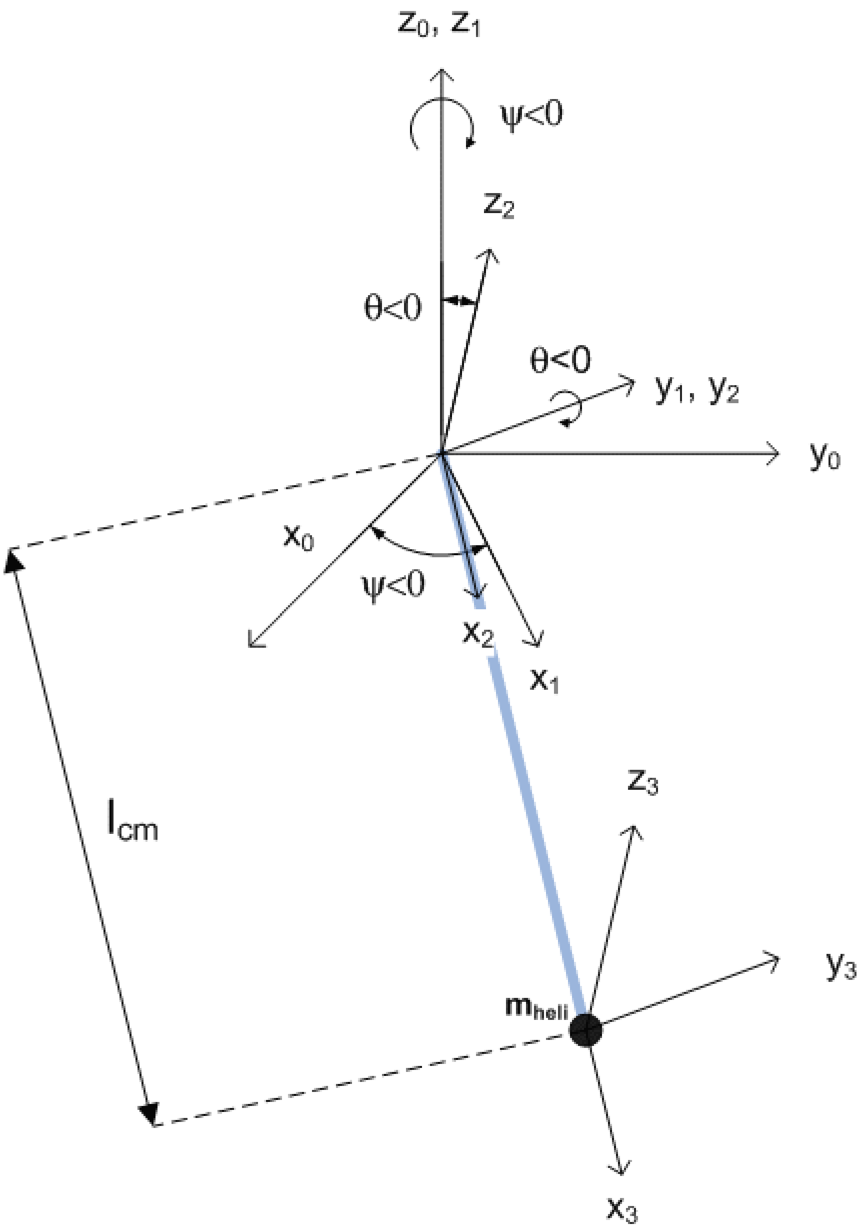


Figure 18: Kinematics of the 2-DOF Helicopter.

The thrust forces acting on the pitch and yaw axes from the front and back motors are then defined. Using the Euler-Lagrange formula, the nonlinear equations of motion of the 2-DOF Helicopter system are derived. These equations are linearized about zero and the linear state-space model (A,B,C,D) describing the voltage-to-angular joint position dynamics of the system is found. Given the state-space representation

$$\frac{\partial}{\partial t} x = A x + B u \qquad [2]$$

and

$$\frac{\partial}{\partial t} y = C y + D u \qquad [3]$$

the state vector for the 2-DOF Helicopter is defined

$$x^T = \left[\theta, \lambda, \frac{\partial}{\partial t}\theta, \frac{\partial}{\partial t}\lambda \right] \qquad [4]$$

and the output vector is

$$y^T = [\theta, \lambda] \qquad [5]$$

where θ and λ are the pitch and yaw angles, respectively. The corresponding helicopter state-space matrices (as derived in the Maple worksheet) are

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{B_p}{J_{eq\_p} + m_{heli} l_{cm}^2} & 0 \\ 0 & 0 & 0 & -\frac{B_y}{J_{eq\_y} + m_{heli} l_{cm}^2} \end{bmatrix}, \qquad [6]$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{pp}}{J_{eq\_p} + m_{heli} l_{cm}^2} & \frac{K_{py}}{J_{eq\_p} + m_{heli} l_{cm}^2} \\ \frac{K_{yp}}{J_{eq\_y} + m_{heli} l_{cm}^2} & \frac{K_{yy}}{J_{eq\_y} + m_{heli} l_{cm}^2} \end{bmatrix}, \qquad [7]$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \text{ and} \qquad [8]$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \qquad [9]$$

The model parameters used in the (A,B) matrices are defined in Table 3.

6.2. Control Design

In this section a linear proportional-integral-derivative, i.e. PID, controller is designed to regulate the elevation and travel angles of the 2-DOF Helicopter to desired positions. The PID control gains are computed using the Linear-Quadratic Regular algorithm. The state-feedback controller entering the front motor, u_f, and the back motor, u_b, is defined

$$\begin{bmatrix} u_p \\ u_y \end{bmatrix} = K_{PD}(x_d - x) + V_i + \begin{bmatrix} u_{ff} \\ 0 \end{bmatrix}, \quad [10]$$

where

$$K_{PD} = \begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4} \\ k_{2,1} & k_{2,2} & k_{2,3} & k_{2,4} \end{bmatrix} \quad [11]$$

is the proportional-derivative control gain,

$$x_d^T = [\theta_d \quad \lambda_d \quad 0 \quad 0] \quad [12]$$

is the desired state, x is the state defined in Equation [4],

$$V_i = \begin{bmatrix} \int k_{1,5}(x_{d,1} - x_1)\,dt + \int k_{1,6}(x_{d,2} - x_2)\,dt \\ \int k_{2,5}(x_{d,1} - x_1)\,dt + \int k_{2,6}(x_{d,2} - x_2)\,dt \end{bmatrix} \quad [13]$$

is the integral control, and

$$u_{ff} = \frac{K_{ff}\, m_{heli}\, g\, l_{cm} \cos(x_{d,1})}{K_{pp}} \quad [14]$$

is the nonlinear feed-forward control, which compensates for the gravitational torque. The variables θ_d and λ_d, are the pitch and yaw setpoints, i.e. the desired angles of the helicopter. In state-space, the desired pitch is angle $x_{d,1}$ and the desired yaw is $x_{d,2}$. The gains $k_{1,1}$ and $k_{1,2}$ are the front motor control proportional gains and the gains $k_{2,1}$ and $k_{2,2}$ are the back motor control proportional gains. Next, $k_{1,3}$ and $k_{1,4}$ are the front motor control derivative gains and $k_{2,3}$ and $k_{2,4}$ are the back motor control derivative gains. The integral control gains used in the front motor control are $k_{1,5}$ and $k_{1,6}$ and the integral gains $k_{2,5}$ and $k_{2,6}$ are used in the back motor regulator.

The PID control gains are computed using the Linear-Quadratic Regular scheme. The system state is first augmented to include the integrals of the pitch and yaw states,

$$x_i^T = \left[\theta, \lambda, \frac{\partial}{\partial t} p, \frac{\partial}{\partial t} \lambda, \int \theta\, dt, \int \lambda\, dt \right]. \quad [15]$$

Using the feedback law

$$u = -K x_i \quad [16]$$

the weighting matrices

$$Q = \begin{bmatrix} 200 & 0 & 0 & 0 & 0 & 0 \\ 0 & 150 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 200 & 0 & 0 \\ 0 & 0 & 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 & 0 & 50 \end{bmatrix} \quad [17]$$

and

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad [18]$$

and the state-space matrices (A,B) found previously, the control gain

$$K = \begin{bmatrix} 18.9 & 1.98 & 7.48 & 1.53 & 7.03 & 0.770 \\ -2.22 & 19.4 & -0.45 & 11.9 & -0.770 & 7.03 \end{bmatrix} \quad [19]$$

is calculated by minimizing the cost function

$$J = \int_0^\infty x_i^T Q x_i + u^T R u \, dt \quad [20]$$

with the Matlab *LQR* command. In terms of the PID control gains described earlier, the full control gain is expressed

$$K = \begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4} & k_{1,5} & k_{1,6} \\ k_{2,1} & k_{2,2} & k_{2,3} & k_{2,4} & k_{2,5} & k_{2,6} \end{bmatrix}. \quad [21]$$

The helicopter system runs the risk of integrator windup. That is, given a large error in the between the measured and desired pitch angle, θ-θ_d, or between the measured and desired yaw angle, ψ-ψ_d, the integrator outputs a large voltage that can saturate the amplifier. By the time the measured angle reaches the desired angle the integrator built-up so much energy that it remains saturated. This can cause large overshoots and oscillations in the response. To fix this, an integral windup protection algorithm is used. Figure 19 illustrates the anti-windup scheme implemented to control the pitch.

