Correction to Flexible Joint Model

Lab 4: Linear Quadratic Regulator (LQR) and Observer Design for Flexible Joint

ECE 758: *Control System Implementation Laboratory*

In the Laboratory Design Challenges document, the state-space model of the flexible joint has a *typo,* and so the state-space model will not produce the MATLAB results shown below it. **To remedy this problem, replace every R with R_m.** The effect of R is already included in the K_{stiff} parameter, and the typo omits any influence of the R_m parameter.

The correct model has

$$x = \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix}, \qquad u = v_{\text{in}}, \qquad \dot{x} = Ax + Bu, \qquad \text{and} \qquad y = Cx \tag{1}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{K_{\text{stiff}}}{J_{\text{hub}}} & \dfrac{-K_m^2 K_g^2}{R_m J_{\text{hub}}} & 0 \\ 0 & \dfrac{-K_{\text{stiff}}\left(J_{\text{load}} + J_{\text{hub}}\right)}{J_{\text{hub}} J_{\text{load}}} & \dfrac{K_m^2 K_g^2}{R_m J_{\text{hub}}} & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ \dfrac{K_m K_g}{R_m J_{\text{hub}}} \\ \dfrac{-K_m K_g}{R_m J_{\text{hub}}} \end{bmatrix}, \qquad \text{and} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}. \tag{2}$$

These corrections have already been applied to the following MATLAB code.

```
A = [0, 0, 1, 0;
     0, 0, 0, 1;
     0, Kstiff/J_hub, -mpower(Km,2)*mpower(Kg,2)/(Rm*J_hub), 0;
     0, -Kstiff*(J_load+J_hub)/(J_hub*J_load), mpower(Km,2)*mpower(Kg,2)/(Rm*J_hub) 0];

B = [0; 0; Km*Kg/(Rm*J_hub); -Km*Kg/(Rm*J_hub)];

C = [1 0 0 0; 0 1 0 0; 1 1 0 0];
```

This code uses `mpower(Km,2)` in place of `Km^2` so that you can copy and paste from this PDF into MATLAB. Using this code, your results should match the ones from the laboratory design challenge. That is,

```
format short e; A, B
```

should give you

```
A =

         0         0    1.0000e+00         0
         0         0         0    1.0000e+00
         0    7.6705e+02   -5.2795e+01         0
         0   -1.0401e+03    5.2795e+01         0


B =

         0
         0
    9.8333e+01
   -9.8333e+01
```