Multi-loop nonlinear control of the single-link manipulator with flexible joint

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1 Introduction

In the first part of the lab, you will implement a setpoint control system for the single-link manipulator with flexible joint using a multiloop controller based on feedback linearization (inner loop) and linear state-feedback control (outer loop). Then, you will modify the multiloop controller to solve a trajectory tracking problem.

2 Laboratory procedures

The procedures are basically the same as those in Lab. #4. Remember to put the link in a vertical position.

2.1 Necessary Equipment

Same as Lab. #4.

2.2 Connections

Same as Lab. #4.

3 Multiloop setpoint controller

1. From the previous lab, you should have a block whose outputs are the position θ and the velocity $\dot{\theta}$ of the motor, and a block that acquires α and produces $\dot{\alpha}$ via high-pass filtering.

2. Remember that since you are using the tachometer and the encoders, you have to use the appropriate blocks to acquire these values.

3. Use the "Safety Stop Angle" block set at $\gamma = 50°$.

4. The regulated output of your system is $y = \theta + \alpha$, and all the state variables $x = (\theta, \alpha, \dot{\theta}, \dot{\alpha})$ are available for feedback. First, you need to implement the inner loop (feedback linearization). Build a block that produces the change of coordinates $z = \Phi(x)$, and a block the implements the inner loop controller $u = \alpha(x) + \beta(x)v$. Then, implement the outer loop control $v = Kz$, where K has been obtained using your favorite linear design method.

5. To regulate $y(t)$ to y_d, you need to send to zero the error $e_1 = z_1 - y_d$ (remember that $y = z_1$). The best way to do this, in preparation for the tracking problem, is to change coordinates once again as $e = z - z_d$, where z_d is the set point for the state, and close the outer loop controller

as $v = Ke$. Since $z_2 = \dot{y}$, $z_3 = \ddot{y}$, and $z_4 = y^{(3)}$, the setpoint for the other variables z_2, z_3, z_4 is zero.

6. As a setpoint y_d, use the same square wave with an amplitude of 30° and a frequency of 0.05 Hz used in the previous experiment.

7. Fix the sampling time as 1 ms, and compile your experiment.

8. Create a new experiment in dSPACE. This experiment will have the file that you already compiled, and a layout with a plotter, four displays that show the gains, and one display that will show the actual position in degrees. You can put more elements in your experiment if you want.

9. Run the experiment the first time. Is the value of the gain vector K that you computed in the pre-lab a good one? If not, try to tune until you will have a better response.

10. Once you find the correct values for these gains, save the closed-loop response data as a Matlab file.

4 Multiloop tracking controller

In the second part of the experiment, the outer loop controller will be modified to let the output $y(t)$ of the system track a time-varying reference trajectory $y_d(t)$. In order to have $z_(t)$ track $y_d(t)$, we need to drive the state $z(t)$ to an appropriate time-varying equilibrium (a reference state trajectory), function of $y_d(t)$ and its derivatives. Assume that $y_d(t)$ and its derivatives upto order 4 are known. Remember that, after feedback linearization, the system is given by

$$\begin{aligned} \dot{z} &= Az + Bv \\ y &= Cz \end{aligned}$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}.$$

Define

$$z_d = \begin{pmatrix} y_d \\ \dot{y}_d \\ \ddot{y}_d \\ y_d^{(3)} \end{pmatrix}$$

and note that z_d satisfies

$$\begin{aligned} \dot{z}_d &= Az_d + By_d^{(4)} \\ y_d &= Cz_d. \end{aligned}$$

Then, defining the error $e = z - z_d$, we obtain the *error dynamics*

$$\dot{e} = Ae + B[v - y_d^{(4)}].$$

In order to regulate $e(t)$ to zero, we need to choose an outer-loop control input v that performs two distinct actions:

1. Rendering the point $e = 0$ an equilibrium for the error dynamics

2. Stabilizing the newly created equilibrium at the origin.

The first action is performed by the feedforward control $v_{\text{ff}} = y_d^{(4)}$, while the second is performed by the usual state-feedback $v_{\text{fb}} = Ke$. Letting $v = v_{\text{ff}} + v_{\text{fb}}$, we obtain the closed loop system

$$\dot{e} = (A + BK)e$$

which has an asymptotically stable equilibrium at the origin, as required. Note that the stabilizing controller is exactly the same as before, only the way the error is defined changes, and a feedforward control action is needed.

The last thing to do is to generate y_d and its derivatives. The best way to do so, is to implement a *reference model*, that is, to filter a reference signal through a stable linear systems. Let y_d be given in the Laplace domain as the output of the transfer function

$$y_d(s) = W_r(s)y_r(s)$$

where $y_r(s)$ is an external reference signal, and $W_r(s)$ is the reference model. Assume that

$$W_r(s) = \frac{a_0}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

where the roots of $s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$ are all in $\text{Re}[s] < 0$. The state space realization in controller form of $W_r(s)$ is

$$\begin{aligned} \dot{z}_d &= A_r z_d + B_r y_r \\ y_d &= C_r z_d\,, \end{aligned} \tag{1}$$

where

$$A_r = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 \end{pmatrix}, \quad B_r = \begin{pmatrix} 0 \\ 0 \\ 0 \\ a_0 \end{pmatrix}, \quad C_r = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}.$$

The state-space realization (1) yields directly the required reference y_d and its derivatives, as

$$z_d = \begin{pmatrix} y_d \\ \dot{y}_d \\ \ddot{y}_d \\ y_d^{(3)} \end{pmatrix}, \qquad y_d^{(4)} = \begin{pmatrix} -a_0 & -a_1 & -a_2 & -a_3 \end{pmatrix} z_d$$

and, as before,

$$\begin{aligned} \dot{z}_d &= A z_d + B y_d^{(4)} \\ y_d &= C z_d. \end{aligned}$$

4.1 Tracking controller experiment

As a reference model, consider the transfer function

$$W_r(s) = \frac{a^4}{(s+a)^4}, \qquad a = 4$$

and as external reference input

$$y_r(t) = A \sin(2\pi f\, t)$$

with $A = 30°$, and $f = 0.2$ Hz.

IMPORTANT: make sure you implement the given sinusoidal signal correctly: a high value for the angular velocity may damage the motor!

1. Create a block that implements the reference model. Make all the state z_d and $y_d^{(4)}$ available to the outer loop controller.

2. Modify the outer loop control to generate the correct error variables , and perform the feed-forward and the stabilizing controls.

3. Make sure the "Safety Stop Angle" block is set at $\gamma = 50°$.

4. Fix the sampling time as 1 ms, and compile your experiment.

5. Create a new experiment in dSPACE. This experiment will have the file that you already compiled, and a layout with a plotter, four displays that show the gains, and one display that will show the actual position in degrees. You can put more elements in your experiment if you want.

6. Run the experiment the first time. Is the value of the gain vector K that you computed in the pre-lab a good one? If not, try to tune until you will have a better response.

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