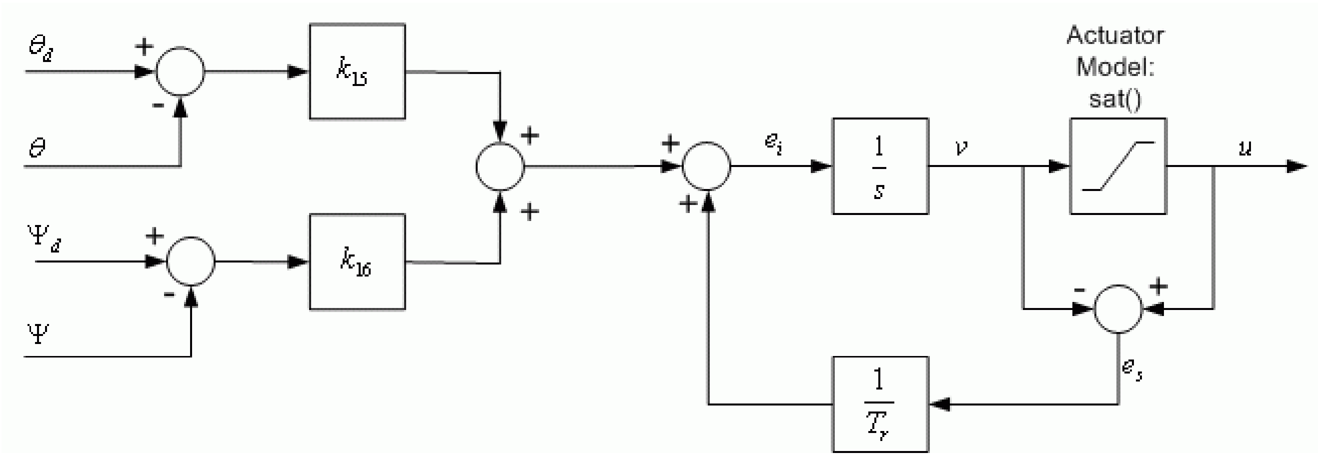


Figure 19: Anti-windup loop.

The integrator input shown in the windup loop is

$$e_i = k_{1,5}(\theta_d - \theta) + k_{1,6}(\psi_d - \psi) + \frac{u - v}{T_r} \quad [21]$$

When the integrator output voltage, *v*, is larger than the imposed integral saturation then the saturation error becomes negative, $e_s < 0$. The saturation error gets divided by the reset time, T_r, and its result is added to the integrator input. This effectively decreases the integrator input and winds-down the integrator. In the simulation and experimental results the saturation limit of the integrator is set to 5 Volts and the reset time to 1 second for maximum wind-down speed.

7. In-Lab Procedure

7.1. 2-DOF Helicopter Library

The heli_2d lib.mdl Simulink model shown in Figure 20 is the Quanser 2 DOF Helicopter Simulink library and it contains various subsystem that are used in the supplied Simulink models listed in Table 1. The *2DOF HELI: FF+LQR+I Controller* subsystem implements the feed-forward control and the LQR PID position controller discussed in Section 6.2. The *2DOF HELI: FF+LQR Controller* is a feed-forward and proportional-derivative control. Thus it is the same as the FF+LQR+I algorithm developed except there is no integral action. The nonlinear feed-forward control is constructed in the *Pitch feed-forward controller* block. The *2DOF HELI: Nonlinear Model* contains the nonlinear model summarized in Section 6.1.

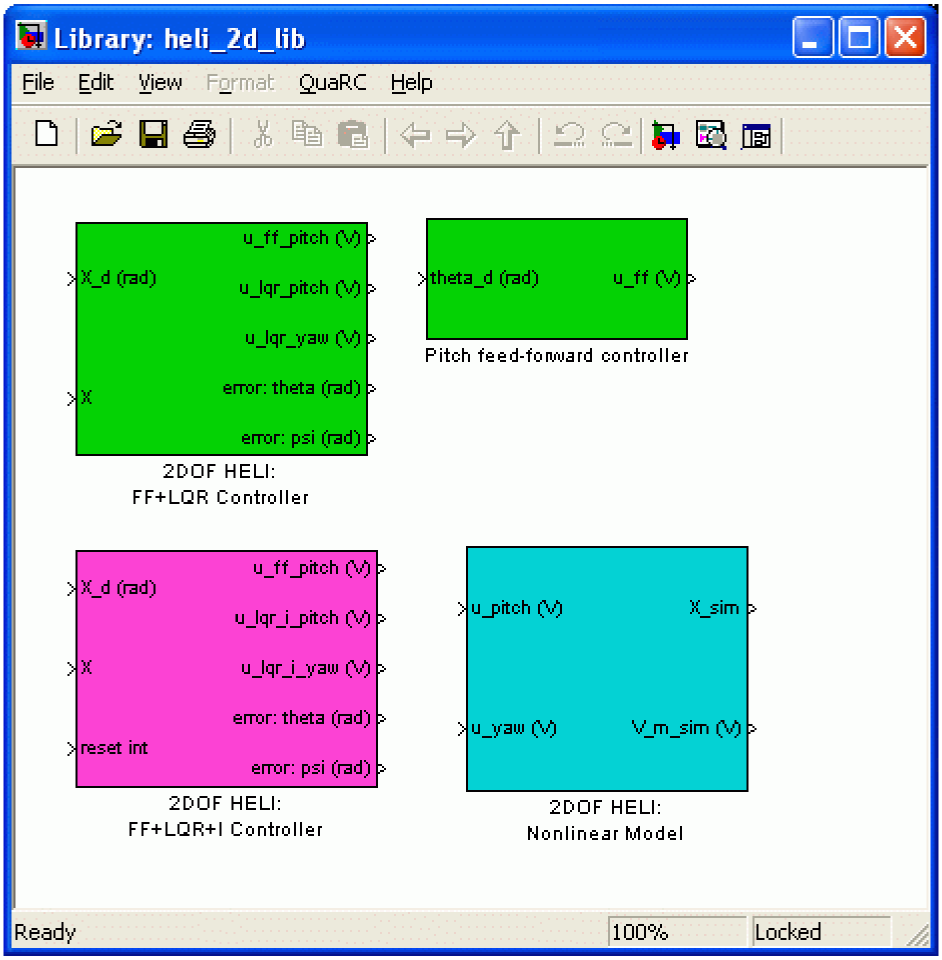


Figure 20: 2-DOF Helicopter Simulink library.

The interior of the *2DOF HELI: FF+LQR+I Controller* subsystem is displayed in Figure 21. As discussed in Section 6.2, the position and velocity states are multiplied by the corresponding elements of control gain K. The state includes the integral of the pitch and yaw angles and those are multiplied by the integral gains in K. Further the anti-windup scheme shown in Figure 19 is implemented in the *Pitch Integral Antiwindup* and *Yaw Integral Antiwindup* blocks. The *Pitch feed-forward controller* and *2DOF HELI: FF+LQR Controller* subsystems blocks are linked to 2-DOF Helicopter Simulink library.

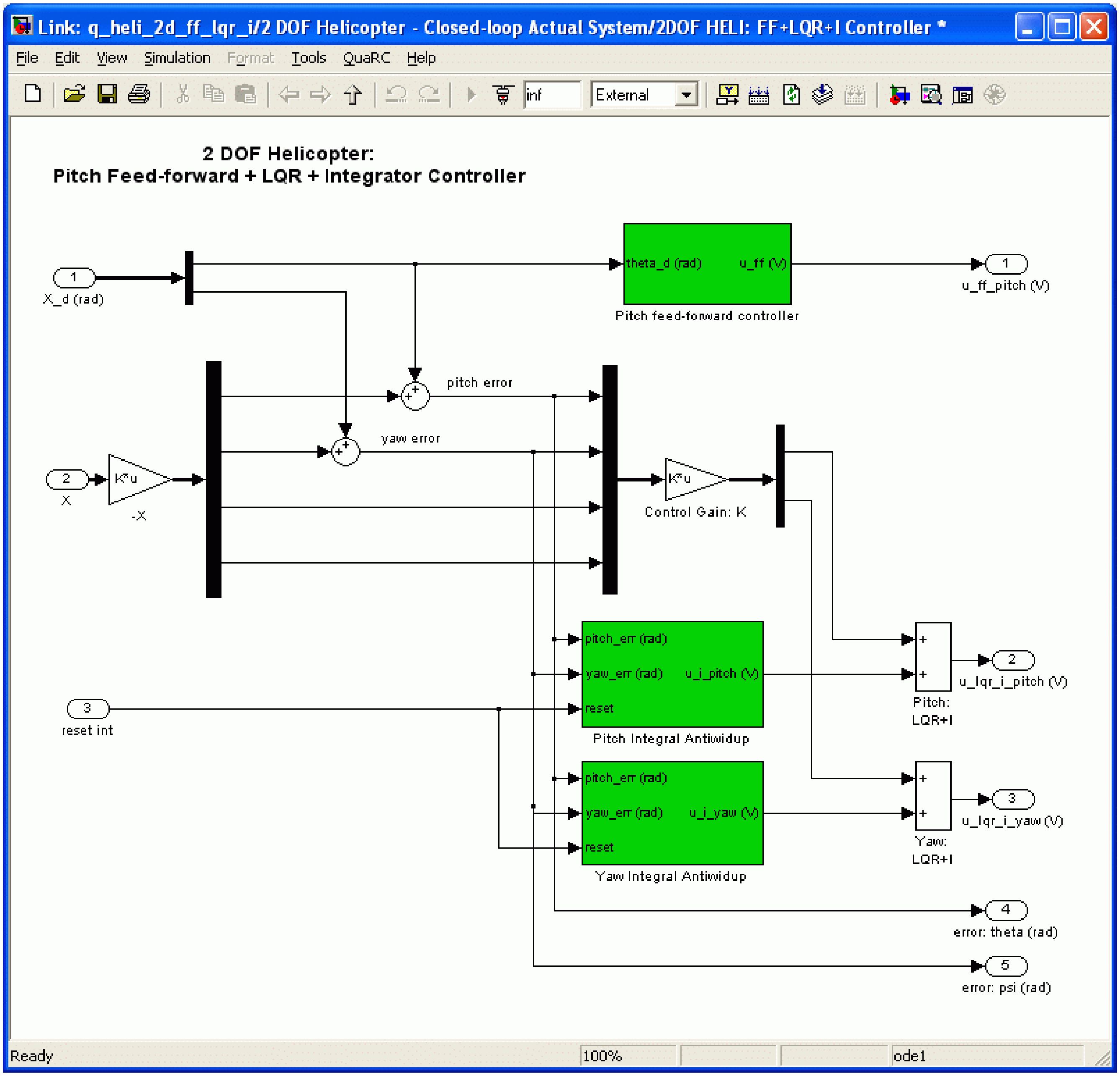


Figure 21: Subsystem that implements the FF+LQR+I controller.

7.2. Controller Simulation

7.2.1. Objectives

- Investigate the closed-loop position control performance of the FF+LQR and FF+LQR+I using a nonlinear model of the 2-DOF Helicopter system.
- Ensure the controller does not saturate the actuator.

7.2.2. Procedure

Follow these steps to simulate the closed-loop response of the 2-DOF Helicopter:

1. Load the Matlab software.
2. Open Simulink model *s_heli_2d_ff_lqr_i.mdl* shown in Figure 22.

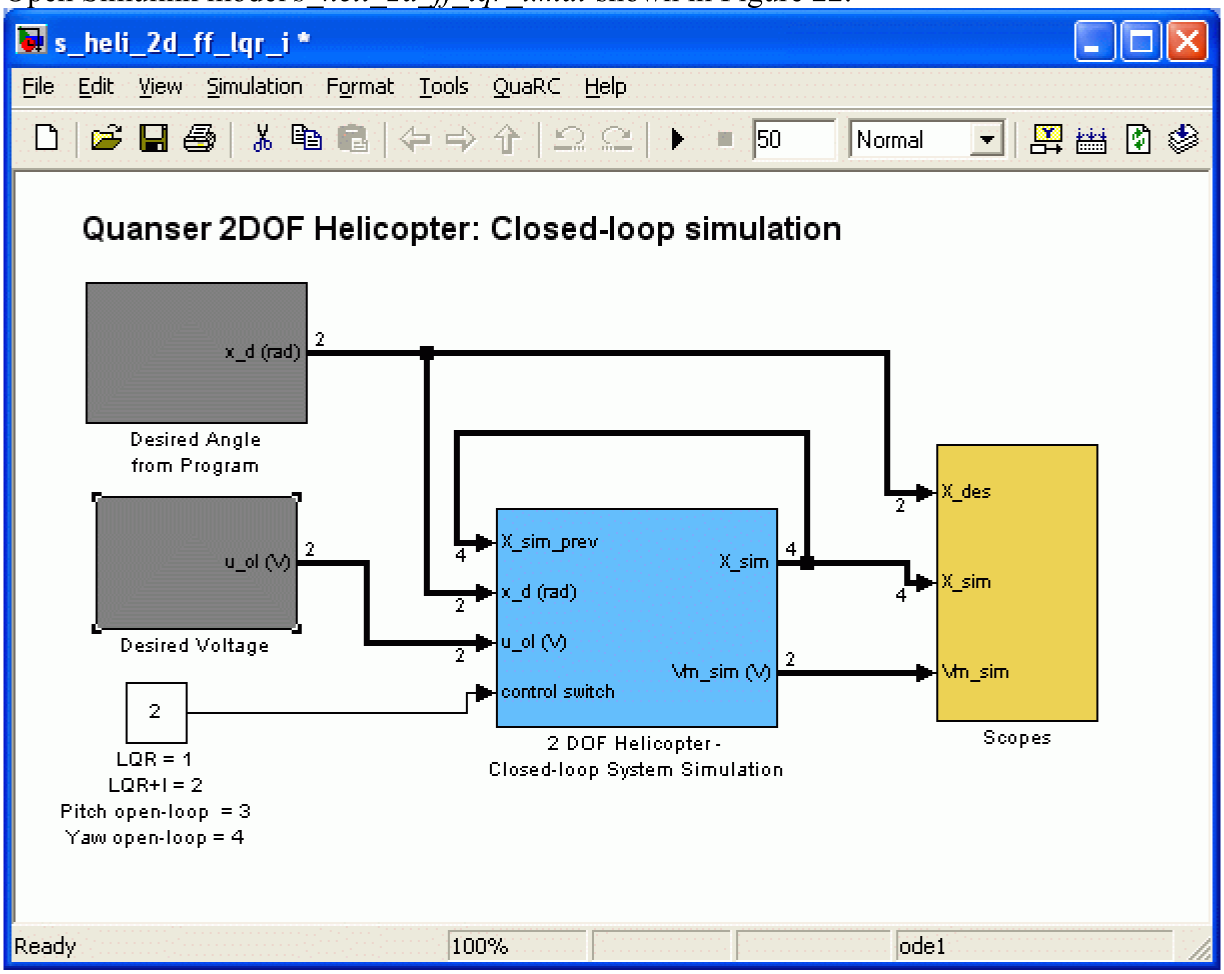


Figure 22: Simulink diagram used to simulate 2-DOF Helicopter system.

3. The subsystem labeled *Desired Angle from Program* is used to generate a desired pitch and yaw angle while the *Desired Voltage* block feeds open-loop voltages. The *Controller Switch* block implements the following switching logic:
   (a) switch = 1: FF+LQR closed-loop control.
   (b) switch = 2: FF+LQR+I closed-loop control.
   (c) switch = 3: Apply open-loop voltage to pitch motor.
   (d) switch = 4: Apply open-loop voltage to yaw motor.

   When the switch is 1 or 2 the system runs in closed-loop and when it is 3 or 4 the user can command voltages directly to the actuators. When the switch is made from the closed-loop mode to open-loop mode the controller voltage values are latched and the *Desired Voltage* block shown in Figure 22 is enabled. This is particularly useful when performing model validation and parameter tuning

4. The interior of the *2DOF Helicopter - Closed-loop System Simulation* subsystem is shown in Figure 23. The LQR and LQR+I control blocks along with the nonlinear model are all linked to the *2 DOF Helicopter Library* and are described in Section 7.1. The *Controller Switch* subsystem implements the logic to switch between the FF+LQR and FF+LQR+I controllers and between the pitch and yaw open-loop modes.

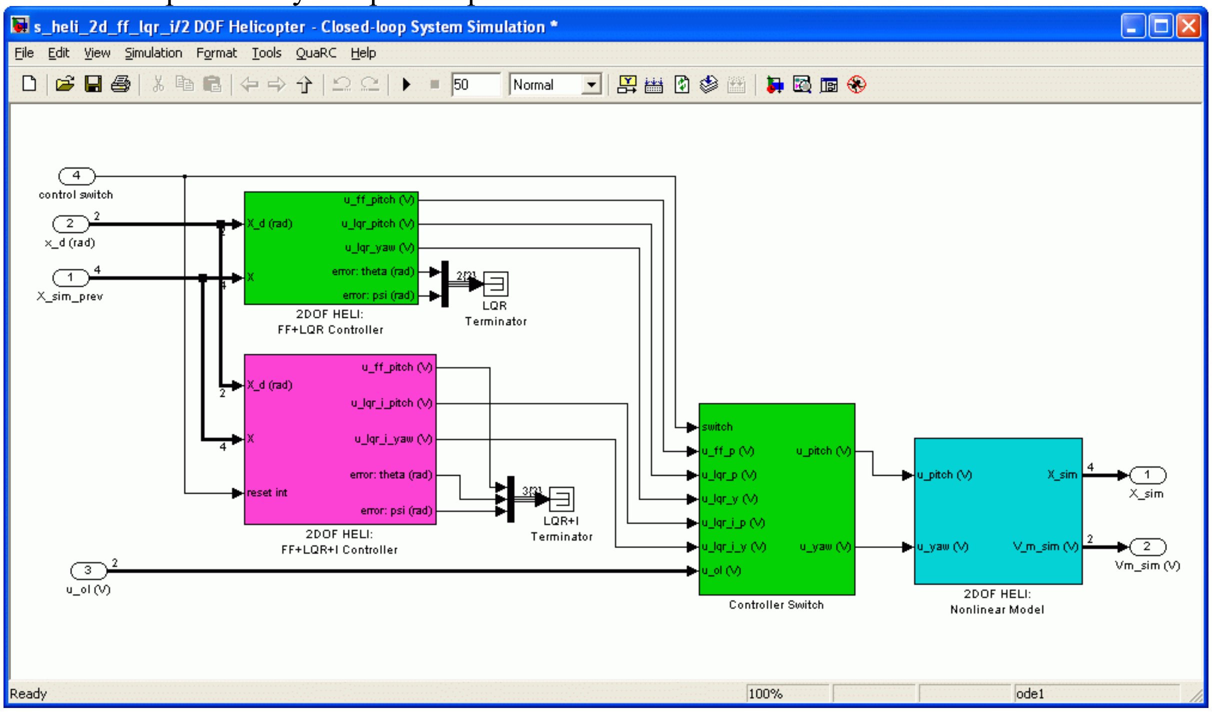


Figure 23: Closed-loop simulation of 2-DOF Helicopter.

5. Open the Matlab script called setup lab heli 2d.m. This script sets the model parameters, control gains, amplifier limits, and so on that are used in the 2 DOF Helicopter Simulink models supplied, such as s_heli_2d_lqr i.mdl. By default, K CABLE P, K CABLE Y, VMAX UPM P, VMAX UPM Y, K EC P, and K EC Y is set to match the configuration used in Section 5.3. If, for example, a gain cable of 3 is used with the UPM-2405 instead of 5 then the script parameter K CABLE P must be changed to 3. The parameter theta_0 initializes the integrator labeled *theta* in the *2DOF HELI Nonlinear Model* subsystem that calculates the pitch position.
6. The saturation limit of the integrators that are used in the FF+LQR+I controller are set using the variables SAT_INT_ERR_PITCH and SAT_INT_ERR_YAW. The reset time of the anti-windup loop can be changed using Tr_p and Tr_y. For more information on the anti-windup algorithm see Section 6.2.
7. Ensure the CONTROLLER TYPE is set to 'LQR AUTO' to generate the controller automatically. Set the feed-forward gain K_{ff} = 1 V/V and the LQR and LQR+I Q and R weighting matrices as already given in the script.
8. Run the Matlab script *setup_lab_heli_2d.m* to load the state-space model matrices, control gains, and various other parameters into the Matlab workspace. The LQR and LQR+I controls gains should be displayed in the Matlab Command Window.
9. Open the subsystem labeled *Desired Angle from Program,* shown in Figure 24, below

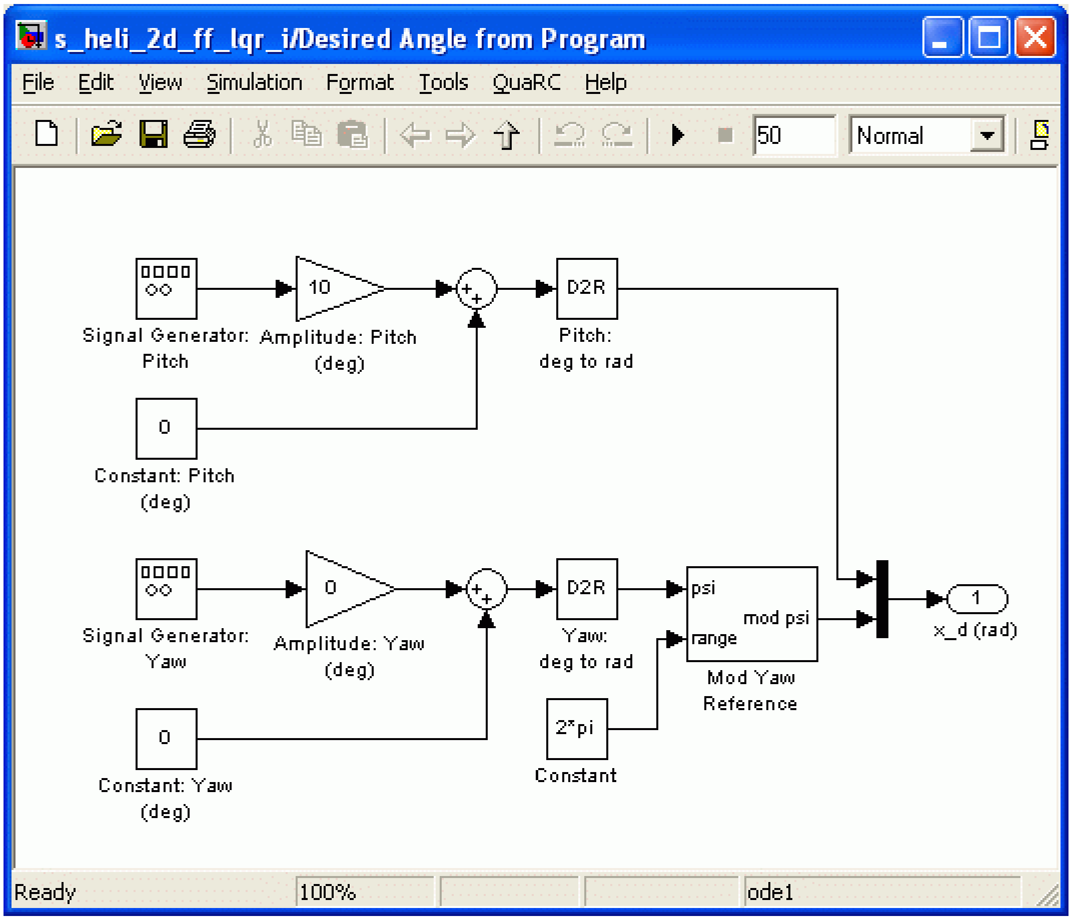


Figure 24: Desired Angle from Program subsystem.

10. Ensure the pitch scope, *theta (deg)*, the yaw scope, *psi (deg),* and the motor input voltage scope, *Vm_sim (V)*, are open. If not, go into the *Scopes* subsystem and double-click on those sinks.
11. To generate a desired pitch step of 20.0 degrees at 0.05 Hz frequency, set the *Amplitude: Pitch (deg)* gain block to 7.5 degrees and *Frequency* input box in the *Signal Generator: Elevation* block to 0.05 Hz.
12. Click on the *Start* simulation button, or on the *Start* item in the *Simulation* menu, to run the closed-loop system using LQR+I and the scopes should read as shown in figures 25, 26, and 27. In each scope, the simulated pitch and yaw angles (purple trace) should track the corresponding desired position signals (yellow trace). Also, examine the voltage in the *Vm_sim (V)* scope and ensure the front (yellow plot) and back motor (purple plot) are not saturated. Recall that the maximum peak voltage that can be delivered to the front motor by the UPM-2405 is ±24 V and to the back motor by the UPM-1503 is ±15.0 V.

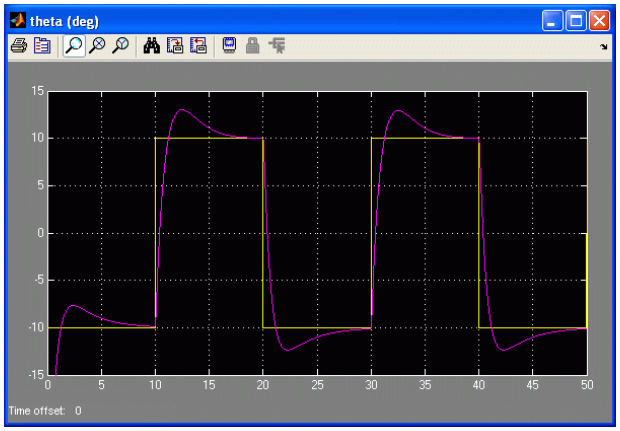


Figure 25: Simulated pitch response under pitch reference step using LQR+I.

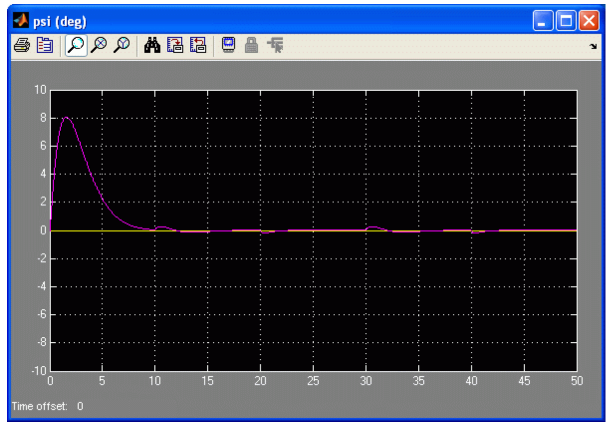


Figure 26: Simulated yaw response under pitch reference step using LQR+I.

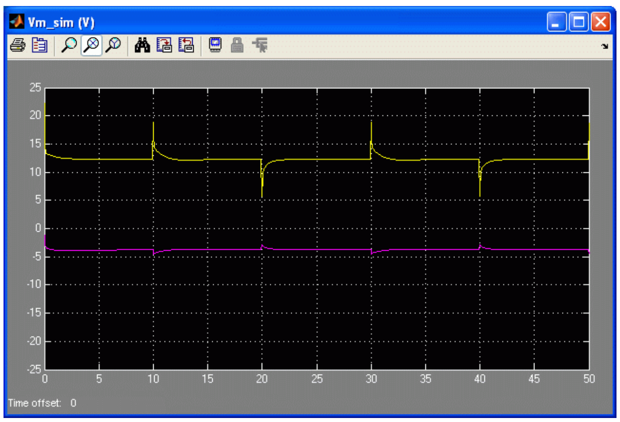


Figure 27: Simulated front and back motor voltage under pitch reference step using LQR+I.

13. To generate a desired yaw step of 100 degrees at 0.05 Hz frequency set the *Amplitude: Yaw (deg)* block to 50.0 degrees and the *Frequency* input box in the *Signal Generator: Yaw* block to 0.05 Hz.
14. Run the simulation and the responses shown in figures 28, 29, and 30 should be obtained. The yaw voltage saturates the back amplifier at -15 V

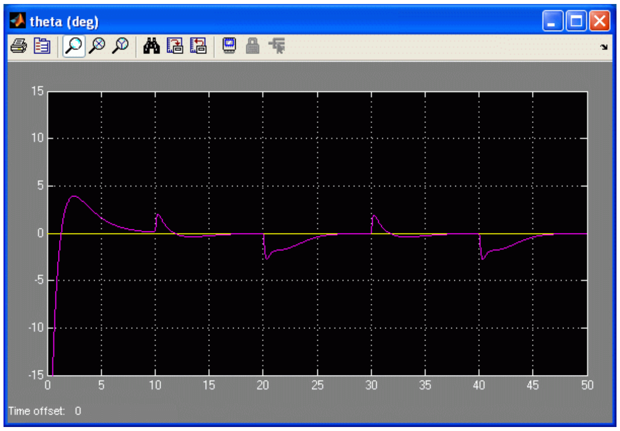


Figure 28: Simulated pitch response under a desired yaw reference step using LQR+I.

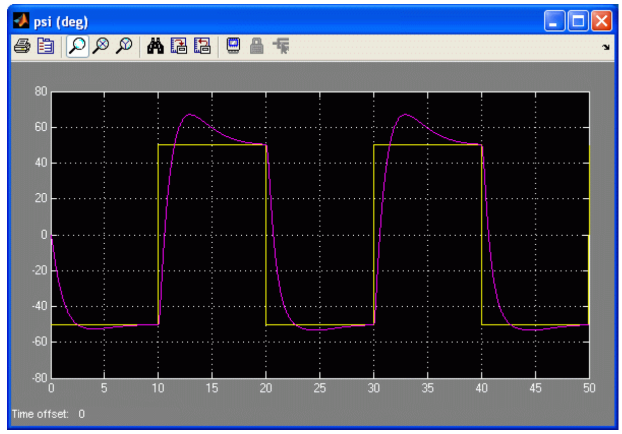


Figure 29: Simulated yaw response under a desired yaw step using LQR+I.

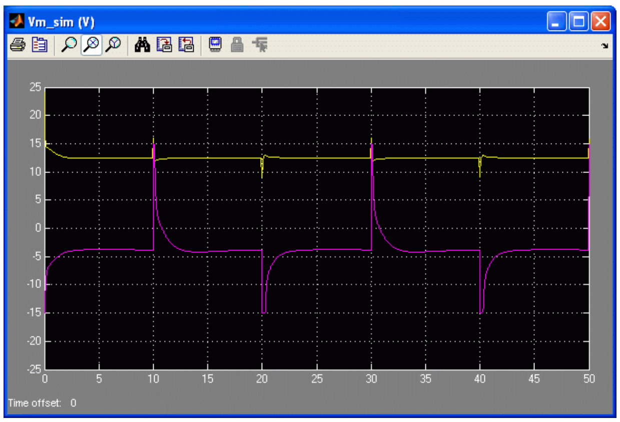


Figure 30: Simulated front and back motor voltage under yaw reference step using LQR+I.

15. Try changing the desired elevation and travel angles to familiarize yourself with the controller. Observe that rate limiters are placed in the desired position signals to eliminate any high-frequency changes. This makes the control signal smoother which places less strain on the actuator.

7.3. Open-loop Implementation

7.3.1. Objectives

The objectives of running the 2-DOF Helicopter in open-loop are to:

- Gain an intuition on the dynamics of the system, in particular the coupling effect that exists between the pitch and yaw actuators.
- Obtain an idea on how difficult it is to control the apparatus in order to compare human operator performance with computer control.

7.3.2. Procedure

1. Load Matlab.
2. Open Simulink model *q_heli_2d_open_loop.mdl* shown in Figure 31. The model runs your actual 2-DOF Helicopter plant by directly interfacing with your hardware through the QuaRC blocks, described in Reference [2].

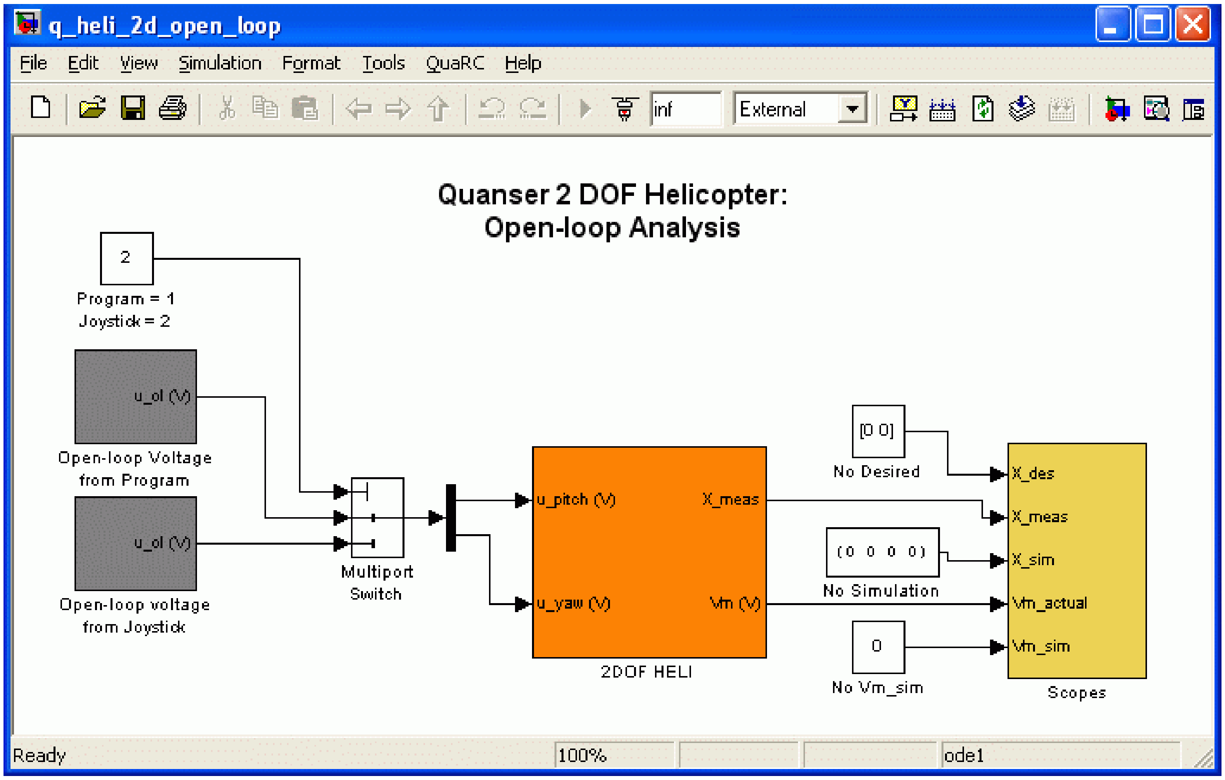


Figure 31: Simulink diagram to run 2-DOF Helicopter in open-loop with QuaRC.

3. **Configure setup script**: Open the design file *setup_lab_heli_2d.m* and ensure everything is configured properly. The K_JOYSTICK_V_X and K_JOYSTICK_V_Y parameters control the rate that a voltage command is generated. The JOYSTICK_X_DZ and JOYSTICK_Y_DZ specify the deadzone of the joystick. The deadzone is used to remove negligible joystick outputs due to noise or from small motions in the joystick handle.
4. Execute the *setup_lab_heli_2d.m* Matlab script to setup the workspace before compiling the diagram and running it in real-time with QuaRC.
5. Open the 2*DOF HELI* subsystem. Its contents are shown in Figure 32. This subsystem contains the QuaRC blocks that interface with the hardware of the actual plant. The Analog Output block sends the voltage computed by the controller to the DACB which drives the actuators and the Encoder Input block reads the encoder measurements.
6. **Configure DAQ**: Double-click on the HIL Initialize block in the Simulink diagram and ensure it is configured for the DAQ device that is installed in your system. By default the block shown in Figure 32 is setup for the Quanser Q8 hardware-in-the-loop board.

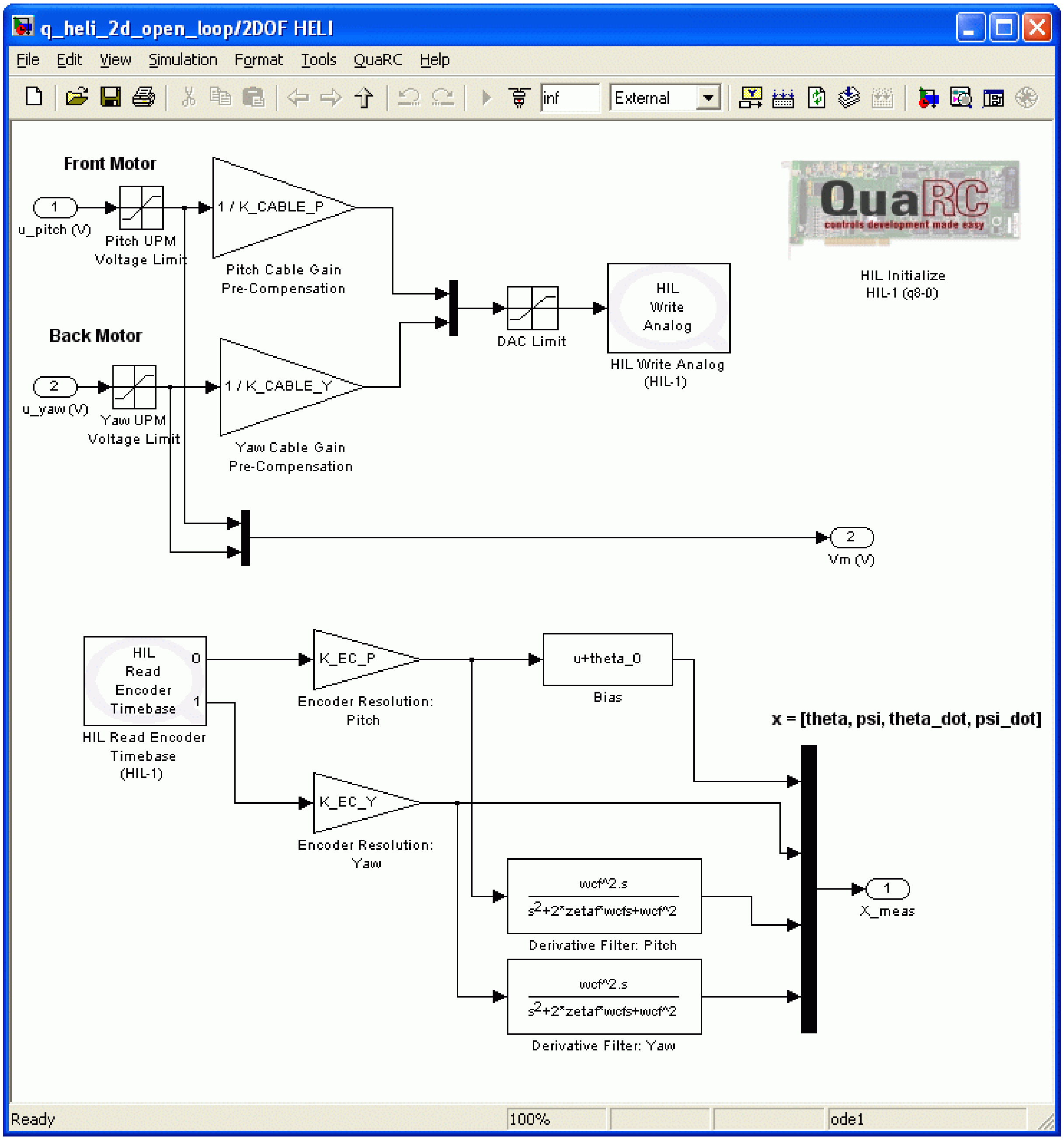


Figure 32: 2DOF HELI subsystem contains QuaRC blocks that interface with hardware.

7. The voltage sent to the Analog Output block is amplified by the UPMs and applied to the power amplifier's attached motor. Note that the control input is divided by the amplifier gain, K_CABLE, before being sent to the DACB. This way, the amplifier gain does not have to be included in the mathematical model as the voltage output from the controller is the voltage being applied to the motor. The UPM and DACB saturation blocks limit the amount of voltage that can be fed to the motor. In this case, since K_CABLE_P = 5 and K_CABLE_Y = 3, the voltage is only saturated by the UPM and not by the DACB.
8. Open the *theta (deg)*, *psi (deg)*, and *Vm (V)* scopes in the *Scopes* folder. These scopes display

both the desired and measured angles of the Helicopter as well as the voltages being applied to the front and back motors.

9. Ensure the helicopter has been setup and all the connections have been made as instructed in Section 5.
10. Turn the power of the two Universal Power Amplifiers on. The red LED on the upper-left corner of the each UPM should be lit.
11. In the *q_heli_2d_open_loop* Simulink diagram, make sure the *Program/Joystick* block shown in Figure 31 is set to 2 in order to generate the desired voltage from the joystick.
12. Click on Quarc | Build to compile the code from the Simulink diagram.
13. Select Quarc | Start to begin running the controller.

NOTE: Click on the STOP button on the Simulink tool bar at any time to stop running the controller.

14. Try to bring the helicopter body to a horizontal by pulling the joystick handle toward you. This supplies a positive voltage to the pitch motor and causes the pitch angle to increase
15. As depicted in Figure 33, you will notice that the yaw angle moves clockwise in the positive direction as the voltage in the pitch motor increases. Compensate for this coupling effect by moving the joystick arm to the left and apply a negative voltage to the yaw motor. As illustrated, the pitch is eventually stabilized at about 0 degrees when the pitch motor voltage is approximately 12.5 V and the yaw begins to stabilize when feeding a voltage of about -6.0 V to the yaw motor. The voltage does not surpass the limit of the power amplifiers: ±24 V for UPM-2405 and ±15 V for UPM-1503.

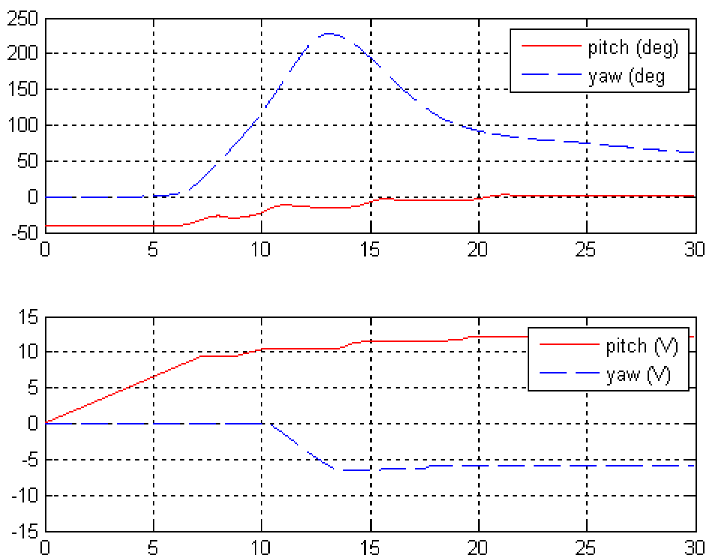


Figure 33: Effect of pitch on yaw.

16. From this point, now try decreasing the yaw voltage and observe its effect on the pitch angle.
17. As depicted in Figure 34, applying a negative voltage to the yaw motor causes the pitch angle to decrease, i.e. the helicopter nose goes down. Similarly to the effect the pitch motor has on the yaw motion, the voltage applied to the yaw motor generates a torque on the pitch axis

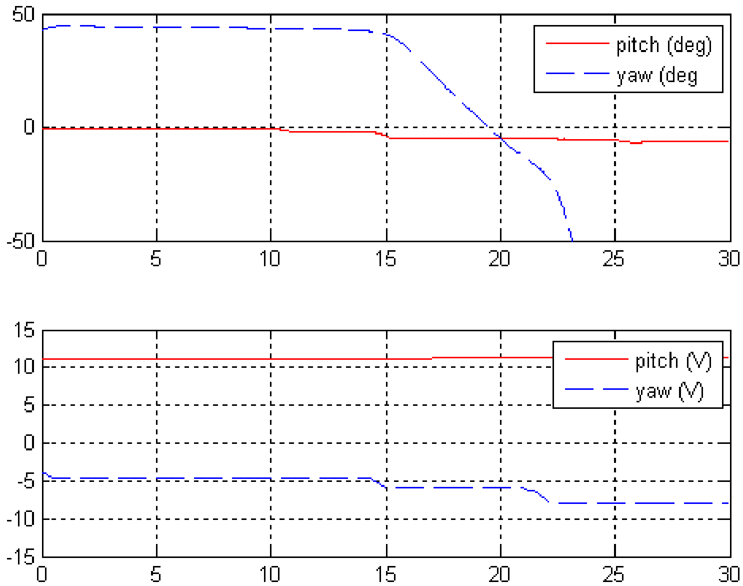


Figure 34: Effect of yaw on pitch.

18. Gradually bring the helicopter back to starting position.
19. Click on the *Stop* button on the Simulink diagram tool bar (or select Quarc | Stop from the menu) to stop running the controller.
20. If the laboratory session is complete, power off both UPM units.

7.4. Closed-loop Position Control Implementation

7.4.1. Objectives

The objectives of running the 2 DOF Helicopter in closed-loop are to:

- Investigate the closed-loop performance between the FF+LQR and the FF+LQR+I controllers running on the actual 2 DOF Helicopter plant.
- Compare the measured closed-loop response with the simulated response

7.4.2. Procedure: 2-DOF Helicopter

Follow this procedure to run the FF+LQR and the FF+LQR+I controllers on the actual helicopter plant:

1. Load Matlab.
2. Open Simulink model *q_heli_2d_ff_lqr_i.mdl* shown in Figure 35 that implements the closed-loop LQR and LQR+I position controllers.

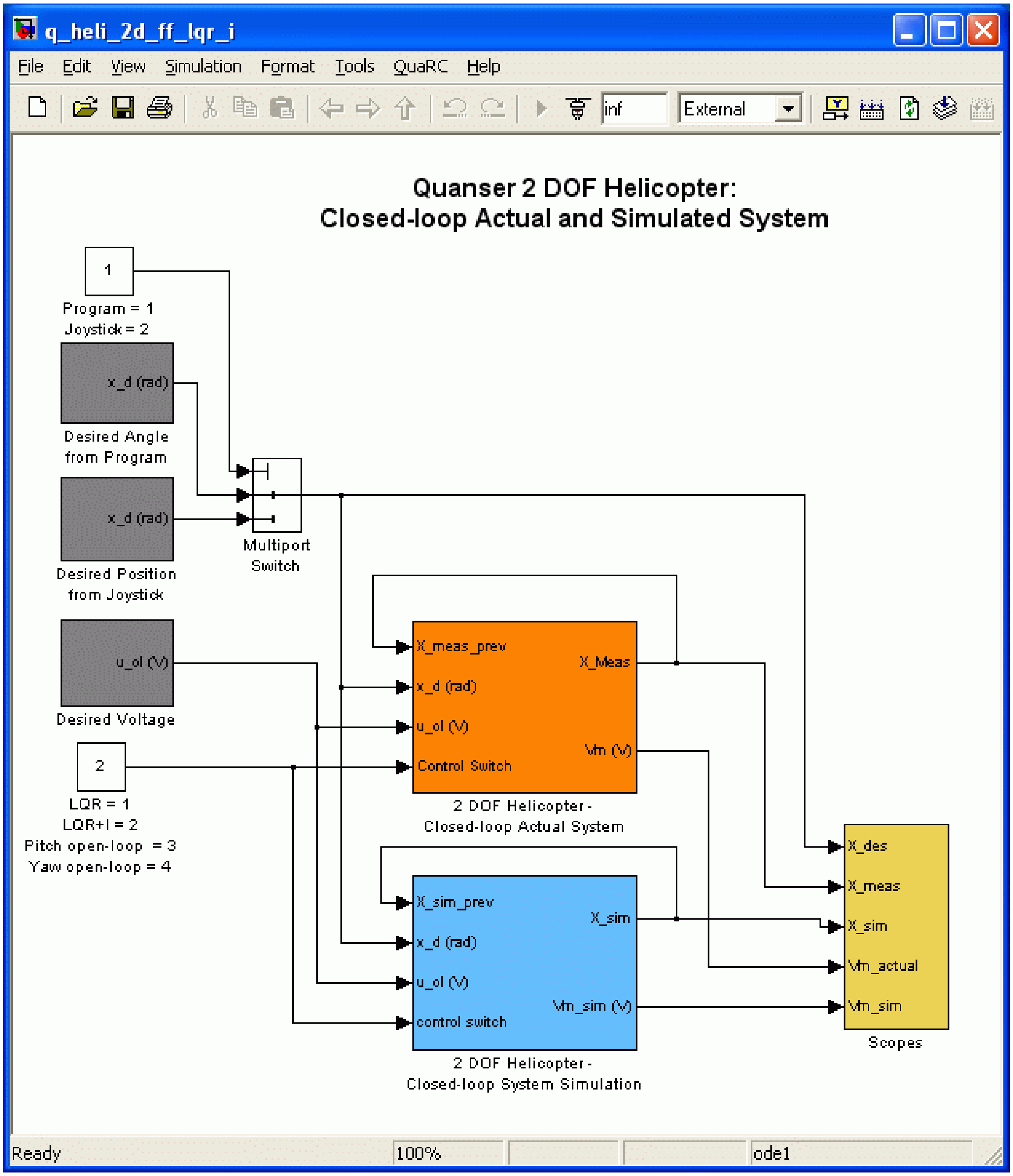


Figure 35: Simulink model used with QuaRC to run the closed-loop controller on the 2-DOF Helicopter.

3. **Configure setup script**: Open the design file *setup_lab_heli_2d.m* and ensure everything is configured properly.
4. Execute the *setup_lab_heli_2d.m* Matlab script to setup the workspace before compiling the diagram and running it in real-time with QuaRC. This file loads the model parameters of the 2-DOF Helicopter system, calculates the LQR and LQR+I feedback gains, and sets various other parameters that are used such as the filter cutoff frequencies, amplifier gain, encoder sensitivities, and the UPM and DACB limits.

5. Open the 2-*DOF HELI* subsystem shown in Figure 36. It contains the QuaRC blocks that interface with the hardware of the actual plant. The Analog Output block outputs the voltage computed by the controller to the DACB and the Encoder Input block reads the encoder measurements.
6. **Configure DAQ**: Double-click on the HIL Initialize block located in the *2 DOF Helicopter – Closed-loop Actual System \ 2 DOF HELI* subsystem, shown in Figure 32, above, and ensure it is configured for the DAQ device that is installed in your system. By default the block is setup for the Quanser Q8 hardware-in-the-loop board.
7. Ensure the helicopter has been setup and all the connections have been made as instructed in Section 5.
8. Turn the power of the two Universal Power Amplifiers on. The red LED on the upper-left corner of the each UPM should be lit.
9. In the *q_heli_2d_ff_lqr_i* Simulink diagram, make sure the *Program/Joystick* block shown in Figure 35 is set to 1 in order to generate the desired angle from Simulink.
10. Click on Quarc | Build to compile the code from the Simulink diagram.
11. Select Quarc | Start to begin running the controller. The pitch propeller should start turning lightly.

NOTE: Click on the STOP button on the Simulink tool bar at any time to stop running the controller.

12. Initially the helicopter pitch angle is -30.0 degrees. Inside the *Desired Angle from Program* subsystem, set the *Amplitude: Pitch (deg)* to 10 and the *Pitch: Constant (deg)* to 0. Set the pitch constant gradually, i.e. increment it by steps of 10 degrees. The helicopter should be tracking this reference position about its horizontal.
13. Open the *theta (deg)*, *psi (deg)*, and *Vm_actual (V)* scopes in the *Scopes* folder. In the *theta (deg)* scope, the desired pitch position is the yellow line, the measured pitch position is the purple plot, and the simulated pitch position is light blue trace. Similarly in the yaw scope, *psi (deg)*, yellow is the desired yaw, purple is the measured yaw, and light blue is the simulated yaw. The *Vm_actual (V)* scope plots the voltage being applied to the pitch motor in yellow and yaw motor in purple.
14. Figure 36 depicts the typical measured and simulated pitch and yaw response under a desired step pitch angle. The measured response is the solid red line (-), the simulation is the dashed blue line (--), and the reference is the solid green line (-). Notice in both the measured and simulated responses the pitch angle has a steady-state error.

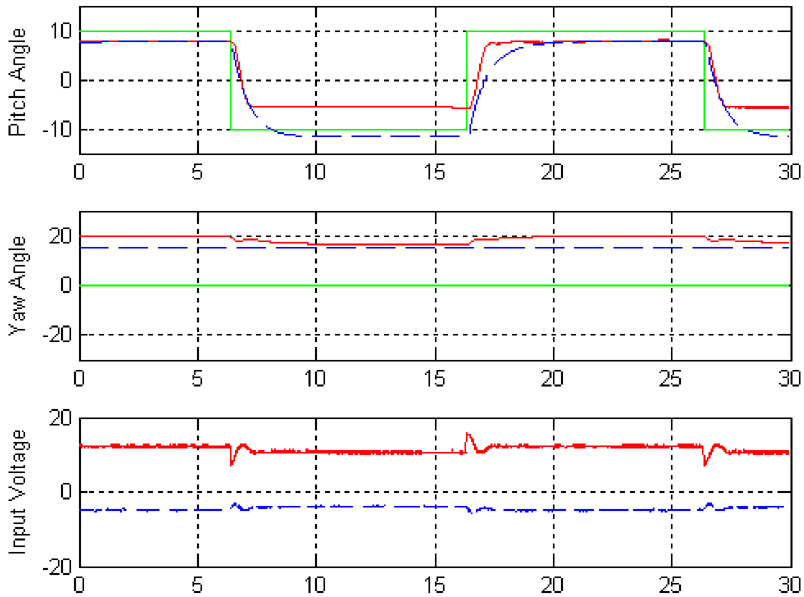


Figure 36: Closed-loop LQR response under pitch reference step.

15. Inside the *Desired Angle from Program* set the *Amplitude: Pitch (deg)* to 0 and the *Amplitude: Yaw (deg)* to 50. The helicopter should tracking the commanded yaw angle.
16. Figure 37 depicts the typical measured and simulated pitch and yaw response given a desired step yaw angle. There is a steady-state error in the yaw angle.

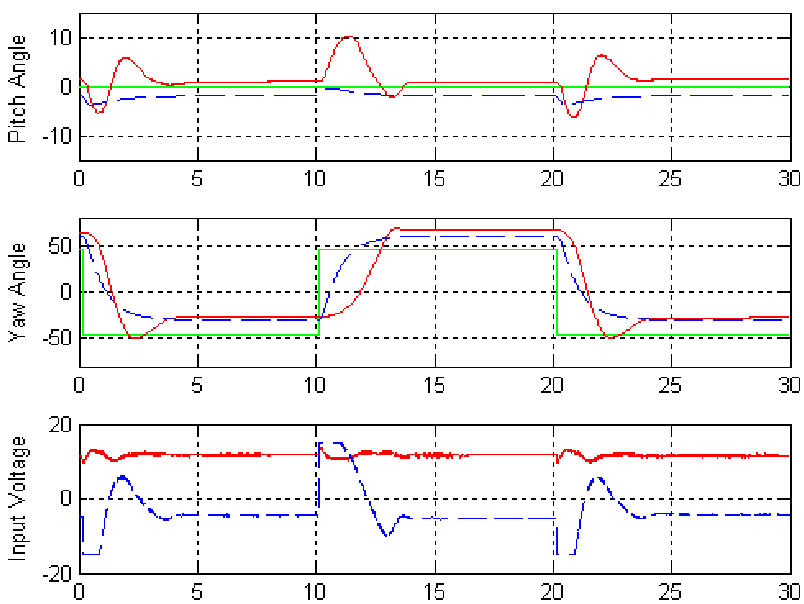


Figure 37: Closed-loop LQR response under yaw reference step.

17. Set *Amplitude: Yaw (deg)* to 0. Both the pitch and yaw commands should now be zero.
18. The steady-state error can be removed using integral action. Switch to the FF+LQR+I control by setting the control switch source block to 2. The pitch and yaw angle should both, eventually, converge to 0 degree.
19. Set the *Amplitude: Pitch (deg) to 10* and the *Pitch: Constant (deg) to 0* to observe the LQR+I response under a step pitch reference. The result should be similar to the response shown in Figure 38. Note that the steady-state error is removed.

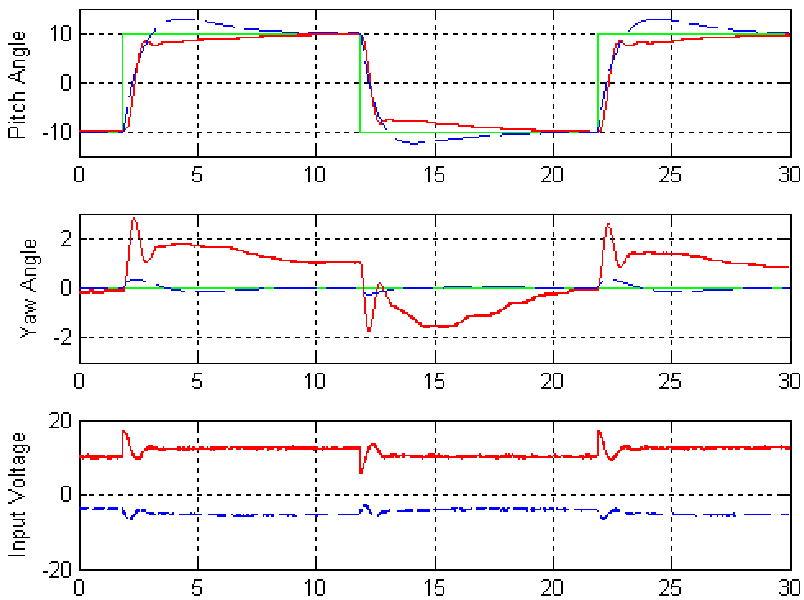


Figure 38: Closed-loop LQR+I response under pitch reference step.

20. Set the *Amplitude: Pitch (deg)* to 0 and *Amplitude: Yaw (deg)* to 50 observe the LQR+I response under a desired yaw step. The obtained response should be similar to Figure 39. There is no steady-state error but the controller saturates the yaw amplifier due to the large step reference.

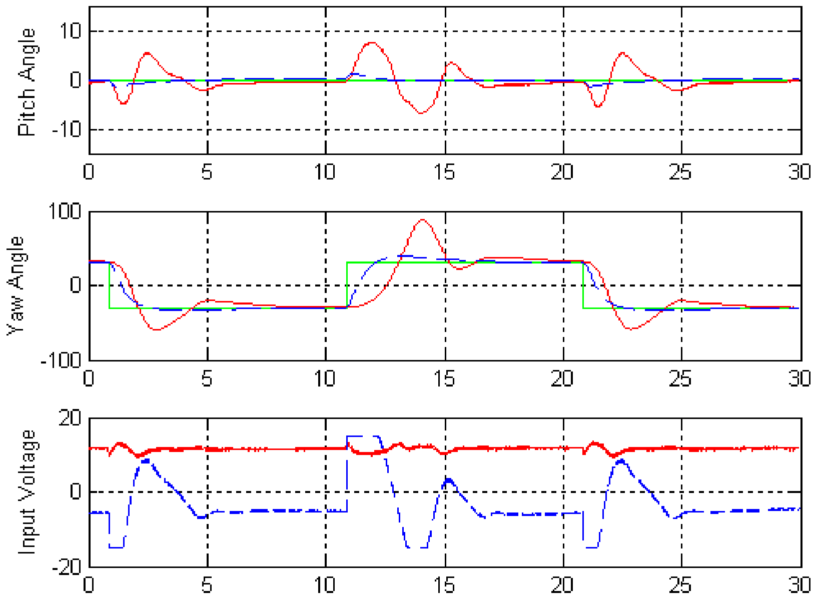


Figure 39: Closed-loop LQR+I response under yaw reference step.

21. Alternatively, the desired angle can be generated using the joystick, as described in Section 5.4. To use the joystick, set the *Program/Joystick* switch shown in Figure 35 to 2. The rate at which the desired angle increases or decreases given a joystick position can be changed using the K_JOYSTICK_X and K_JOYSTICK_Y variables that are set in the *setup_lab_heli_2d.m* script file and the using the *Rate Command* knob. When starting, set the *Rate Command* knob on the joystick to the midpoint position.

CAUTION: Do not switch from the Program to the Joystick (from 1 to 2) when the controller is running. Set the program/joystick switch to 2 before starting QuaRC if the joystick is to be used.

22. Gradually bring the helicopter back to starting position.
23. Click on the *Stop* button on the Simulink diagram tool bar (or select Quarc | Stop from the menu) to stop running the code.
24. Power off the two UPMs.

7.5. Model Validation Implementation

7.5.1. Objectives

The objectives of running the model validation controller on the 2 DOF Helicopter are to:

- Verify that the nonlinear model, which is summarized in Section 6.1, represents the actual device with reasonable accuracy.
- Roughly identify the rotary viscous friction parameter about pitch and yaw axis.

7.5.2. Procedure

Follow this procedure to identify the viscous rotary friction on the yaw axis and do model validation of the pitch:

1. Follow the steps 1 to 11 given in Section 7.4 to run the *q_heli_2d_ff_lqr_i* QuaRC controller. The helicopter should be at the starting point, i.e. pitch of -30 degrees and yaw of 0 degrees.
2. Run the LQR+I controller by setting the control switch source block to 2.
3. Inside the *Desired Angle from Program* set the *Amplitude: Pitch (deg)* to 0 and the *Pitch: Constant (deg)* to 0. The helicopter should be stabilized about a pitch and yaw angle of zero.
4. In the *Scopes* subsystem, double-click on the *theta (deg)*, *psi (deg)*, and *Vm Actual (V)* sinks. In the *theta (deg)* scope, the desired pitch position is the yellow line, the measured pitch position is the purple plot, and the simulated pitch position is light blue trace. Similarly for the yaw *psi (deg)* scope. The *Vm_actual (V)* scope plots the voltage being applied to the pitch motor in yellow and yaw motor in purple.
5. To identify the viscous rotary friction parameter in the yaw axis a voltage step command must be applied to the yaw motor. In the *Desired Voltage* subsystem, set the *Signal Generator: Yaw (V)* block to 2.5.
6. Change the control switch to 4. The yaw input motor voltage is now the control voltage used to stabilize the yaw angle, $u^*_{lqr,y}$, added to the commanded open-loop voltage, i.e. $V_{m,p} = u^*_{lqr,y} + 2.5$. The plot should resemble Figure 40.

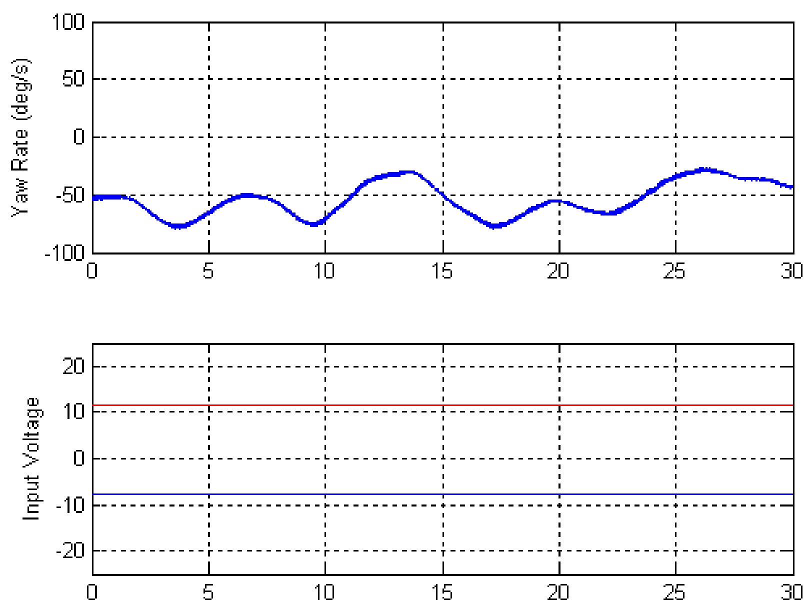


Figure 40: Identifying the viscous damping about the yaw axis.

7. For calculating the viscous damping parameter B_y, consider the linear equation describing the yaw motion in the 2-DOF Helicopter Maple worksheet (or its HTML equivalent). Given that the helicopter is rotating more or less at a constant speed, it can be assumed that the acceleration is zero, thus

$$\frac{d^2}{dt^2}\psi(t) = 0 \quad [22]$$

Setting the acceleration to zero as well as the pitch angle to zero and then solving for the viscous damping term gives the expression

$$B_y = \frac{K_{yy} V_{m,y} + K_{yp} V_{m,p}}{\left(\frac{d}{dt}\psi(t)\right)_{avg}} \quad [23]$$

where the bottom term is the average velocity of the yaw angle. From Figure 40, the voltage and average velocity found are:

$$V_{m,y} = -7.65 \, [V], \quad [24]$$

$$V_{m,p} = 11.36 \, [V], \quad [25]$$

$$\left(\frac{d}{dt}\psi(t)\right)_{avg} = -0.9481 \left[\frac{rad}{s}\right], \quad [26]$$

and the parameter K_{yy} and K_{yp} are defined in Table 3. Substituting these values into Equation [23] gives the equivalent viscous damping acting about the yaw axis:

$$B_y = 0.319 \left[\frac{N\,m\,s}{rad}\right].$$

8. Bring the helicopter back to ($\theta = 0$, $\psi = 0$) by setting the control switch block to 2 to run the LQR+I controller.
9. To apply a voltage directly to the pitch motor set the control switch source block to 3.
10. In the *Desired Voltage* subsystem, set the frequency of the *Signal Generator: Pitch (V)* block to 0.4 Hertz, the *Amplitude: Pitch (V)* gain block to 0.2, and the *Constant Pitch (V)* to 0.
11. As shown in Figure 41, the measured and simulated pitch angles are quite close. The pitch viscous damping term B_p was estimated by tuning its value online as the controller is running. To do this enter a value for B_p in the Matlab command window, for example $B_p = 0.8$. Because the parameter change is made in Matlab (and not directly in the Simulink model), the controller that is running must be updated for the changes of this parameter to take effect. The change can be applied by clicking on the *Edit* menu in the Simulink model and selecting the *Update Diagram* item. Alternatively, controller parameters can be updated by use the keystroke CTRL-D whenever the Simulink model is active.

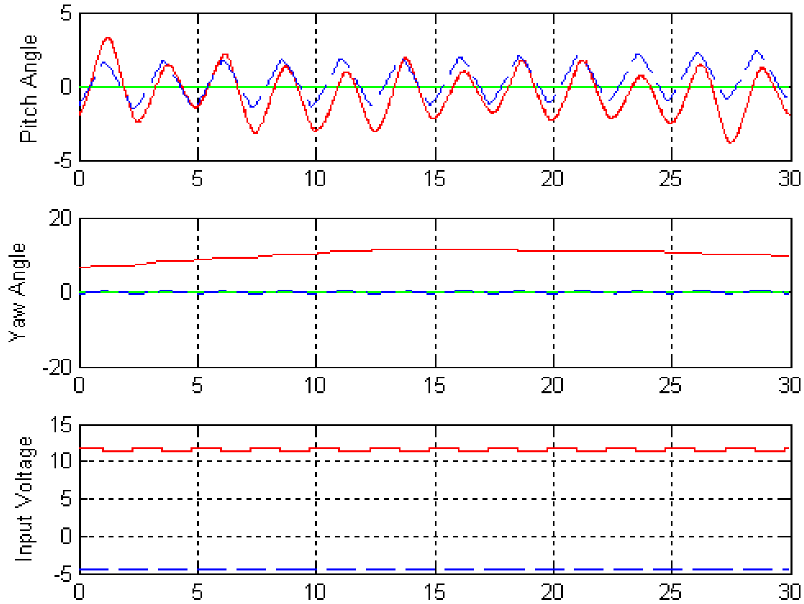


Figure 41: Open-loop pitch step.

12. Gradually bring the helicopter back to starting position.
13. Click on the *Stop* button on the Simulink diagram tool bar (or select Quarc | Stop from the menu) to stop running the code.
14. Make sure both the UPM-1503 and UPM-2405 are powered off if the session is complete.

8. References

[1] Quanser. *Q4/Q8 User Manual*.
[2] Quanser. *QuaRC User Manual* (type `doc quarc` in Matlab to access).
[3] Quanser. *QuaRC Installation Manual*.
[4] Quanser. *UPM User Manual*.
[5] Pittman. *Pittman LO COG DC Servo Motor Series 8000, 9000, and 14000*.
[6] Faulhaber. *Faulhaber Series 2842 Model 006C motor*.